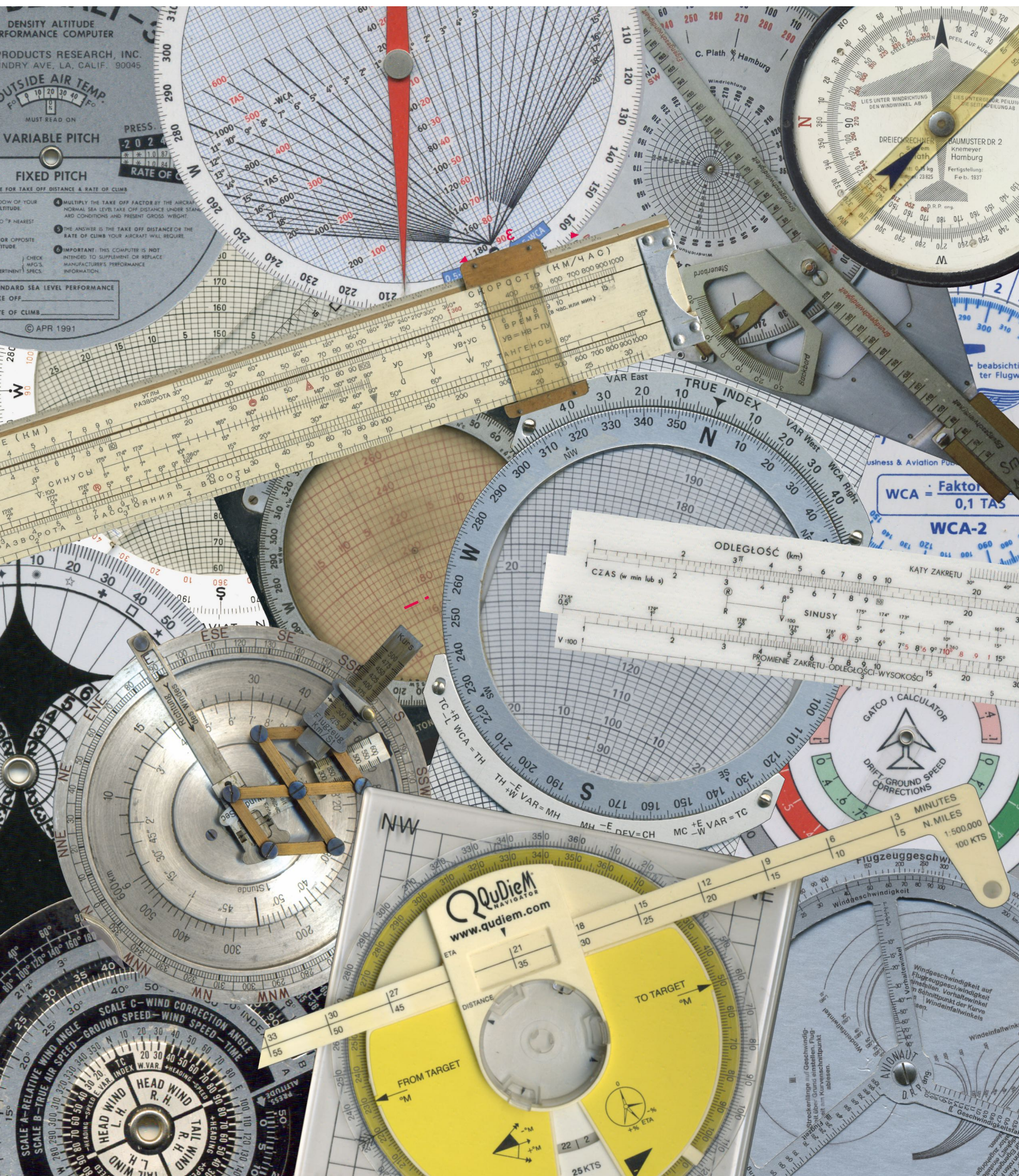


# A CLOSER LOOK AT MECHANICAL AIR NAVIGATION COMPUTERS

Klaus Petzold





## Preface

The invention of the electronic pocket calculator brought the slide rule, once an indispensable tool in many professions, to a very quick end. Only in one niche have mechanical computing devices been able to assert themselves to this day, namely in aviation, although this is an area that embodies technical progress like hardly any other. In the cockpit of a modern commercial aircraft, however, the old dead reckoning computers no longer have any reason to exist. Even in General Aviation, the mechanical computer has a shadowy existence, which is related to the triumph of satellite navigation. However, these old parts are still indispensable for the navigation training of pilots and will therefore continue to be produced. This is certainly also due to the fact that the mechanical solutions for the spatial problems occurring here are particularly demonstrative. This is why the electronic navigation computers, which came onto the market very quickly, were not really able to establish themselves.

That the extraordinary variety of the mechanical navigation computers invented in the course of the relatively short history of aviation might not be too well known, led me to the idea of putting these pages on the net. Perhaps in this way I can save some of the knowledge about historical aviation computing technology from being forgotten. Above all, I wanted to pay tribute to the human ingenuity that is particularly evident here.

This is not a reading material in the conventional sense, but rather a reference work. It is also not a textbook, but is aimed at readers who already have basic knowledge of flight navigation. In many thousands of flying hours I have gained practical experience, so that I can judge such products regarding their suitability in practice. Regardless of my personal evaluation I wanted to set a monument to the numerous inventors. Unfortunately, I do not know all their names. I am always grateful for hints at necessary additions, errors or mistakes (eMail: [info@flight-computer.de](mailto:info@flight-computer.de)).

It is due to the nature of the task that the calculation methods are sometimes the same or at least similar, which does not necessarily mean that the inventors have been sloppy with intellectual property. With obvious solutions it is not unlikely that they have been found several times and that the creators have worked independently of each other.

To make the descriptions as simple and intelligible as possible, I almost always work with examples. These are chosen arbitrarily, which is not explicitly mentioned in the text. In order to be able to estimate the accuracy of the calculation method, the analytically found exact results are also given. These are indicated by decimal places.

The methods are primarily described without comment and the judgement about their utility value, originality and suitability in practice is left to the reader. Rare exceptions to this rule have been necessary for scientific reasons. The order of the individual chapters is purely arbitrary and in no way an expression of my appreciation.

I am fully aware that many readers who are interested in such a special topic may want to know more about the inventors, how the various inventions are linked to each other and what the influence of technical progress was on the design of these computing devices. Unfortunately, I have to disappoint these readers. Since I am no science historian, it was neither my intention nor am I able to write about it. Therefore, no historical details about individual products and their manufacturers can be found here, nor information about distribution and production figures or biographical information about their creators. For readers who want to learn more about the very interesting history of navigation computers, I recommend a forthcoming book by Prof. Alexander Piel, whom I would like to thank for his encouragements and valuable hints for this work.

Capt. (ret.) Klaus Petzold

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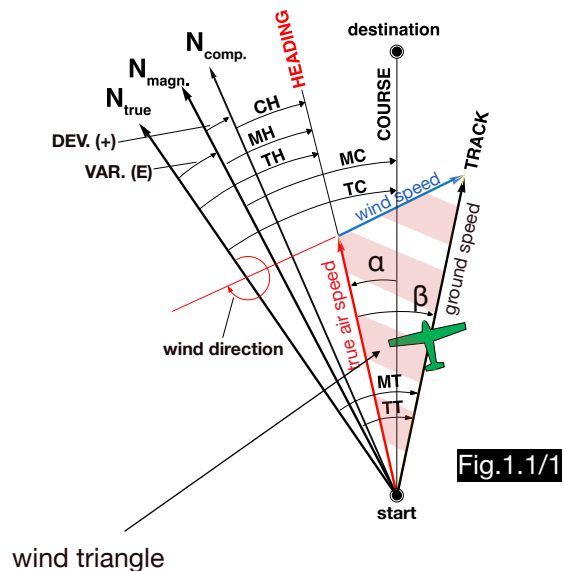
# 1. Calculating of the elements of the wind triangle

In the wind triangle, the laws of vector addition apply, since three of the variables occurring (airspeed, wind speed and ground speed) are vectors. The figures shown under 1.1. are intended to help one to understand the calculation methods described in chapter 1.

## 1.1. Definitions

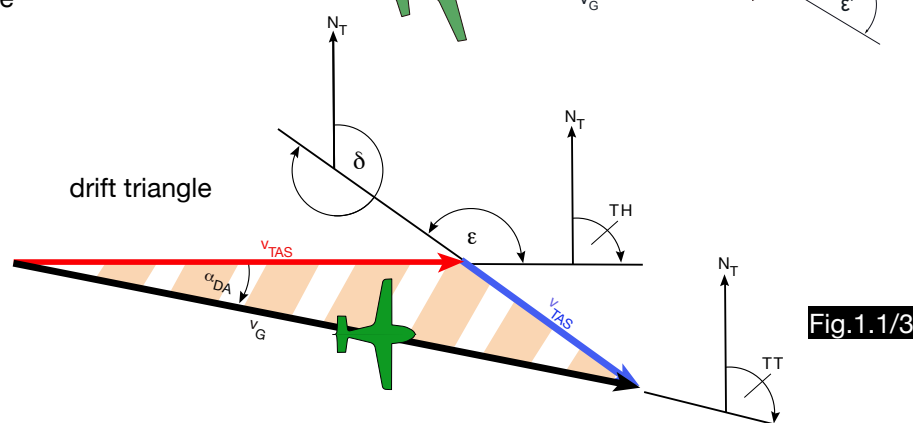
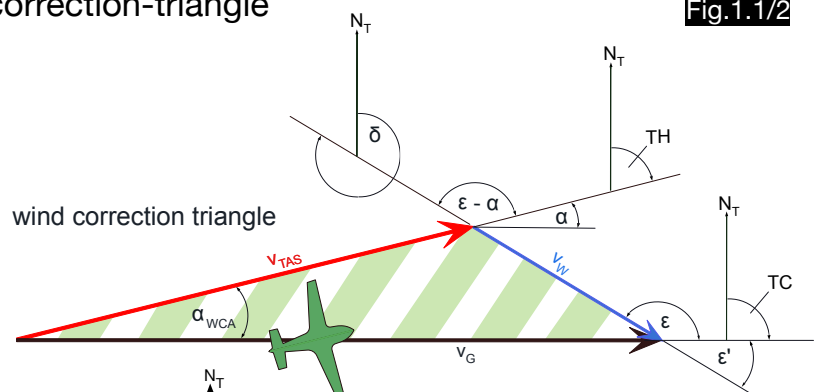
### 1.1.1. Course, heading and track

$N_{true}$	meridian of the geographical north Pole
$N_{magn.}$	meridian of the magnetic north pole
$N_{comp.}$	north direction of the compass
VAR.	variation
DEV.	deviation
HEADING	angle between a reference meridian and the longitudinal axis of the aircraft.
TRACK	angle between a reference meridian and the trajectory projected onto the earth's surface.
COURSE	angle between a reference meridian and the route on the map.
CH	compass heading
MH	magnetic heading
TH	true heading
MC	magnetic course
TC	true course
MT	magnetic track
TT	true track
$\alpha$	wind correction angle
$\beta$	drift angle



### 1.1.2. Drift-triangle and wind-correction-triangle

$N_T$	true north
TH	true heading
TC	true course
TT	true track
$V_{TAS}$	true airspeed
$V_G$	ground speed
$V_W$	wind speed
$\delta$	wind direction
$\alpha_{WCA}$	wind correction angle
$\epsilon$	wind angle
$\epsilon - \alpha$	relative wind angle
$\alpha_{DA}$	drift angle



According to the sine theorem, the following relations apply in the wind correction triangle:

$$\sin \alpha_{WCA} = v_W \frac{\sin \varepsilon}{v_{TAS}} \quad [1.1/1]$$

$$v_G = v_W \frac{\sin (\varepsilon - \alpha_{WCA})}{\sin \alpha_{WCA}} \quad [1.1/2]$$

Note: Computers of type NL work with the wind angle  $\varepsilon'$ . The formula for the ground speed changes accordingly (the formula for the wind correction angle remains unchanged, as  $\sin \alpha = \sin (180 - \alpha)$ ):

$$\sin \alpha_{WCA} = v_W \frac{\sin \varepsilon'}{v_{TAS}} \quad [1.1/3]$$

$$v_G = v_W \frac{\sin (\varepsilon + \alpha_{WCA})}{\sin \alpha_{WCA}} \quad [1.1/4]$$

These relations apply in the drift triangle:

$$v_G = \sqrt{v_{TAS}^2 + v_W^2 - 2 v_{TAS} v_W \cos \varepsilon} \quad [1.1/5]$$

$$\sin \alpha_{DA} = v_W \frac{\sin \varepsilon}{v_G} \quad [1.1/6]$$

In the arbitrarily assumed case of a  $v_{TAS} = 200$  kt, a wind speed  $v_W = 80$  kt and a wind angle  $\varepsilon = 45^\circ$ , a drift angle of  $21.5^\circ$  and a ground speed of 154.2 kt is obtained in the drift triangle. In the wind correction triangle the wind correction angle is  $16.4^\circ$  and the ground speed 135.5 kt. If one flies with a wind correction angle of  $21.5^\circ$  instead of  $16.4^\circ$ , one gets a new wind triangle with a drift angle of  $14.1^\circ$  and a groundspeed of 130.6 kt. The lateral displacement at a target 50 NM away is then 6.5 NM and the calculated flight time is 4 minutes too small. However, the example is rather an exceptional case. With headwind and tailwind (i.e. with wind angles around  $0^\circ$ ) no drift occurs and with pure side wind (i.e. with wind angles around  $90^\circ$ ) drift triangle and wind correction triangle are very similar.



## 1.2. Calculation of the wind triangle according to Dalton

### 1.2.1. Computer with diagram slide

#### 1.2.1.1.General

The wind triangle calculator invented by Philip Dalton exists in two versions. The type shown in Fig.1.2/1, later referred as type E6-B, has a diagram slide with polar coordinates representing speed circles and radial lines for various drift lines. The version shown in Fig. 1.2/2, later referred to as Model G corresponds to the representation of Dalton in his patent specification. Instead of using the diagram slide, it uses an endless band which can be moved by a knob.

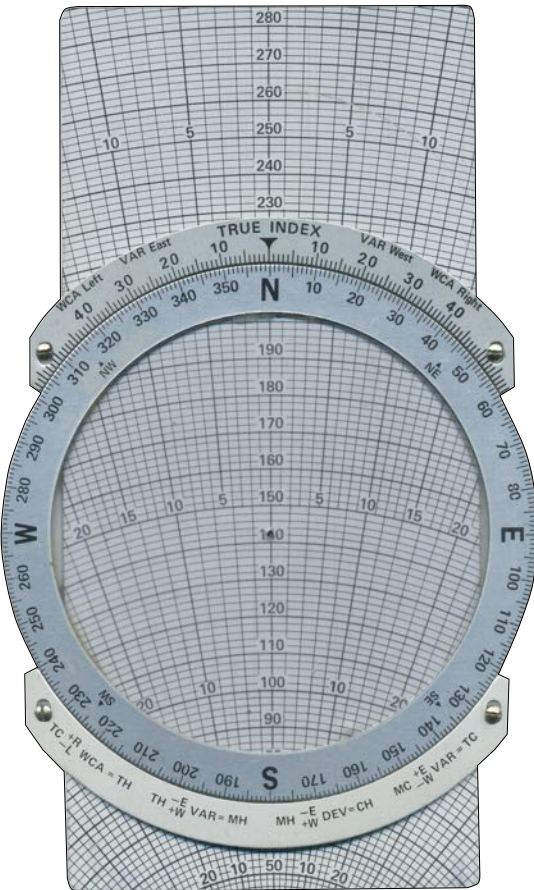


Fig.1.2/1



Fig.1.2/2

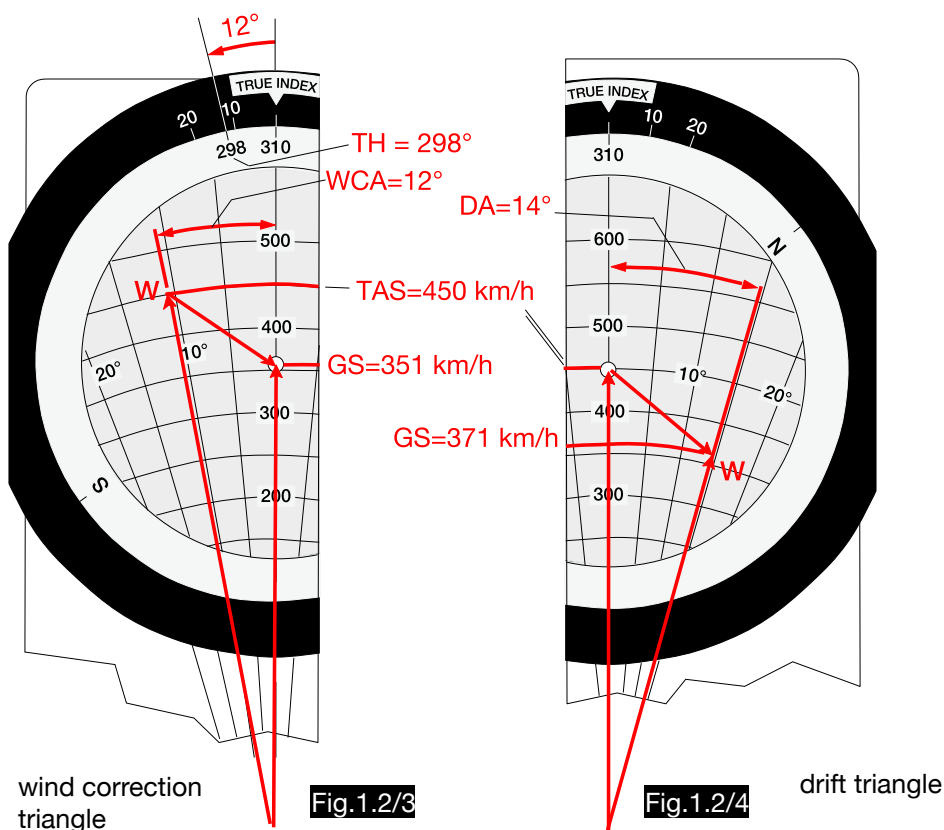
#### 1.2.1.2. Use of the computer for drift-, wind correction- and ground speed calculations

Figures 1.2/3 and 1.2/4 illustrate the calculation of the wind correction- and the drift triangle with a Dalton calculator.

##### Example:

true course	310°
true wind	265°/125 km/h
true airspeed	450 km/h

The calculation starts with the entry of the wind point W. After setting the compass rose at 265° this point must be entered on the middle axis 125 speed units upwards (wind correction triangle) or downwards (drift triangle). The image illustrates the necessary inputs and the results.



In his patent specification Philip Dalton describes a method of changing the relative wind angle in the drift triangle and its step-by-step approach to the wind correction triangle presented with the help of Figures 1.2/5 -7.

#### Step 1 (Fig.1.2/5):

The initial situation is the drift triangle as shown in Fig.1.2/4. If the drift from 14° to the right is determined, a course reduction is recommended which is estimated at 7°, for example.

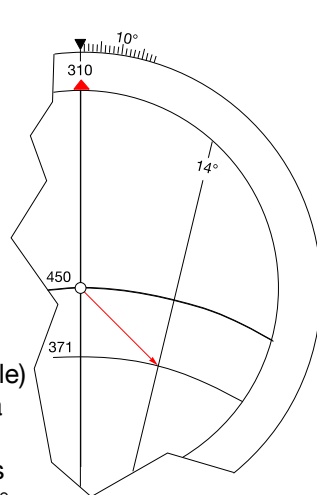


Fig.1.2/5

#### Step 2 (Fig.1.2/6):

After marking the desired course of 310° (red triangle) set the compass rose to 310° - 7° = 303° and read a drift angle of 13°. At the upper correction scale one can see that with this drift angle the target course is still missed. The red arrow (13°) ends at course 313° instead of the red mark (310°).

From this it must be concluded that the selected heading is too small.

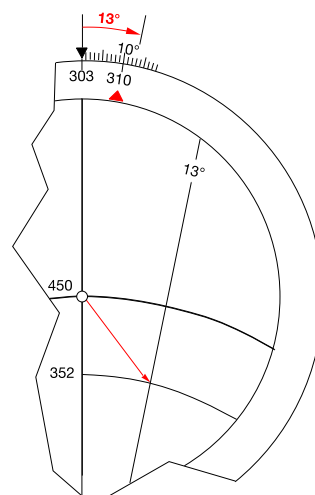


Fig.1.2/6

#### Step 3 (Fig.1.2/7):

The heading is reduced by another 5° to 298°. The drift angle is now 12°. As you can see from the upper scale, this drift leads to the desired track of 310°, because drift angle and wind correction angle now match. The corresponding ground speed of 351 km/h is as expected the same as in the wind correction triangle in Fig.1.2.3.

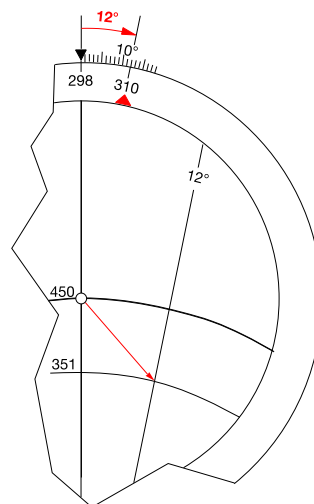


Fig.1.2/7



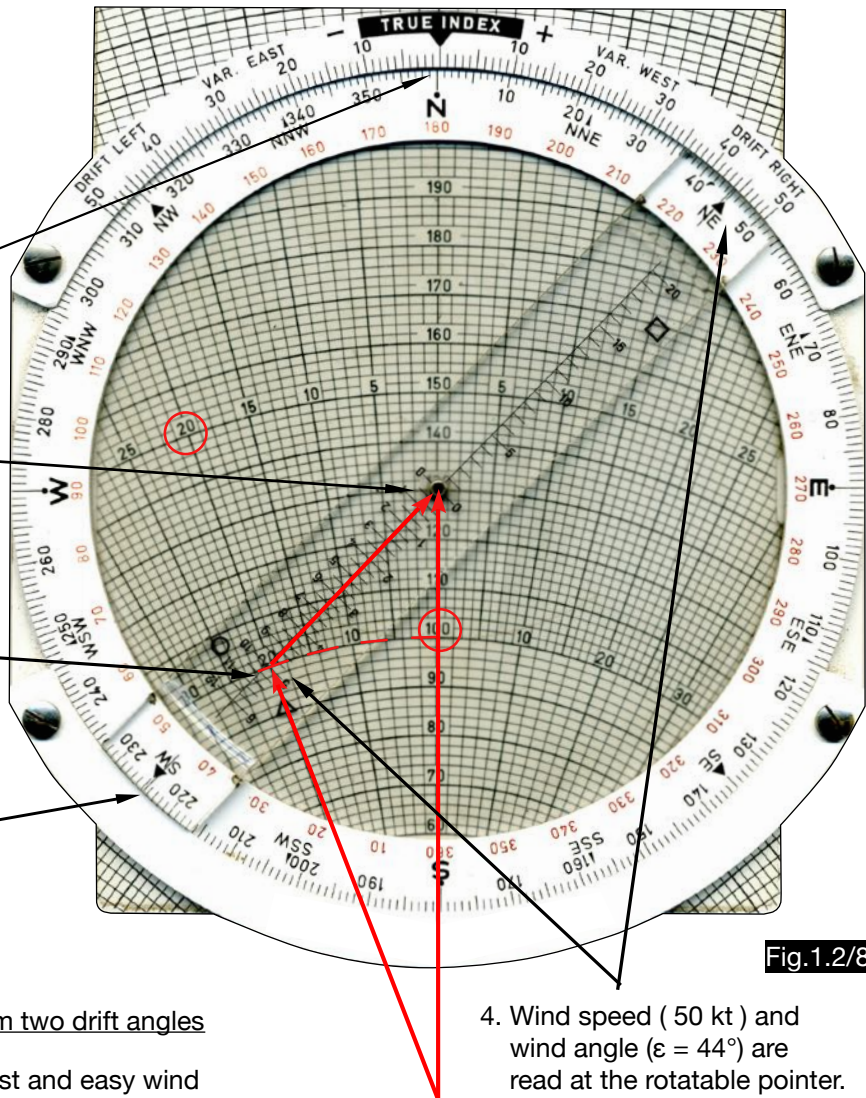
### 1.2.1.3. Use of the computer for wind calculations

#### 1.2.1.3.1. Wind determination from drift and ground speed

Due to the graphical simulation of the wind triangle in the Dalton calculator, wind determinations are particularly quick and easy, as the following example shows (ground speed 130 kt, true air speed 100 kt, wind correction angle 20° left, true course 360°).

##### Solution:

1. The true course 360° (N) is set to TRUE INDEX.
2. The diagram slide is set so, that the speed circle 130 kt (ground speed) is below the centre of the rose.
3. The line of the rotatable pointer is now set to the intersection point of the speed circle 100 kt (true airspeed) with the line 20° (wind correction left).
5. The wind direction 224° (true) can be read directly on the wind rose.



4. Wind speed ( 50 kt ) and wind angle ( $\epsilon = 44^\circ$ ) are read at the rotatable pointer.

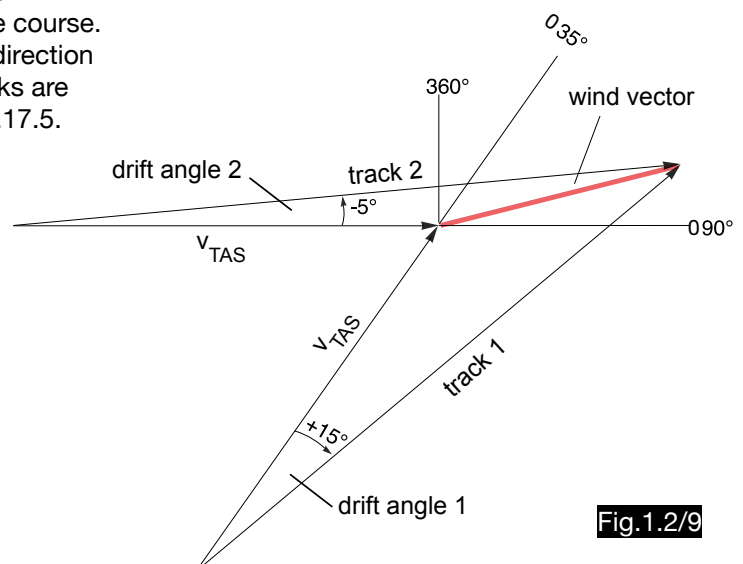
#### 1.2.1.3.2. Wind determination from two drift angles

Dalton computers allow a very fast and easy wind determination if the drift is determined on two courses (which should differ by at least 45°). The calculation principle consists of the graphical reproduction of the two wind triangles on the course. One can also determine the wind vector by direction and size in the same way if two or more tracks are known. This procedure is described under 1.17.5.

##### Example:

true heading 1	035°
true heading 2	090°
drift angle 1	+15°
drift angle 2	-5°
TAS 1 = TAS 2	350 km/h

wind direction (true)	255°
wind speed	200 km/h



### Solution:

1. After setting the wind rose to true course  $035^\circ$  and the diagram slide to 350 km/h the first ground speed vector can be marked with a pencil (Fig. 1.2/10).
2. After setting the wind rose to true course  $90^\circ$  (with unchanged  $v_{TAS}$  the diagram slide remains at 350 km/h) the second ground speed vector is drawn by marking the intersection of the drift angle  $-5^\circ$  with the previously found ground speed vector. Thus the wind vector is determined (Fig.1.2/11).
3. On computers with a rotatable pointer (Fig.1.2.8) the pointer bar is rotated to become aligned with the found wind point. The true wind direction ( $255^\circ$ ) is read at the pointer end and the wind speed (200 km/h) at the pointer scale (Fig.1.2/11). On computers without rotatable pointer, the wind point is turned downwards until it is above the centre line of the diagram slide and the wind direction is read at the TRUE INDEX. The wind speed is then equal to the difference between the values below the centre point ( $v_{TAS}$ ) and below the wind point (Fig.1.2.12).

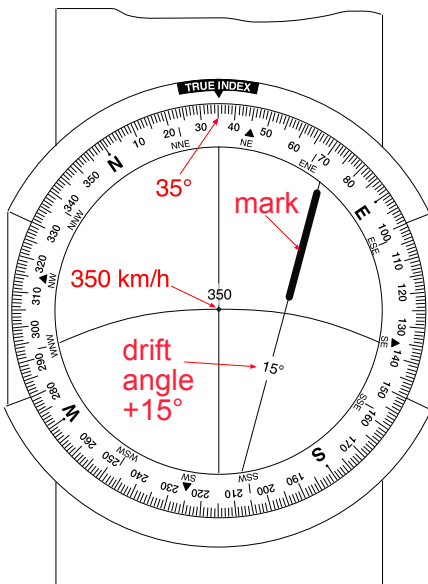


Fig.1.2/10

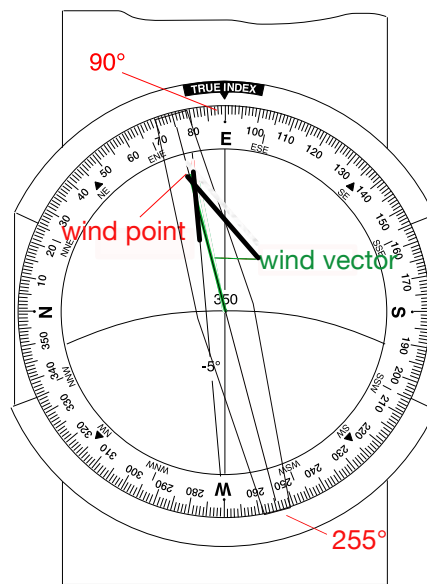


Fig.1.2/11

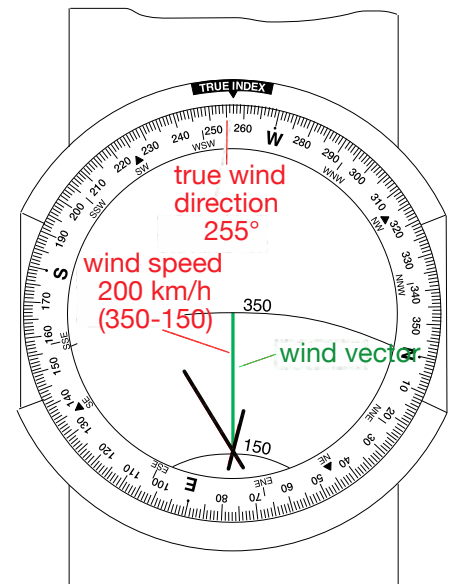


Fig.1.2/12

## 1.2.2. Computer Dalton Mark VII

### 1.2.2.1. Use of the Computer for wind correction- and ground speed calculations

With the Mark VII calculator, wind correction angle and ground speed are calculated graphically using the longitudinal component  $v_{WL}$  of the wind vector.

#### Example:

true course  $180^\circ$   
 true wind  $315^\circ/40$  kt  
 true airspeed 150 kt  
 variation  $15^\circ$  W

wind correction angle  $10.86^\circ$   
 ground speed 175.6 kt  
 true heading  $191^\circ$   
 magn. heading  $206^\circ$

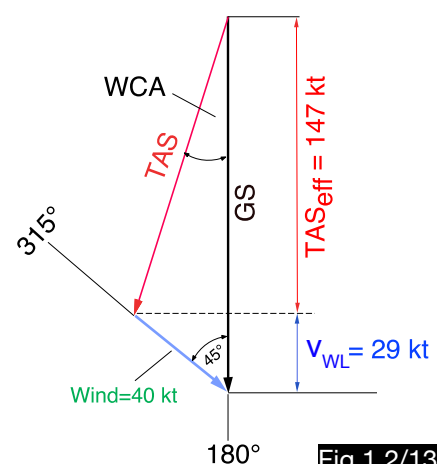


Fig.1.2/13



- 
- Diagram of a circular slide rule with numbered callouts 1 through 7. Callout 1 points to the outer scale, 2 to the inner scale, 3 to the grid, 4 to the drift index, 5 to the true heading index, 6 to the true heading scale, and 7 to the drift scale. A green line is drawn across the grid, and a red line is drawn along the drift index.

The diagram illustrates a flight computer, specifically a circular slide rule used for aviation calculations. It consists of several concentric rings or scales. The outermost ring is labeled "True Airspeed" and has markings from 0 to 360. Below this is a ring labeled "Indicated Airspeed". Inside these are rings for "Altitude" and "Pressure Altitude". The center of the slide rule contains smaller scales for "Fuel Flow" and "Engine RPM". A red arrow points to the "True Airspeed" scale, and another red arrow points to the "Indicated Airspeed" scale. A green line connects two points on the "True Airspeed" scale, passing through the center. Labels "a", "b", and "c" are placed around the slide rule, corresponding to specific parts of the device.

7

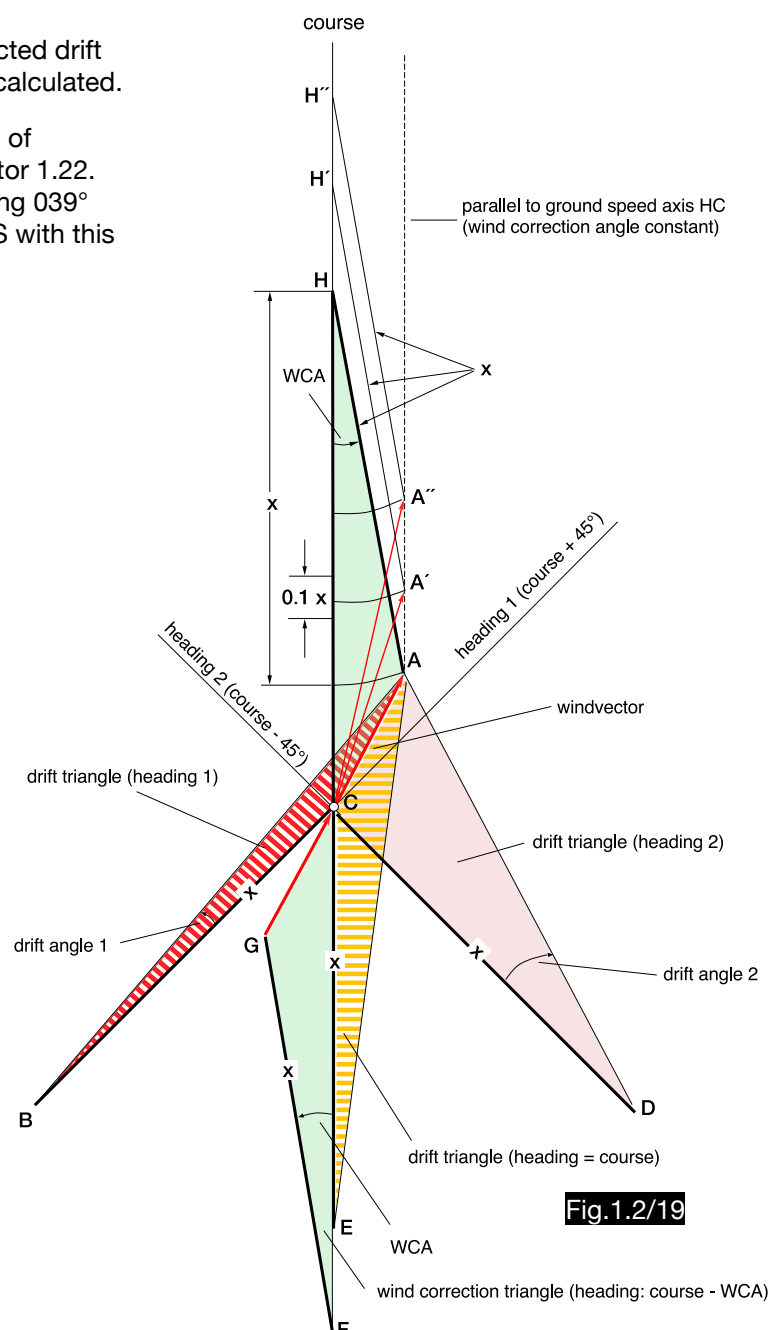
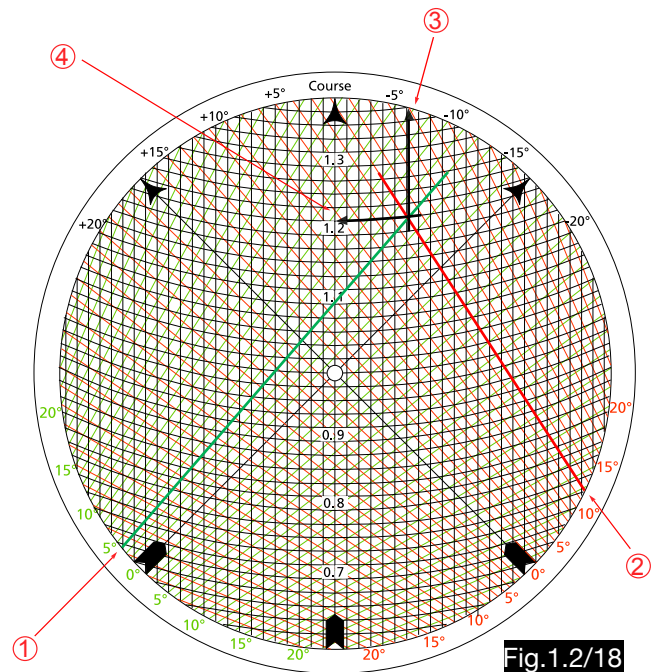
With the calculator E-1A (see Fig.1.2/18) ground speed and wind correction angle can be determined without have to calculate the wind. For this purpose, the drift angle is determined on two headings (45° larger and 45° lower than the course). The following example describes the handling.

true course (TC)	045°
true air speed (TAS)	120 m.p.h.
drift angle at heading 090° (TC + 45)	- 4°
drift angle at heading 360° (TC - 45)	+12°
wind correction angle	- 5.8°
ground speed	146.2 m.p.h.

Soluton:

1. In the green drift angle diagram (valid for the heading TC + 45°), the drift line - 4° is found (marked in the figure by a thick green line ).
2. In the red drift angle diagram (valid for the heading TC - 45°), the drift line +12° is found (marked in the figure by a thick red line ).
3. Vertical above the intersection of the two selected drift lines the wind correction angle WCA = - 6° is calculated.
4. Along the arc through the point of intersection of drift lines is shown on the speed scale the factor 1.22. The ground speed when flying with true heading 039° (45 - 6) is obtained by multiplication of the TAS with this factor to 146 m.p.h. ( $1.22 \times 120 = 146$ ).

In Fig.1.2/19 the drift angles 1 and 2 are contained in the drift triangles ABC and ADC and the wind vector then results as the line A-C. The triangle CGF is the wind correction- and the triangle CEA is the drift triangle for the flight along the course. In both, the TAS is of the arbitrarily selected length  $x$  ( $= 100\%$ ). The ground speed is received as a percentage of the TAS. The clear difference between the two triangles shows that a drift angle determined on the course is not suitable as a wind correction angle. The wind correction triangles CGF and CAH are congruent, so the angle C-H-A is equal to the WCA. The ground speed of the flight with this wind correction angle is the vector H-C. Its length as a percentage of the TAS is determined on a scale along the line H-C, the graduation of which is  $0.01 x$ . To read the ground speed scale, parallel circular arcs of radius  $x$  (no concentric circles) have been provided. The wind correction angle WCA contained in the triangle HCA retains its size even if the peaks of different wind vectors lie along a parallel to the axis of symmetry H-F. Because the size  $x$  remains the same, the length of the ground-speed-distance H - C changes to H' - C, but the WCA remains unchanged. Therefore the lines of the same WCA could be drawn parallel to the line H - F.



## 1.2.4. Computer VC-2

The computer VC-2 consists of a base plate with a compass rose (see Fig.1.2/20), a middle transparent disk with a grid and speed scales (see Fig.1.2/21) and an upper transparent disk with concentric arcs (see Fig.1.2/22).

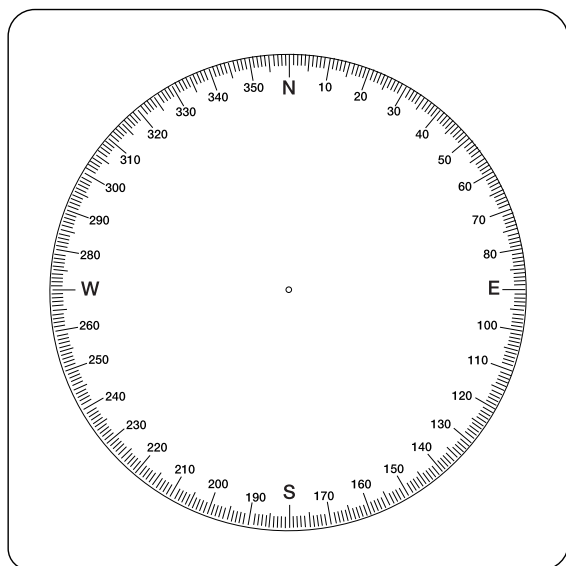


Fig.1.2/20

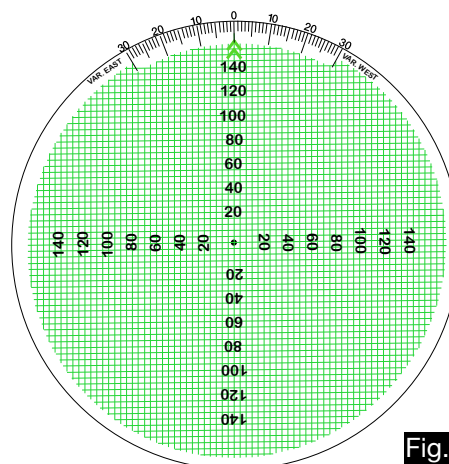


Fig.1.2/21

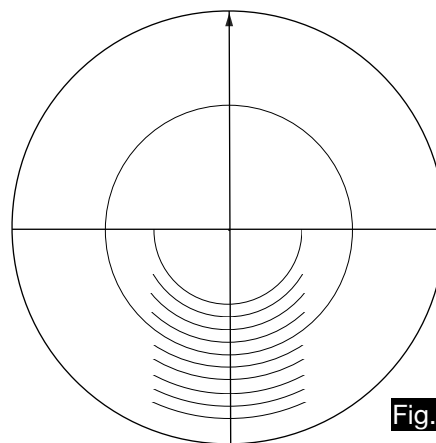


Fig.1.2/22

### 1.2.4.1. Use of the computer for wind correction and ground speed calculations

The handling of the computer is explained by the following example (see Fig.1.2/23):

VTAS            100 kt  
true wind       315°/25 kt  
true course     360°  
variation       10° W

drift angle       +12.1°  
 $V_{G1}$             84.2 kt  
true heading    349.8°  
magn. heading   359.8°  
wind corr. angle -10.2°  
 $V_{G2}$             80.8 kt

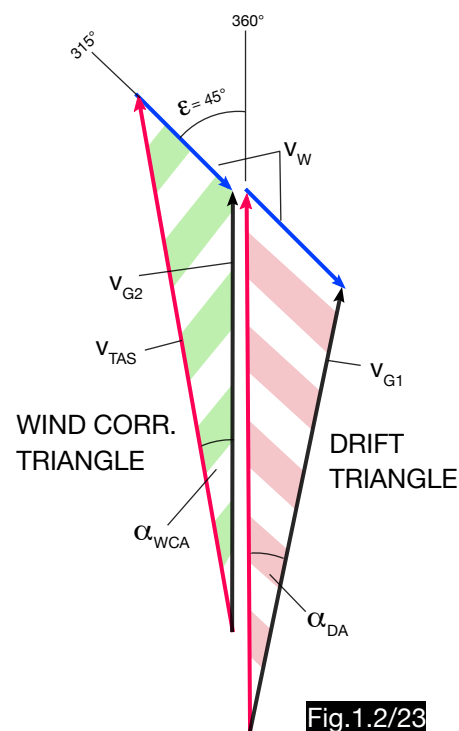


Fig.1.2/23



### Solution:

1. The upper disc is set to the true course 360°.
2. The wind point W is marked on the upper disc in the usual way using the speed scales and the wind rose.
3. Along a vertical grid line through the wind point, point A is found at the circular arc of speed 100 ( $v_{TAS}$ ). Thus the triangle AWM corresponds to the wind correction triangle. In this the distance AM corresponds to the true airspeed and the distance AW to the ground speed. Their length can be determined by counting in the grid as 80 kt (see Fig.1.2/24).
4. In a second setting, the position of the upper disc remains unchanged, but the vertical speed axis of the middle disc is rotated over the true air speed vector AM so that the true heading 350° can be read opposite the green marker and the magnetic heading 360° can be read at the variation scale opposite the variation 10° W (see Fig.1.2/25).

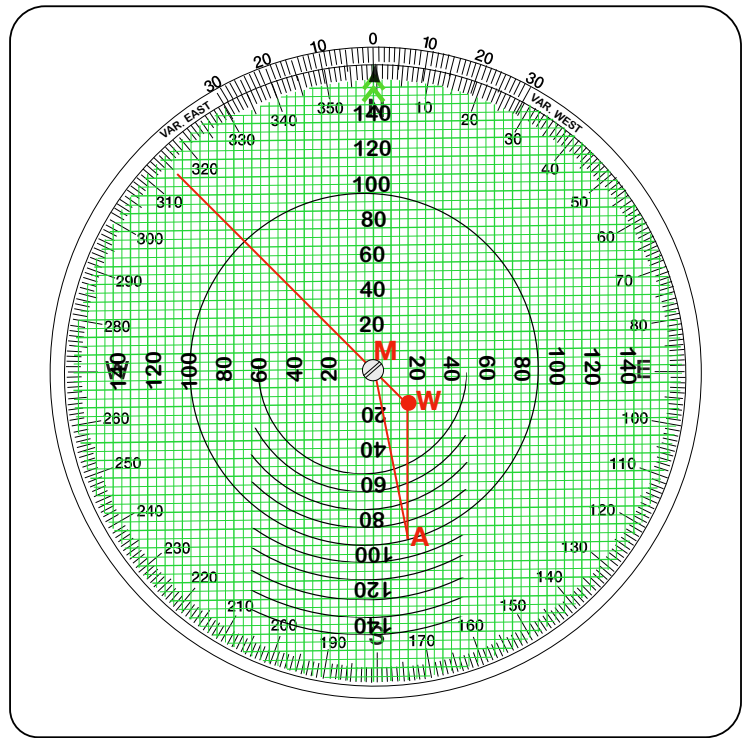


Fig.1.2/24

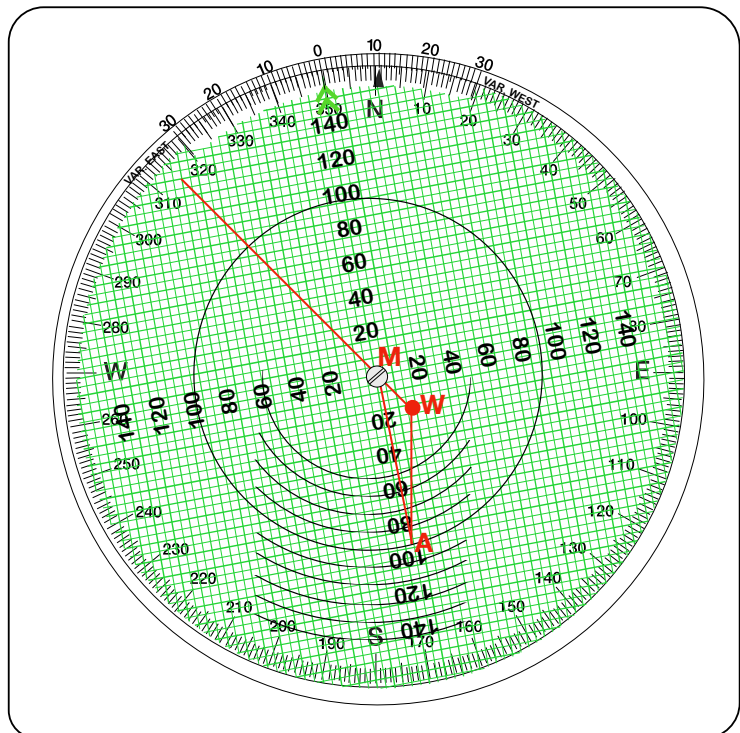


Fig.1.2/25

For the calculation of the drift triangle the procedure is slightly different. The wind point must be drawn in such a way that the peak of the wind vector ends at the center M and the end of the true air speed vector (point A) must be determined by counting in the grid.

#### 1.2.4.2. Use of the computer for wind determination from drift and ground speed

The setting of the calculator when determining the wind from drift angle, heading and ground speed is the same as for the drift triangle described above, which is why a more detailed description is not given here.

### 1.3. Calculation of the wind triangle according to Knemeyer

#### 1.3.1. Use of the computer for wind correction- and ground speed calculations

The handling of the computer (see Fig.1.3/2) is described using an example (see Fig.1.3/1).

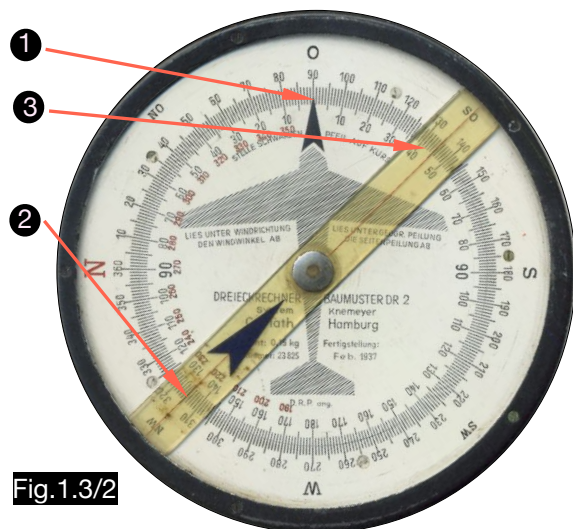


Fig.1.3/2

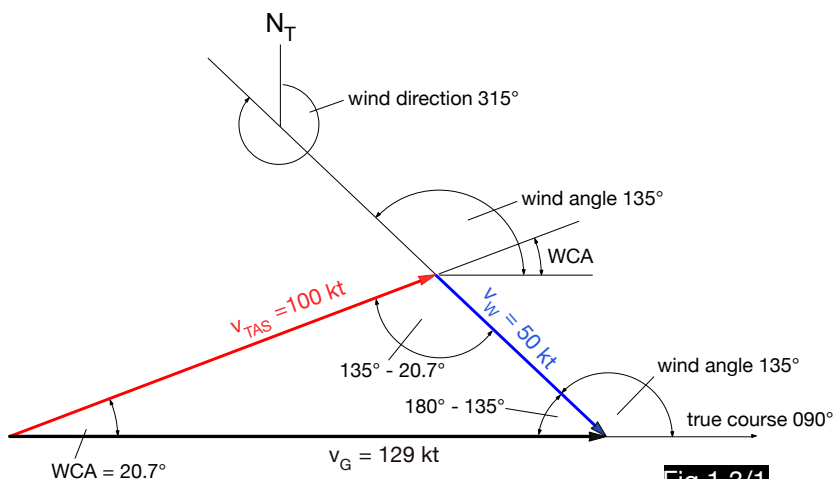


Fig.1.3/1

On one side of the computer (Fig.1.3/2) the wind angle can be calculated as follows:

After setting the true course 090° (1) and the wind direction 315° (2) the wind angle 135° can be read off (3).

The Knemeyer-computer calculates the wind triangle using the sine theorem (see equ. 1.1/1 and 1.1/2) by means of a circular logarithmic slide rule.

Fig. 1.3/3 schematically shows the necessary settings for this example.

Solution:

1. The wind angle (135°) is set opposite the true air speed (100 kt).
2. The wind correction angle (20.7°) is read opposite the wind speed (50 kt).
3. The ground speed (129 kt) is read opposite the angle  $135^\circ - 20.7^\circ = 114.3^\circ$ .

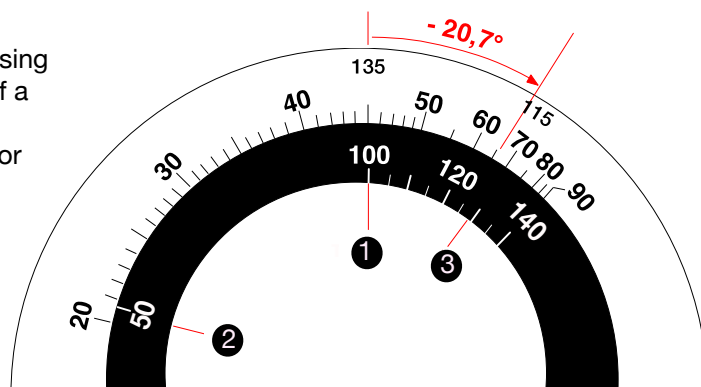


Fig.1.3/3

#### 1.3.2. Use of the computer for wind determination from drift angle and ground speed

To calculate the wind angle and the wind speed, the Knemeyer-computer has to use the method of "fitting" the drift angle between the true air speed and the ground speed. In the example (Fig.1.3/4) a wind speed of 49.6 km/h was determined with the cosine theorem (see equ.1.1/5) and a wind angle of 43.6° was determined with the sine theorem.

Solution:

Between the values 100 km/h (TAS) and 130 km/h (ground speed) on the inner ring, exactly 20° must be "fitted in" on the outer ring. The solution is found, when the angle 43° is opposite the value 100 and the angle 63° is opposite the value 130. Thus the angle  $\epsilon' = 43^\circ$  is clearly determined.  $\epsilon'$  is the complementary angle of the wind angle, which is therefore 137° ( $180^\circ - 43^\circ$ ). Opposite the wind correction angle 20° one will find the wind speed 49 km/h.

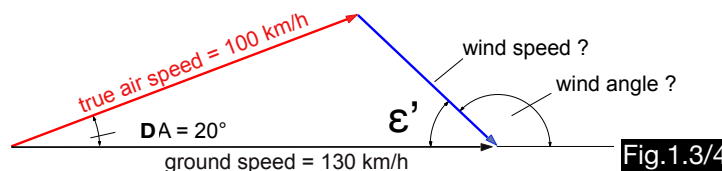


Fig.1.3/4

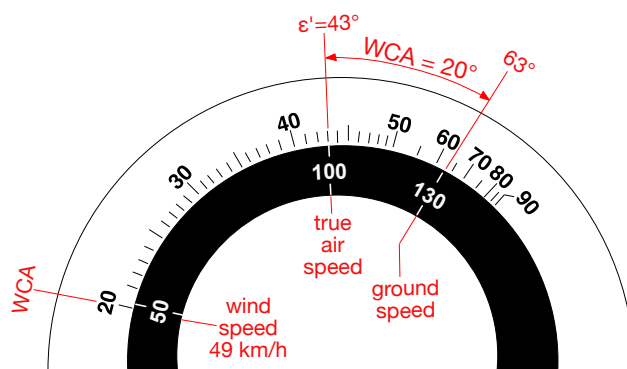


Fig.1.3/5

## 1.4. Calculation of the wind triangle according to Nemtschinow / Popow

Fig.1.4/1 shows a computer of the NL series, invented by W.G.Nemtschinow and L.S.Popow which has logarithmic sine and tangent scales for triangle calculations. Fig. 1.4/2 shows the calculation of the example of chapter 1.3.1.

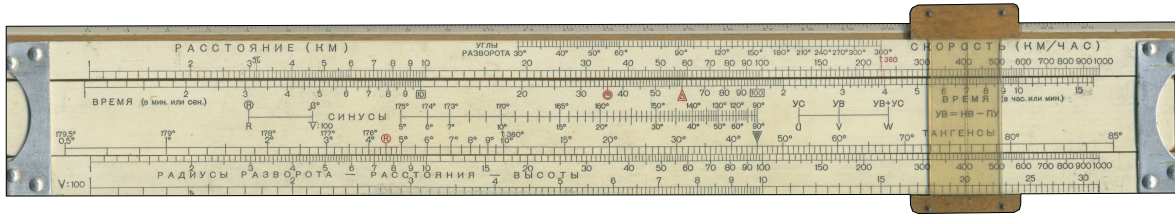


Fig.1.4/1

### 1.4.1. Wind correction angle and ground speed calculations (wind correction triangle)

1. The wind angle  $\epsilon' = 45^\circ$  is set opposite the true airspeed (100 kt).
2. The wind correction angle ( $20.7^\circ$ ) is read opposite the wind speed (50 kt).
3. The ground speed (129 kt) is read opposite the angle  $45^\circ + 20.7^\circ = 65.7^\circ$ .

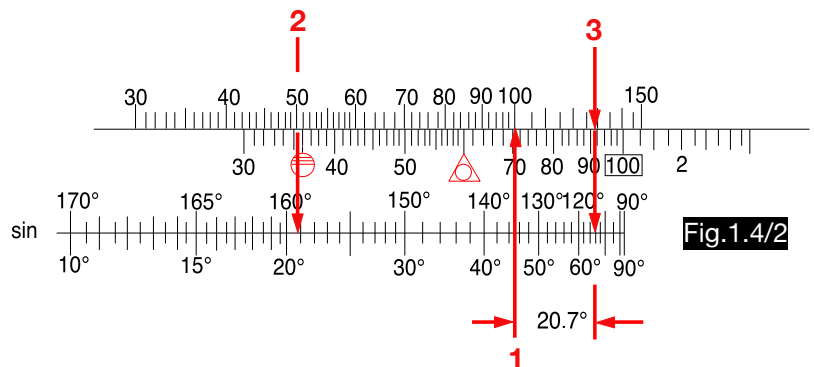


Fig.1.4/2

### 1.4.2. Drift angle and ground speed calculations (drift triangle)

Example:

true airspeed	100 kt
wind angle $\epsilon'$	$40^\circ$
wind speed	50 kt
$V_{WC}$	32.1 kt
$V_{WL}$	38.3 kt
drift angle	$13.1^\circ$
ground speed	141.6 kt

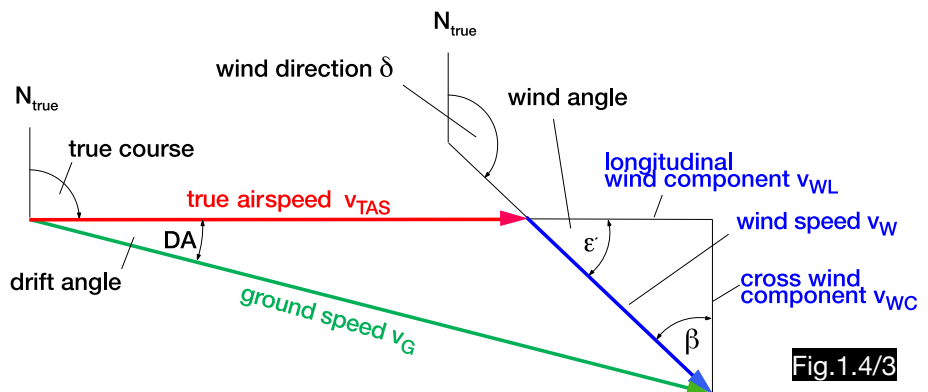


Fig.1.4/3

1. The two wind components  $v_{WC} = 32$  kt and  $v_{WL} = 38$  kt are calculated using the sine scale:

$v_{WC}$	$v_W$	$v_{WL}$	$v_W$
32	50	38	50
$40^\circ$	$90^\circ$	$50^\circ$	$90^\circ$
$\epsilon'$		$\beta$	

2. The drift angle  $DA = 13^\circ$  is calculated using the tangent scale:

DA	
$13^\circ$	
32	138
$v_{WC}$	$v_{TAS} + v_{WL}$

3. The ground speed  $v_G = 142$  kt is calculated using the sine scale:

$v_W$	$v_G$
50	142
$13^\circ$	$40^\circ$
DA	$\epsilon'$

### 1.4.3. Use of the computer for wind determination from drift angle and ground speed

The wind can be calculated approximately from drift angle and ground speed as follows using NL slide rules with a tangent scale.

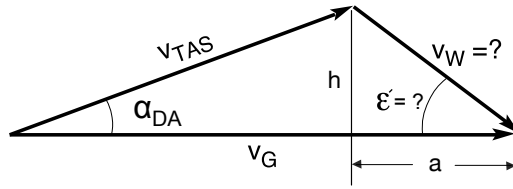


Fig.1.4/4

This applies to small angles  $\alpha$ :

$$a \approx V_G - V_{TAS}$$

$$h \approx V_{TAS} \operatorname{tg} \alpha$$

$$h = a \operatorname{tg} \epsilon'$$

$$a \operatorname{tg} \epsilon' \approx V_{TAS} \operatorname{tg} \alpha$$

$$\frac{\operatorname{tg} \alpha}{V_G - V_{TAS}} \approx \frac{\operatorname{tg} \epsilon'}{V_{TAS}}$$

The calculation of the wind angle  $\epsilon'$  and the wind speed is shown for the following example:

$$V_{TAS} = 100 \text{ km/h}$$

$$V_G = 140 \text{ km/h}$$

$$\alpha = 15^\circ$$

Step 1 (Fig.1.4/5):

After  $15^\circ$  (DA) has been placed opposite 40 ( $V_G - V_{TAS}$ ) in the tangent scale the angle  $\epsilon' = 34^\circ$  can be read opposite 100 (TAS).

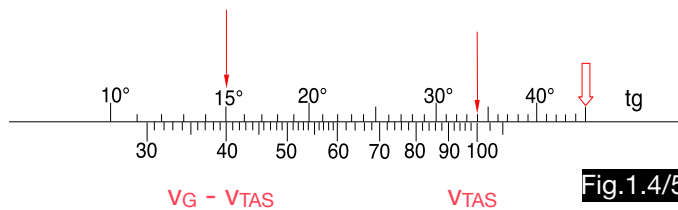


Fig.1.4/5

$$\frac{\operatorname{tg} 15^\circ}{140 - 100} \approx \frac{\operatorname{tg} \epsilon'}{100}$$

Step 2 (Fig.1.4/6):

In the triangle follows after the sine theorem:  $\frac{\sin \alpha}{V_W} = \frac{\sin \epsilon'}{V_{TAS}}$

After the corresponding adjustment of the sinus scale, the wind speed 46 km/h is obtained.

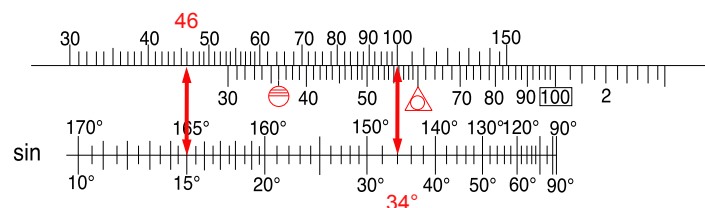


Fig.1.4/6

Analytically were determined the wind angle  $\epsilon' = 30.8^\circ$  and the wind speed 50.5 km/h. The errors in the selected example are 10% for the wind angle ( $34 : 30.8 = 1.10$ ) and 9% for the wind speed ( $46 : 50.5 = 0.91$ ). With a drift angle of  $10^\circ$  the errors would be reduced to 5 or 4.5% for the same true air speed and ground speed values.



## 1.5. Calculation of the wind triangle according to Bygrave

### 1.5.1. Calculating instrument for solving the wind triangle

Fig.1.5/2 shows the computer invented by Leonard Charles Bygrave. The computer can only be used for one true air speed (60 kt, m.p.h. km/h, m/s). Its handling is described by the following example (see Fig.1.5/1) from the patent specification:

true course	350°
true airspeed	60 m.p.h.
true wind	SE (135°) / 15 m.p.h.
wind corr. angle	8.2°
ground speed	71.7 m.p.h.

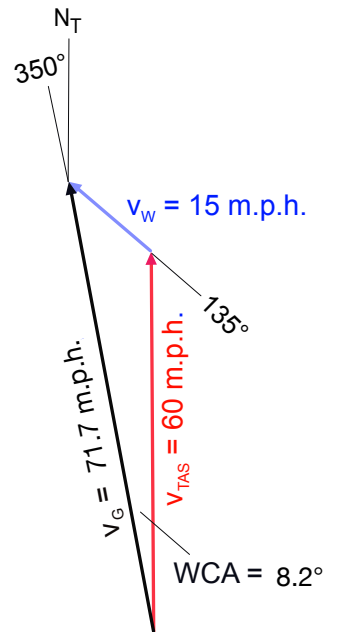


Fig.1.5/1

#### Handling:

1. The compass rose is set to true course TC = 350°.
2. The rotatable pointer is set to the wind direction 135°.
3. The ground speed 72 m.p.h. is read from pointer point 15 (windspeed) parallel to the circular arcs in the speed axis. To calculate the wind correction triangle, the tip of the wind vector must point to the center.
4. Along the parallel lines the wind correction angle 8° is found.

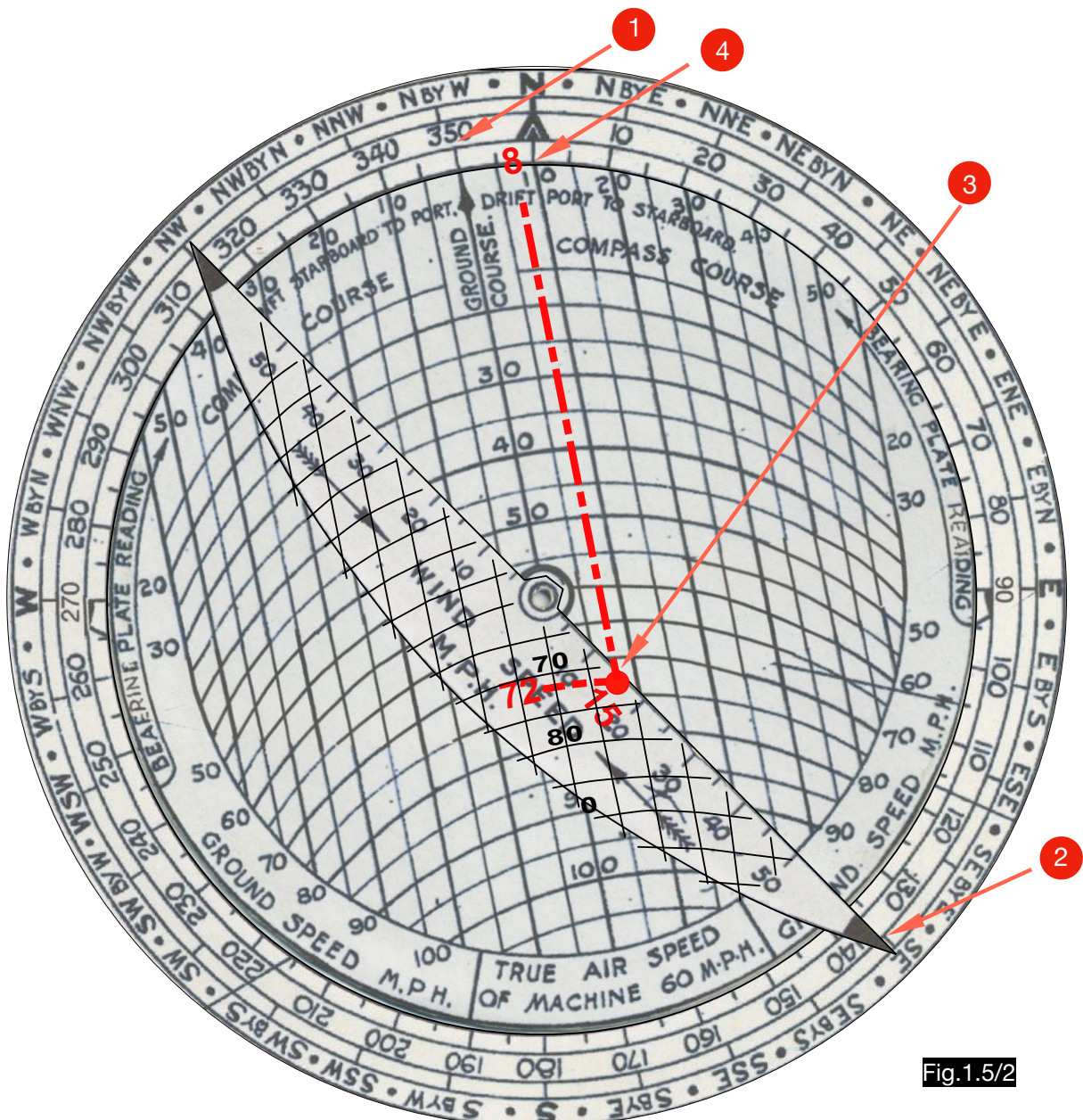
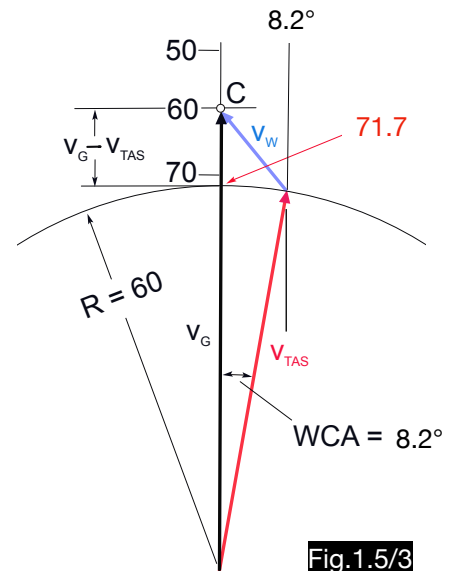


Fig.1.5/2

#### Procedure:

Fig. 1.5/3 shows a schematic representation of the wind triangle in the Bygrave computer. The parallel concentric arcs have a radius of 60. The true air speed vectors have always the length 60 and along the arcs one finds the difference  $V_G - V_{TAS}$  in the speed scale. The speed mark 60 is located at the center C, so that the TAS-arc defines the ground speed. The peaks of all TAS vectors of the same angle move on a parallel to the center axis. Thus the vertical lines form the scale for the wind correction angle.



### 1.5.2. Course and Speed Calculator MK II A

#### 1.5.2.1. Wind correction- and ground speed calculations

##### Example:

true course                     $360^\circ$   
true airspeed                200 kt  
true wind                    NE ( $045^\circ$ ) / 50 kt  
variation                     $15^\circ$  W

##### Results in the wind correction triangle:

WCA                             $+10.2^\circ$   
true heading                 $010^\circ$   
magn. heading               $025^\circ$   
ground speed               161.5 kt

##### Results in the drift triangle:

DA                              $-12.1^\circ$   
ground speed               168.4 kt

##### Handling (see Fig.1.5/4):

1. Set the frame of the compass rose to 200 kt (TAS).
2. Set the wind mark at  $45^\circ$  and 50 kt.
3. Adjust the compass rose so, that  $TC = 360^\circ$  is opposite to the drift mark. As a result of the associated movement of wind point and speed arm the drift mark move also (see design description).
4. Read true heading  $10^\circ$  at the course mark.
5. Read the ground speed 162 kt at the speed arm.
6. Read the magnetic heading ( $25^\circ$ ) opposite the variation ( $15^\circ$  in the SBD-DRIFT-scale).

##### Note:

The drift angle and the ground speed in the drift triangle are calculated when  $TC = 360^\circ$  is set opposite the course mark C.

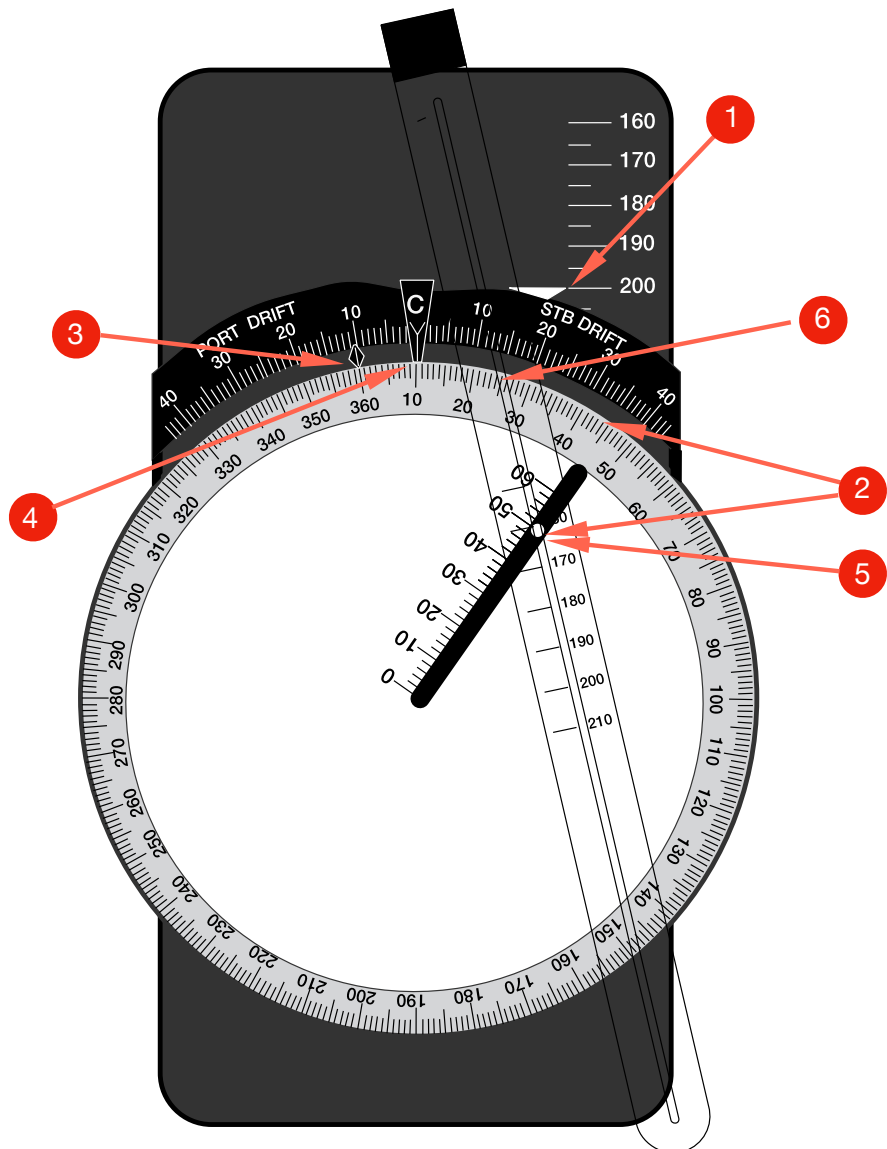
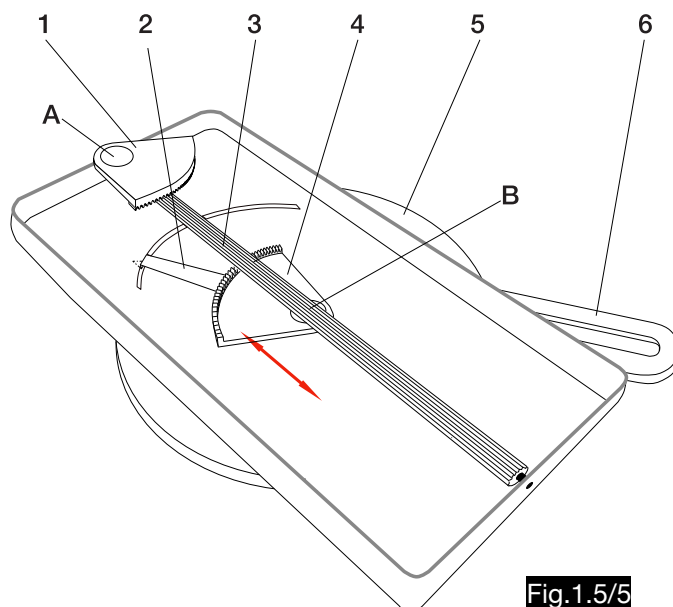


Fig.1.5/4

Design description (see fig.1.5/5):

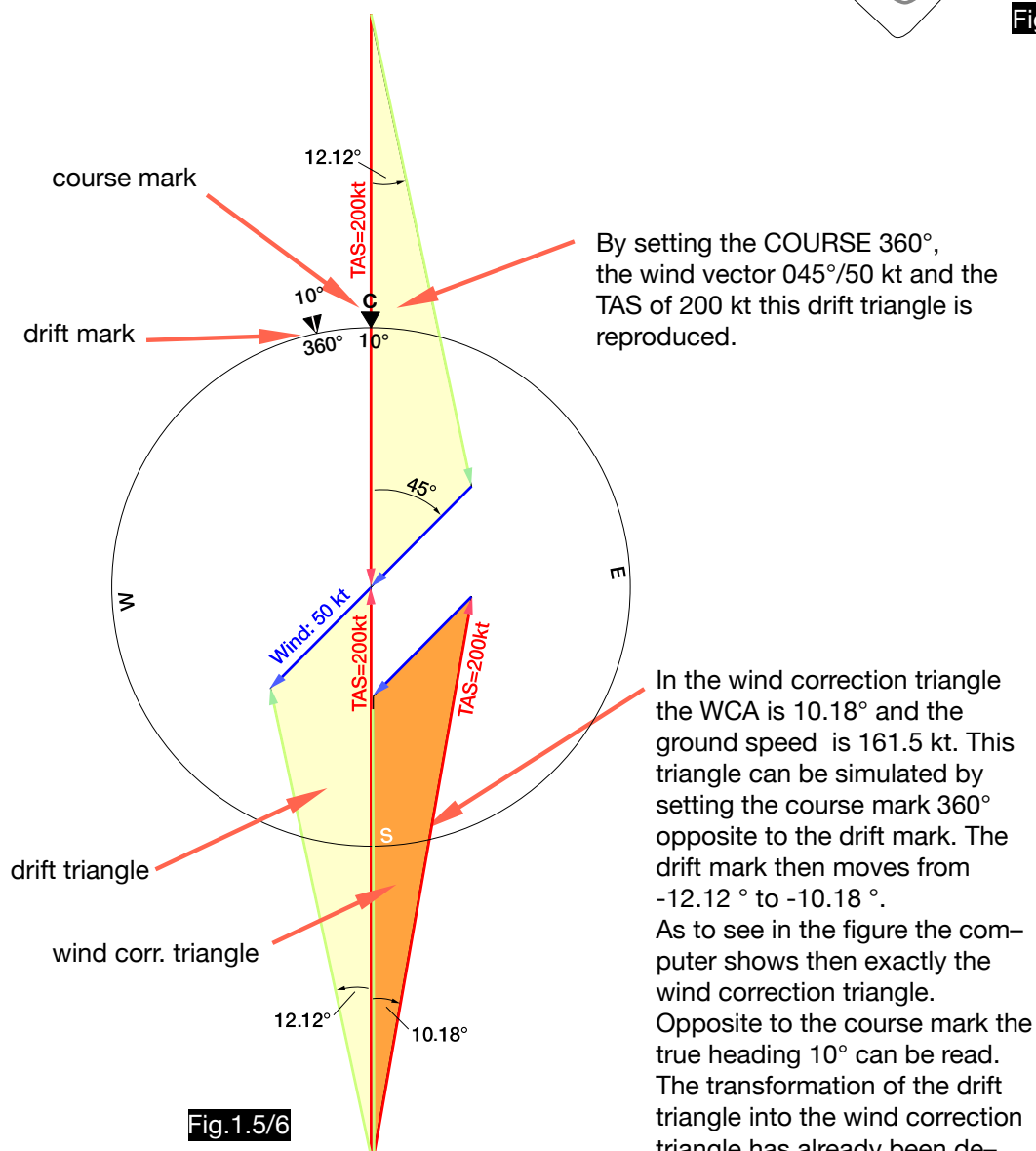
The crown wheel segment 1 is connected to the speed arm 6 so that its rotation around point A is transmitted via the spline shaft 3 to the crown wheel segment 4. The drift mark 2, attached to segment 4, thus rotates at point B by the same angle as the speed arm, but in the opposite direction. The compass rose 5 and the segment 4 are mounted on the axis (B) so that they can rotate independently of each other and can be moved longitudinally together (red arrow) to adjust the true air speed.



**Fig.1.5/5**

Procedure (see fig.1.5/6):

The method of the calculator consists of the mechanical simulation of the wind triangle.



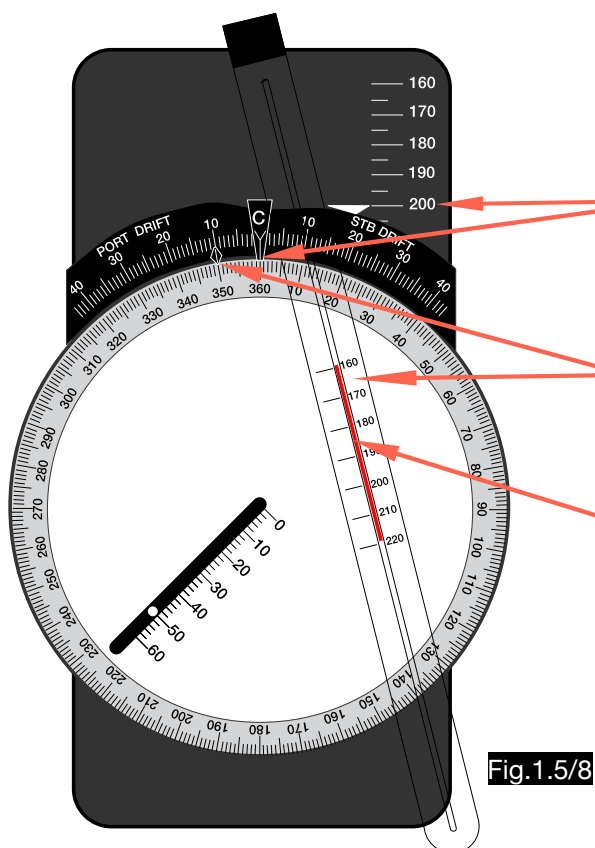
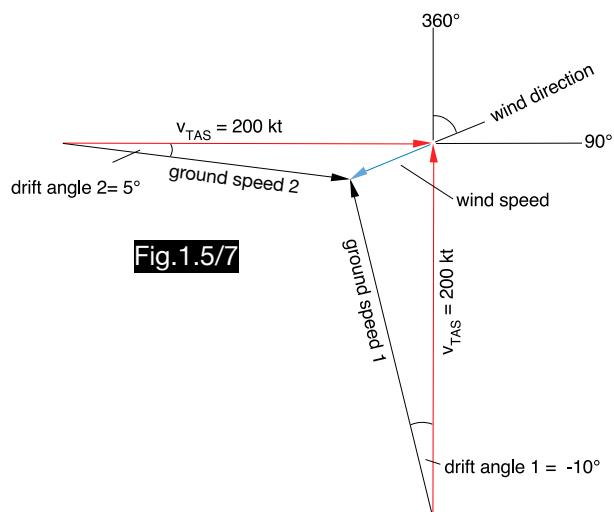
**Fig.1.5/6**

### 1.5.2.2. Wind determinations from two drift angles

Example (see Fig.1.5/7 - 9):

true heading 1	360°
true heading 2	090°
drift angle 1	-10° (left)
drift angle 2	+5° (right)
TAS 1 = TAS 2	200 kt

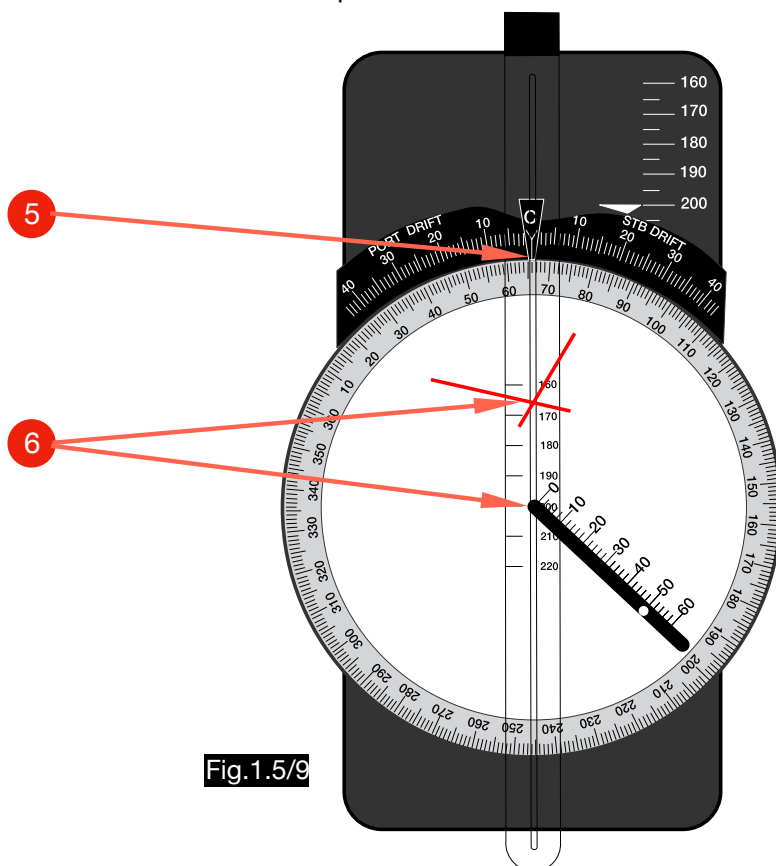
true wind direction	065.8°
wind speed	35.8 kt



- 1 Setting the compass rose on TAS = 200 kt and on TH1 = 360°.
- 2 The speed arm is adjusted so that the drift mark points to the first drift angle of -10°.
- 3 A pencil line is drawn through the groove of the speed arm onto the surface of the compass rose.
- 4 The procedure is used for the second course TH = 90° repeated. The two drawn-in lines cross at the wind point.

After the wind point has been rotated into the symmetry axis, at the course mark the wind direction of 66° can be read.

The wind speed is 35 kt and is determined on the scale of the speed arm between the wind point (at 165 kt) and the centre (at 200 kt).





## 1.6. Calculation of the wind triangle according to Thurston and Sweeney

This computer (see Fig. 1.6/1), invented by Arthur Thurston and Beauregard Sweeney is used in the same way as the Mark IIA of Bygrave (see 1.5.2.), so that a description of the handling is omitted here. The completely different constructive solution is described in Fig. 1.6/ 2.

ground speed arm

locking device 16

wind arm

heading mark 12

switch 15

variation scale

true airspeed scale

drift scale

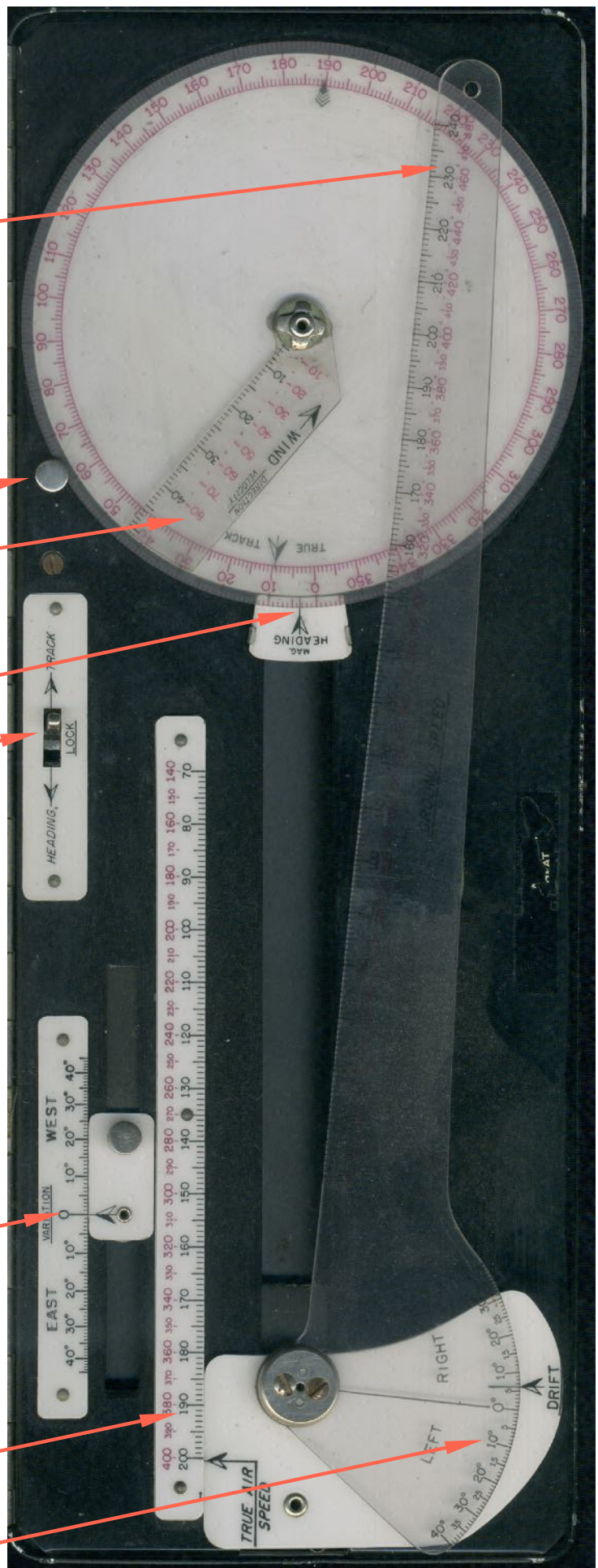
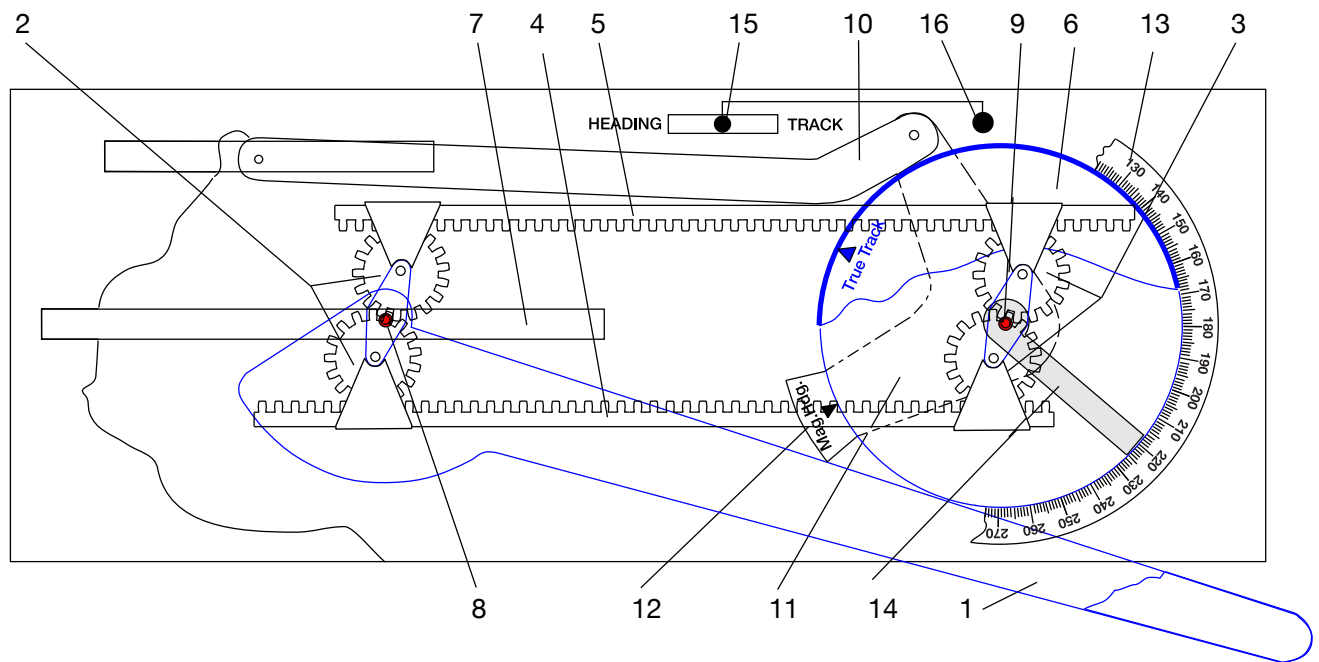


Fig.1.6/1

Design description (see fig.1.6/2):

The rotation of the ground speed arm 1 is transmitted to the true track disc 6 via the pinion pairs 2 and 3 and the racks 4 and 5. The complicated kinematics are necessary, because in case of a longitudinal shift of the arm 1 in the rail 7 (for adjustment of the TAS) the disc 6 must not rotate. When the bearing 8 of arm 1 (which is connected to gear pair 2) in rail 7 is moved to fix the TAS, the pinions of pair 2 rotate around their axes, but the gear racks 4 and 5 remain stationary. When arm 1 is rotated to the wind point, the pinions do not rotate around their axes, but both pinion pairs rotate as a unit. The left pinion pair rotates around the axis 8. The rotation is transmitted to the pinion pair 3 via the gear racks, so that it rotates around the axis 9. Since pinion pair 3 is connected to disk 6 and pinion pair 2 to arm 1, its rotation is transmitted to the true track mark at the correct angle. The variation is set at the left end of the connecting rod 10 and the lever 11 with the mark 12 (true heading) is turned around the axis 9. The compass rose 13 is rotatably arranged on the axis 9 in such a way that it rotates synchronously with the disc 6. When calculating the wind correction triangle, this coupling must be switched off, so that the TC does not change. Switch 15 (LOCK) is used for this purpose and is mechanically connected to the locking device 16, which blocks the compass rose. The wind arm 14 can be turned around axis 9 to determine the wind direction, then it rotates synchronously with the compass rose 13.



**Fig.1.6/2**

## 1.7. Calculation of the wind triangle according to Addison and Luard

### 1.7.1. General

The wind triangle calculator invented by Herbert Addison and William Blaine Luard mechanically reproduces the wind triangle. It exists in several versions. One of these shows Fig.1.7/1.

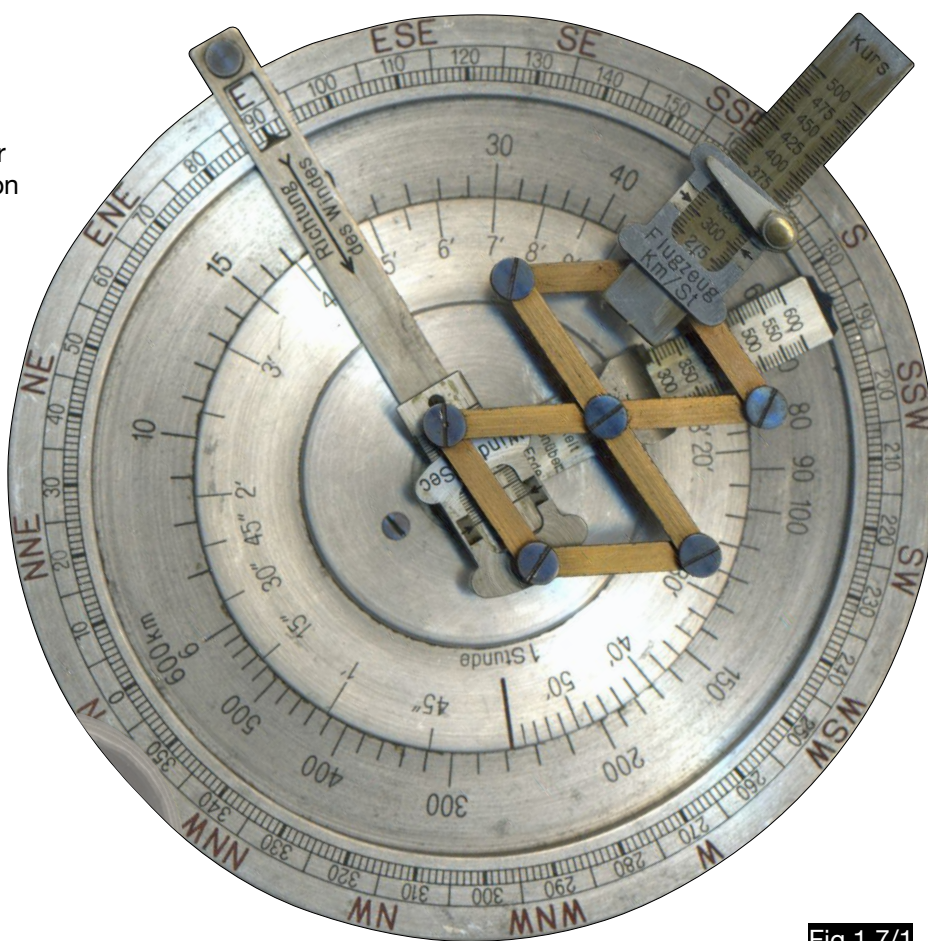


Fig.1.7/1

### 1.7.2. Use in normal triangular calculations

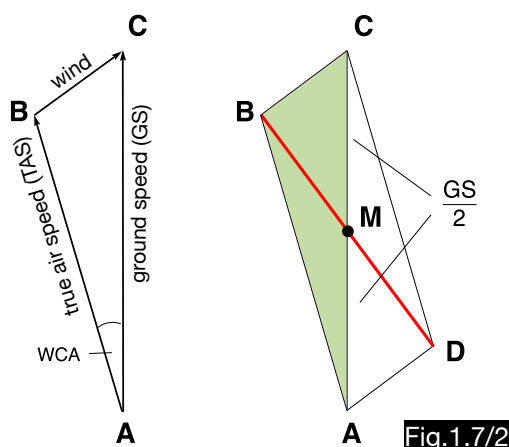


Fig.1.7/2

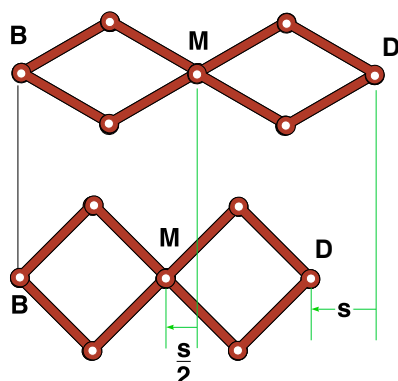


Fig.1.7/3

The wind triangle ABC (Fig.1.7/2) can be supplemented to the parallelogram ABCD, the diagonals of which are exactly halved by the intersection point M.

The Addison-Luard computer therefore has three pointers that can be rotated around the centre A of a compass rose to simulate the wind triangle. Fig.1.7/4 shows the construction schematically, where after input of two vectors always the third one adjusts itself in direction and size. The mechanical connection between the pointers of the true airspeed and the ground speed is made by means of a pantograph mechanism, where the point M is always in the middle of the line DB. If the distance DB is reduced by the distance s, the center M only moves half the way (see Fig.1.7/3). Therefore, only half the amount is displayed at the ground speed pointer. Divisions in scale 1: 2 on this pointer make doubling of the displayed value unnecessary.

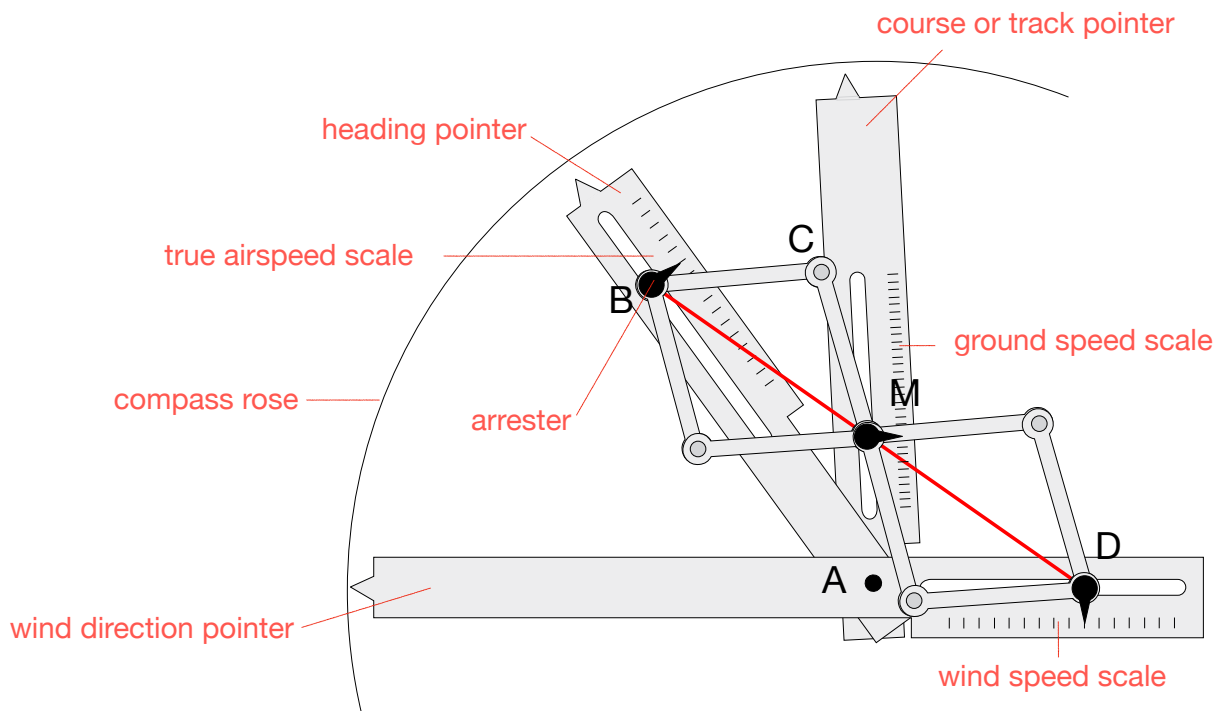


Fig.1.7/4

### 1.7.3. Use with wind determinations from two drift values

The version of the Addison - Luard calculator, described in the patent specification, has two additional pointers for setting two tracks to determine wind direction and wind speed (see Fig.1.7/6). The handling of the computer with wind determinations from two drifts is described using this example (see Fig. 1.7/5):

true heading 1	360°
true heading 2	060°
drift angle $\alpha_1$	+12° (right)
drift angle $\alpha_2$	+17° (right)
TAS 1 = TAS 2	100 kt
true wind direction	329.3°
wind speed	30.7 kt

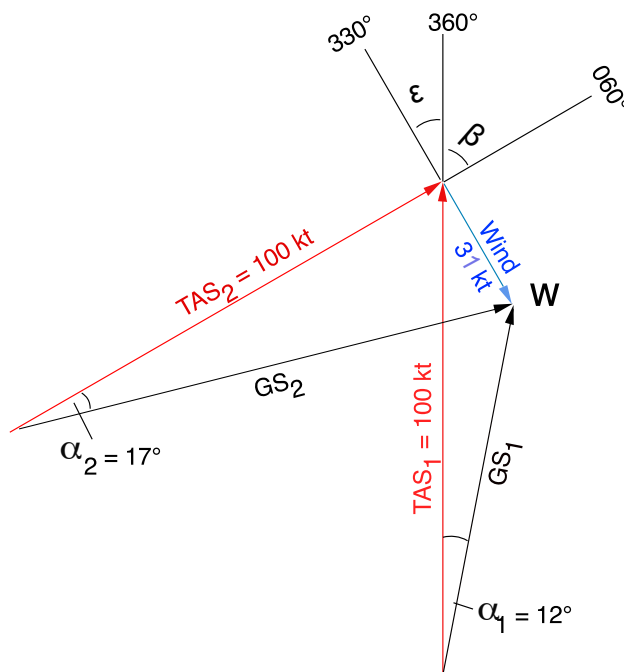


Fig.1.7/5

Handling (see fig.1.7/6):

1. Set speed arm on 360° (HDG 1).
2. Set speed 100 kt (TAS).
3. Set the pointer to 12° (drift 1)
4. Set speed arm on 060° (HDG 2).
5. Set speed 100 kt (TAS).
6. Set the pointer to 17° (drift 2)
7. Adjust the wind arm so, that the wind point W is at the intersection of the drift arms.
8. Read wind direction 330° at wind arm.
9. Read wind speed 31 kt at wind scale.

For the analytical calculation of the wind angle $\epsilon$ and the wind speed $v_w$ these formulas shall be applied:	$\tan \epsilon = \frac{\sin \alpha_2 - \sin(\alpha_2 + \beta)}{\cos(\alpha_2 + \beta) - \frac{\sin \alpha_2}{\tan \alpha_1}}$	[1.7/1]
	$v_w = 100 \frac{\sin \alpha_2}{\sin(\alpha_2 + \beta + \epsilon)} \left[ \% v_{TAS} \right]$	[1.7/2]



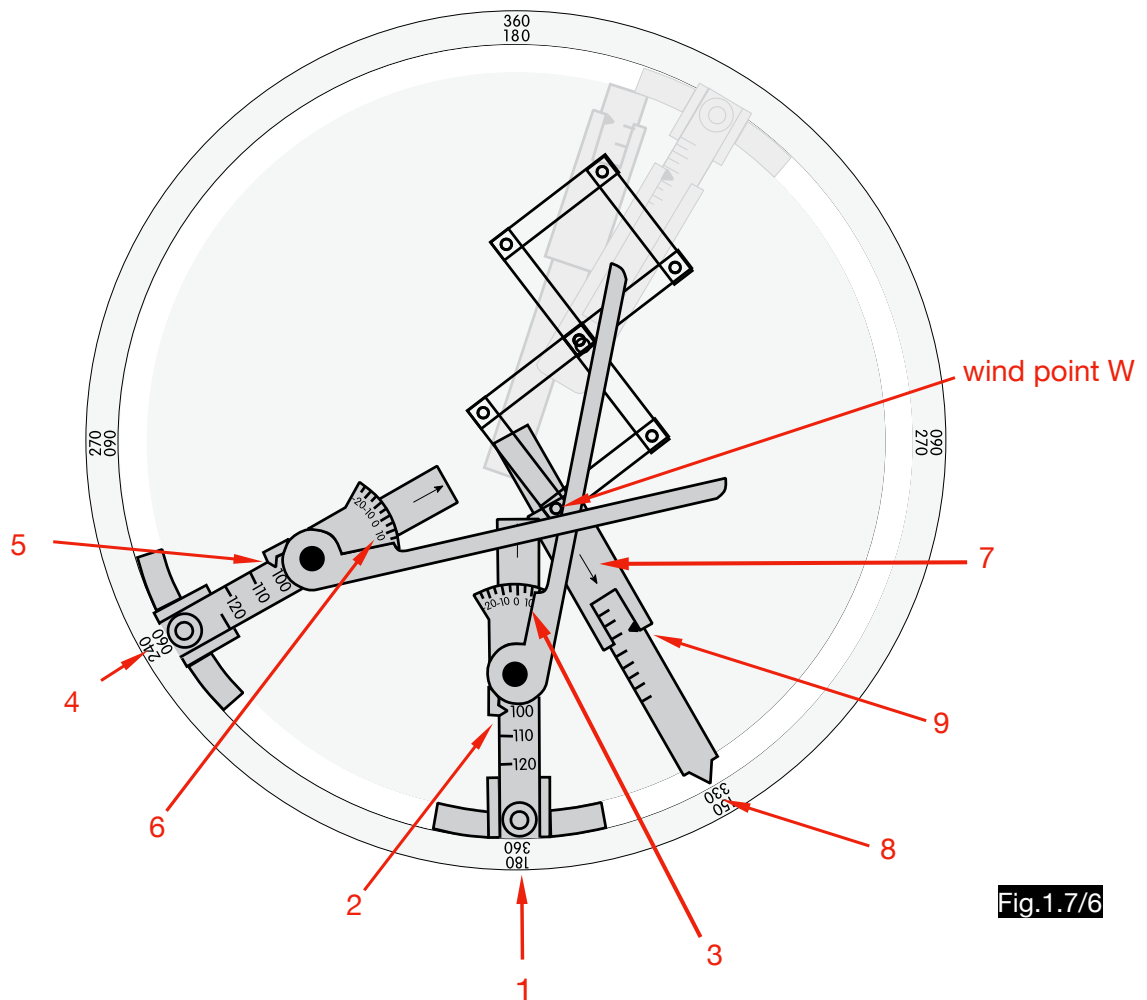


Fig.1.7/6

## 1.8. Calculation of the wind triangle according to Goudime-Levkovitsch

### 1.8.1. Type 1

Paul Goudime-Levkovitsch had several computers patented, all based on the law described in Fig.1.7/2. Fig.1.8/1 shows a drawing of type 1. This variant has the additional adjustable ruler 1 for wind calculations from two drift values (see 1.8.1.2.).

The computer has three wind roses, which are rotatable and arranged next to each other at equal distances. Since the base plate is transparent, the device can also be used as a protractor. The ruler attached to the air speed arm has two linear scales, the beginning of which lies at the pivot point. The two sets of labels on the airspeed arm are arbitrary numbers for the distances  $a$  and  $2a$ , as can be seen below.

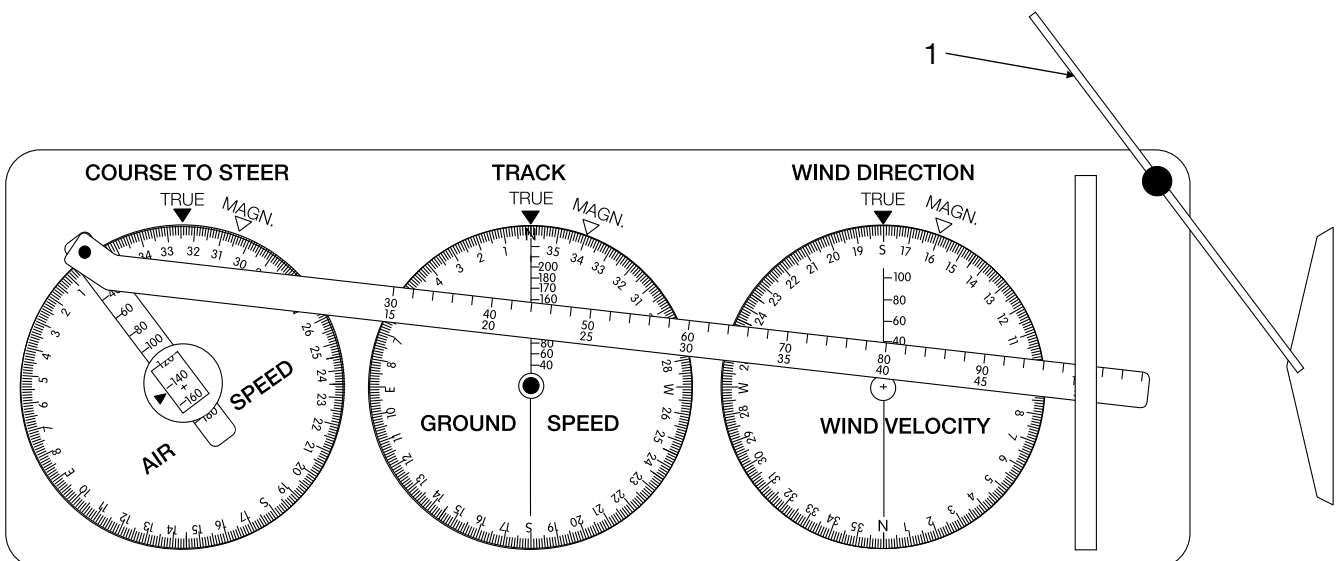


Fig.1.8/1

### 1.8.1.1. Calculation of wind correction angle and ground speed

How to use the computer to calculate wind correction and ground speed is described in this example (see Fig. 1.8/2):

true course	010°
true airspeed	100 kt
true wind	240° / 30 kt
WCA	13.3°
ground speed	116.6 kt

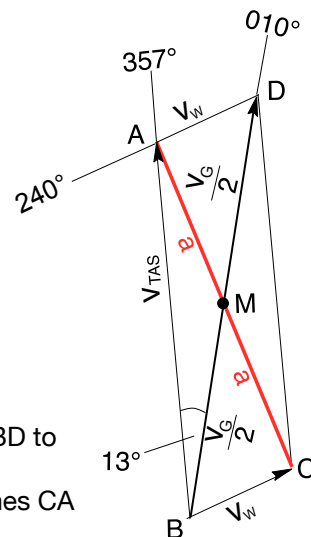


Fig.1.8/2

In Fig.1.8/2 the line BA corresponds to the true airspeed vector, and the line BD to the ground speed vector. The triangle ABD is the wind triangle with the wind correction angle (13°). The diagonals intersect at point M, which halves the lines CA and BD.

Fig.1.8/3 shows the necessary settings on the computer. First the wind direction is set to 240° on the right disc, the course 010° on the middle disc and 100 kt on the speed arm (point A).

Now the ruler has to be set to the value 30 on the wind speed scale (point C) and the speed arm has to be adjusted (point A) so that the distances AM and MC are equal. The coarse and fine scales of the ruler are used for this purpose. The point M is always exactly in the middle between the points A and C, if at point M in the coarse scale the same digit stands as at point C in the fine scale (in the example it is the digit 36). The heading of 357° is read on the left compass rose and the ground speed of 117 kt is read on the middle scale. The division of the ground speed scale is 1:2 to the other two scales, because the distance BM is only half as long as the ground speed vector.

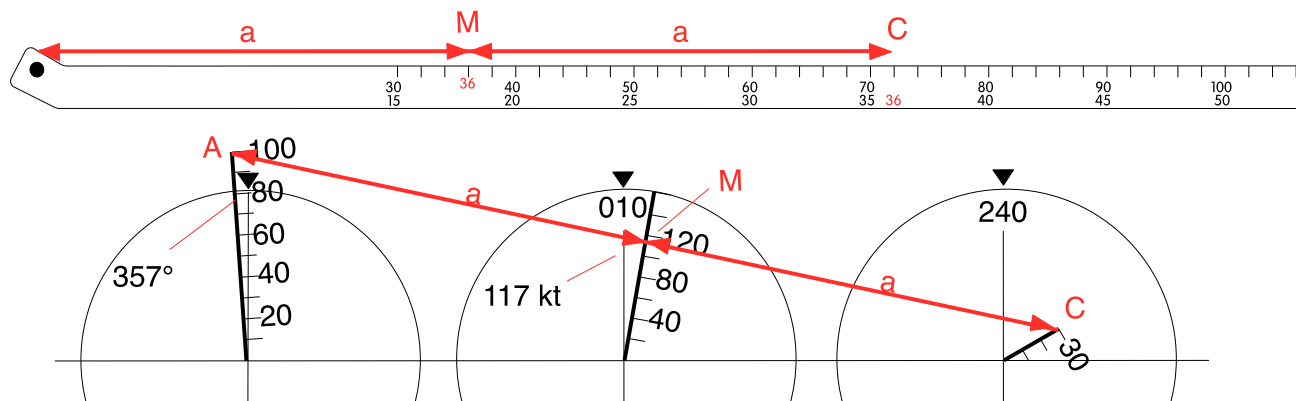


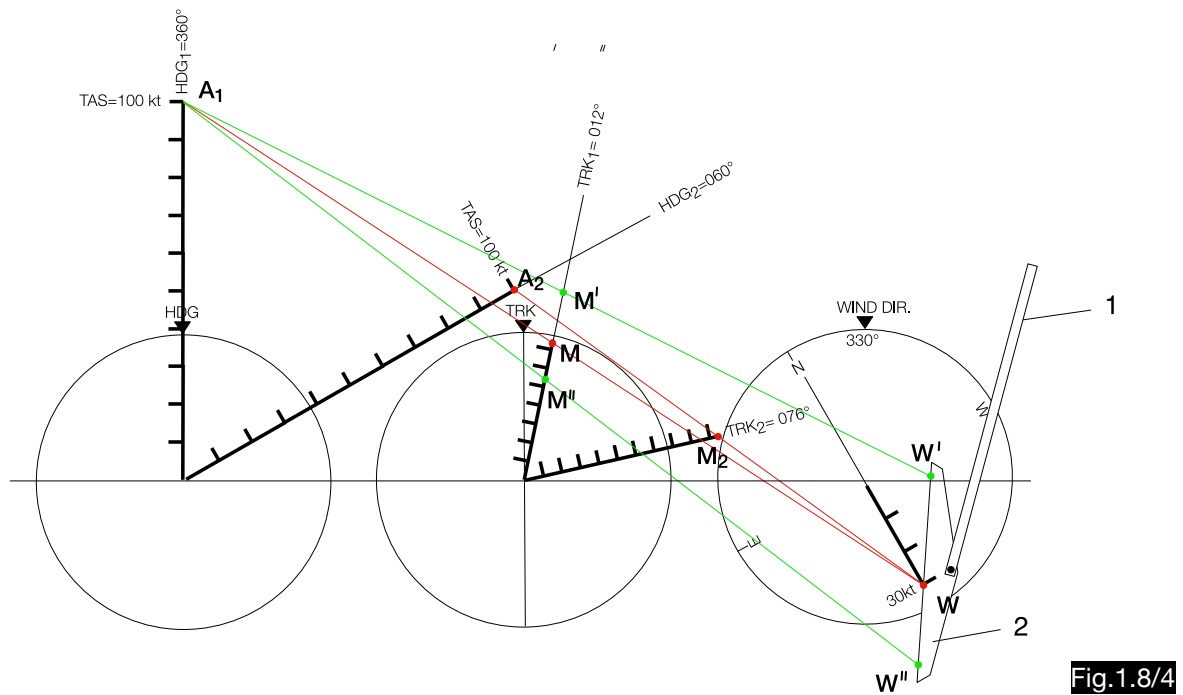
Fig.1.8/3

### 1.8.1.2. Use of the computer for wind determination from two drift angles

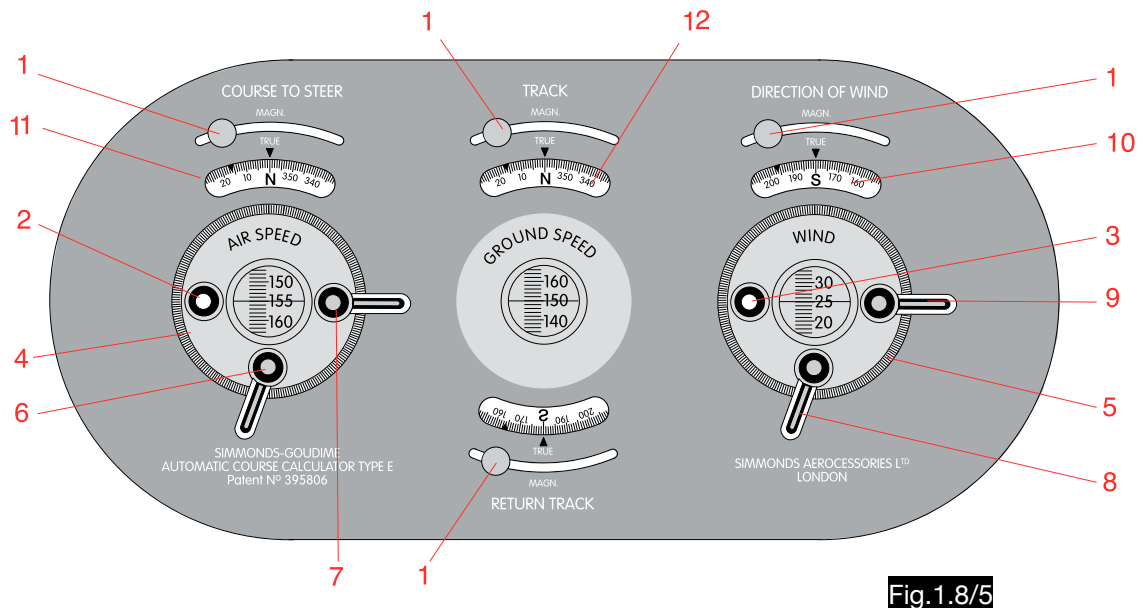
The calculation of wind direction and wind speed is explained by the example shown in Fig.1.7/5 (TAS= 100 kt, DA1 = +12°, DA2 = +17°, TH1 = 360°, TH2 = 060°).

Solution (see Fig.1.8/4):

1. The first heading (360°) is set at the left rose (COURSE TO STEER) and the first track (012°) on the middle rose (TRACK).
2. The wind point W is found, if the connecting line A<sub>1</sub>-M-W is exactly halved by the ground speed vector, i.e. the distance A<sub>1</sub>-M has the same length as the distance M-W (see 1.7.). However, since the ground speed is unknown, this method fails. If, however, one arbitrarily assumes two ground speeds (at points M' and M''), the wind points W' and W'' are found, on whose connecting line the real wind point W lies (see 1.8.2.). The movable ruler 2 is used to mark the connecting line W' - W'' and is fixed at the drawn position.
3. After setting the second heading (060°) on the left rose and the second track (076°) on the middle rose, the ruler 2 can be used to find a point W, which is located on the line W' - W''. The lines A<sub>2</sub> - M<sub>2</sub> and M<sub>2</sub> - W have the same length. The wind point W is used to determine the wind direction 330° on the right rose and the wind speed 30 kt on the wind speed scale.



### 1.8.2. Type 2



#### Design Description

Fig.1.8/5 represents the front view of the computer and Fig.1.8/6 a schematic and side-inverted rear view.

- |  |  |
|--|--|
| 1 variation setting knob                       | 10 right compass rose (wind direction) |
| 2 knob for TAS setting                         | 11 left compass rose (heading)         |
| 3 knob for wind speed setting                  | 12 middle compass rose (track, course) |
| 4 disc for heading setting                     | 13 with knob 3 connected pinion        |
| 5 disc for wind direction setting              | to adjust the wind speed arm 15        |
| 6 lever for locking the TAS setting            | 14 with the knob 2 connected pinion    |
| 7 lever for locking the heading setting        | for adjusting the airspeed arm 16      |
| 8 lever for locking the wind speed setting     | 15 wind speed arm                      |
| 9 lever for locking the wind direction setting | 16 airspeed arm                        |
|  | 17 rack                                |

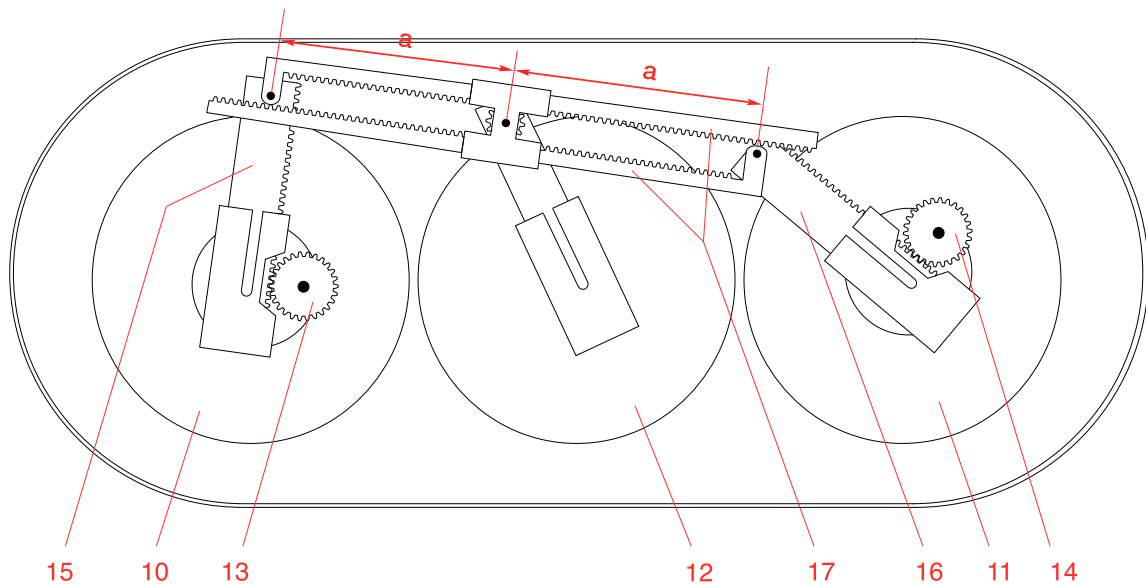


Fig.1.8/6

This schematic representation of the interior of the computer shows its basic structure. The principle of the calculator is based on the procedure described under 1.8.1.1., in which a specially scaled ruler creates the same distance "a" between the peaks of the three vectors true airspeed, wind and track. With the construction used here, the equality of the distances "a" is achieved by two racks which are coupled to each other via a pinion on the ground speed arm. The calculation of wind triangles is thus reduced to the setting of the given values, and the result can then be read off immediately.

### 1.8.3. Type 3

#### 1.8.3.1. General

This type (see Fig. 1.8/7) is a four-vector-computer specially designed to calculate the interception of moving targets in particular vessels. The calculator therefore has four compass roses.

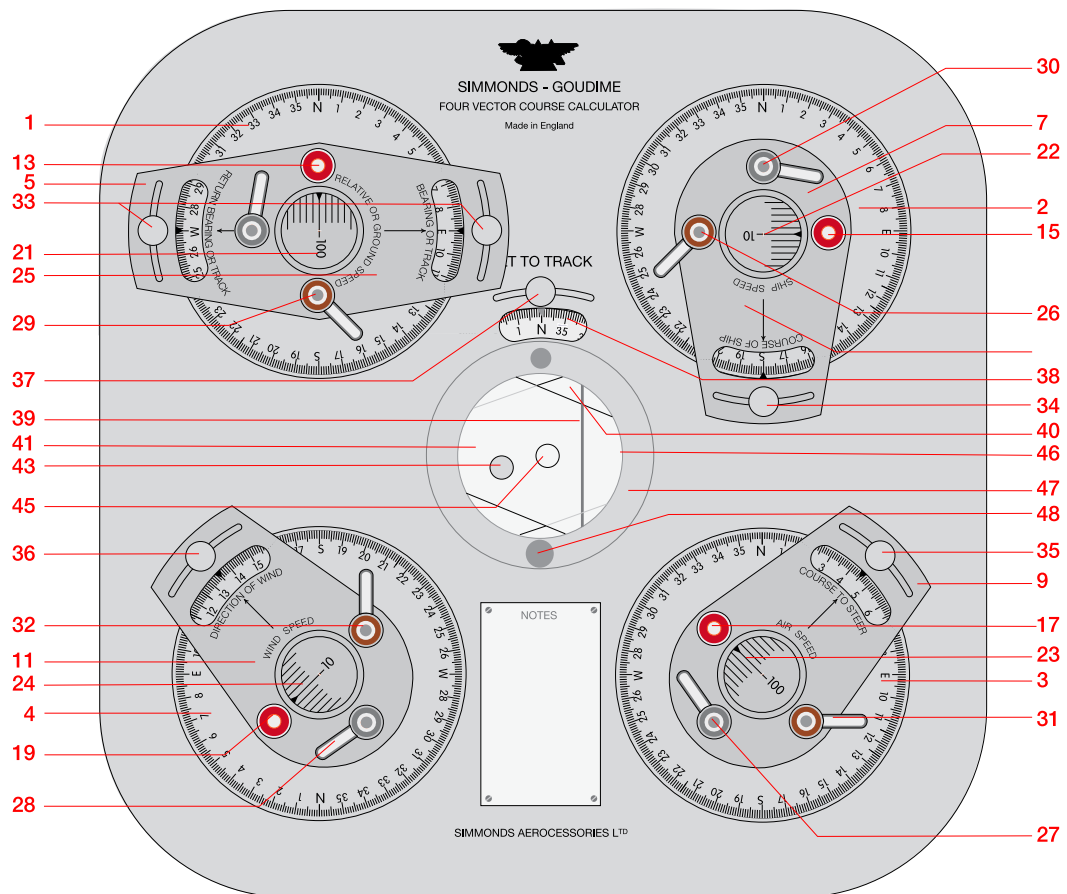


Fig.1.8/7



Fig.1.8/8 is a schematic side-inverted representation of the interior of the computer and shows its basic structure.

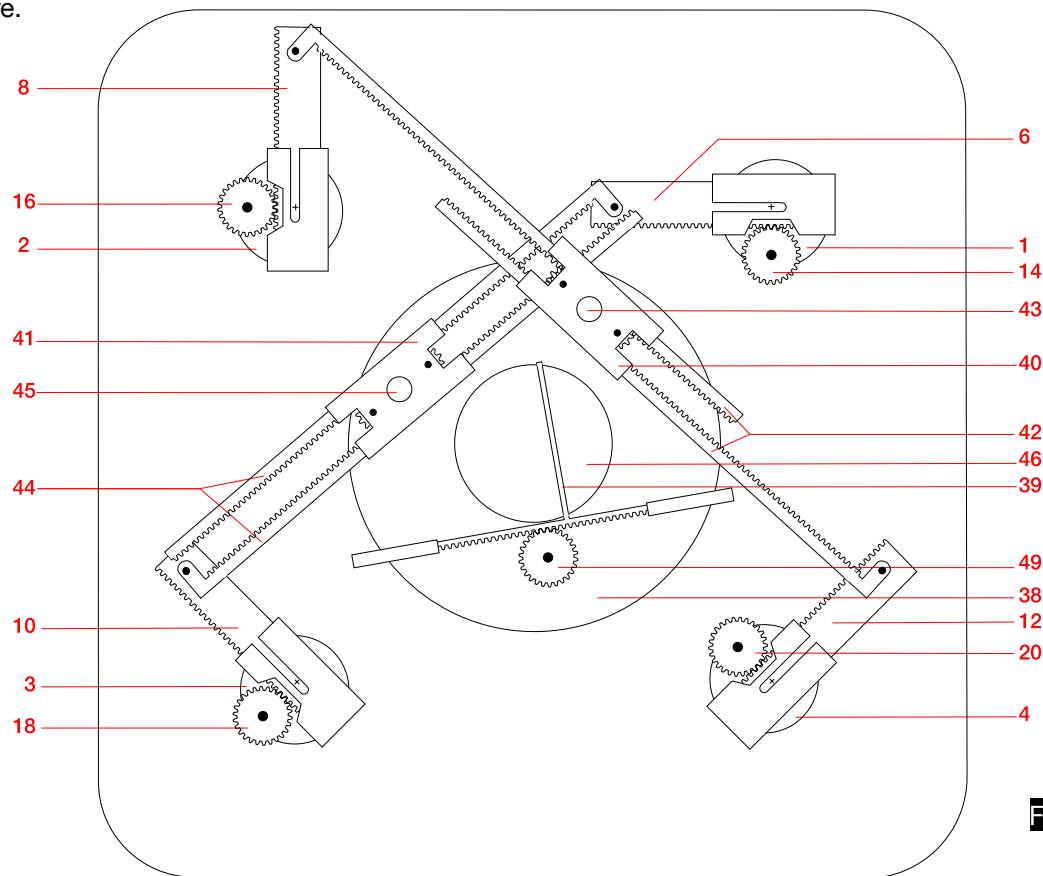


Fig.1.8/8

#### Design Description

- 1 compass rose for setting / reading BEARING or TRACK
- 2 compass rose for setting / reading COURSE OF SHIP
- 3 compass rose for setting / reading COURSE TO STEER (heading)
- 4 course rose for adjustment / reading of the DIRECTION OF WIND
- 5 setting and display element for BEARING OR TRACK and RELATIVE OR GROUND SPEED, connected to arm 6. Using the second course window (RETURN BEARING OR TRACK) one can set BEARING TO (QDM or QUJ) or BEARING FROM (QDR or QTE).
- 6 arm for RELATIVE OR GROUND SPEED
- 7 setting and display element for COURSE OF SHIP (connected to ship speed arm 8)
- 8 arm for SHIP SPEED
- 9 setting- and display-element for COURSE TO STEER (connected to true airspeed-arm 10)
- 10 arm for TRUE AIRSPEED
- 11 setting- and display-element for DIRECTION OF WIND (connected to wind speed arm 12)
- 12 arm for WIND SPEED
- 13 adjustment knob for RELATIVE OR GROUND SPEED (on one axle with pinion 14)
- 14 pinion for longitudinal movement of the arm 6
- 15 adjustment knob for SHIP SPEED (on one axle with pinion 16)
- 16 pinion for longitudinal movement of the arm 8
- 17 adjustment knob for TRUE AIRSPEED (on one axle with pinion 18)
- 18 pinion for longitudinal movement of the arm 10
- 19 adjustment knob for WIND SPEED (on one axle with pinion 20)
- 20 pinion for the longitudinal movement of the arm 12
- 21 scale for RELATIVE OR GROUND SPEED
- 22 scale for SHIP SPEED
- 23 scale for TRUE AIRSPEED
- 24 scale for WIND SPEED
- 25 lever for locking RELATIVE OR GROUND SPEED
- 26 lever for locking SHIP SPEED
- 27 lever for locking TRUE AIRSPEED
- 28 lever for locking WIND SPEED
- 29 lever for locking TRACK / BEARING
- 30 lever for locking COURSE OF SHIP

- 31 lever for locking COURSE TO STEER
- 32 lever for locking DIRECTION OF WIND
- 33 adjustment of the reading line for TRACK / BEARING
- 34 adjustment of the reading line for COURSE OF SHIP
- 35 adjustment of the reading line for COURSE TO STEER
- 36 adjustment of the reading line for DIRECTION OF WIND
- 37 adjustment of the reading line for the track when the double-drift method is used to determine the wind
- 38 compass rose for double-drift method (see note 2)
- 39 pointer for double-drift method
- 40 middle part of the racks 42 with the mark 43
- 41 middle part of the racks 44 is made of transparent material with mark 45
- 42 rack pair 1 (due to the coupling of the racks via two gear wheels the distances between the circle mark and the bearing points at the speed arms are always the same)
- 43 mark on part 40.
- 44 rack pair 2
- 45 mark on part 41
- 46 window for the adjustment of marks 43 and 45 as well as for wind determinations according to the double-drift method
- 47 adjustment ring for the compass rose 38
- 48 adjustment knob for the parallel movement of the pointer 39 (on one axis with the pinion 49)
- 49 pinion for adjustment of the pointer 39

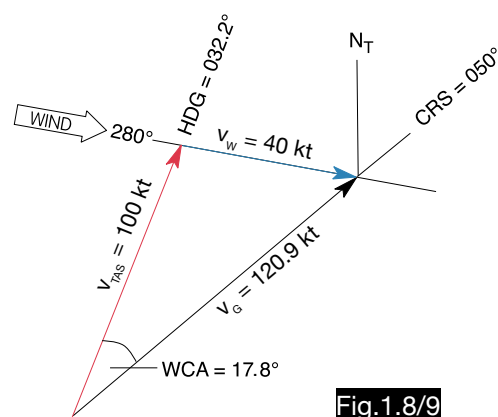
#### Notes:

1. Since it is common practice to indicate the wind according to the direction from which it comes, the course rose is rotated by 180°.
2. For this procedure, the pointer 39 must be aligned parallel to the track. In order to be able to set the adjusting ring 47 to the same value as the tracking element 5, this compass rose is "counter clockwise".
3. The arm 8 for the SHIP-SPEED is mounted in the opposite direction to the adjustment element 7. The explanation for this is given in chapter 2.7.1. (intercepting).

### 1.8.3.2. Use of the computer for wind correction- and ground speed calculations

How to use the computer to calculate wind correction and ground speed is described in this example (see Fig. 1.8/9):

true course	050°
true airspeed	100 kt
true wind	280° / 40 kt
WCA	- 17.8°
ground speed	120.9 kt
true heading	032.2°



The wind correction angle and the ground speed are determined as follows (see Fig.1.8/10):

1. The true course 050° is set and locked at the track element (1).
2. Since a fourth vector does not occur, the ship element is set to zero speed at any course (2).
3. The true air speed 100 kt is set and locked on the airspeed element (3).
4. The wind element is set to 280° and 40 kt and both are locked (4 and 5). With this setting one mark 6 is already fixed in the center window.
5. Only the speed is fixed at the airspeed element and only the direction at the track element. The direction of the airspeed (7) must now be set to a position where the two marks 6 in the window are exactly above each other. To do this, both the adjustment knob 8 and the airspeed element must be turned. Then true heading 032° (at position 7) and ground speed 121 kt (at position 1) can be read.

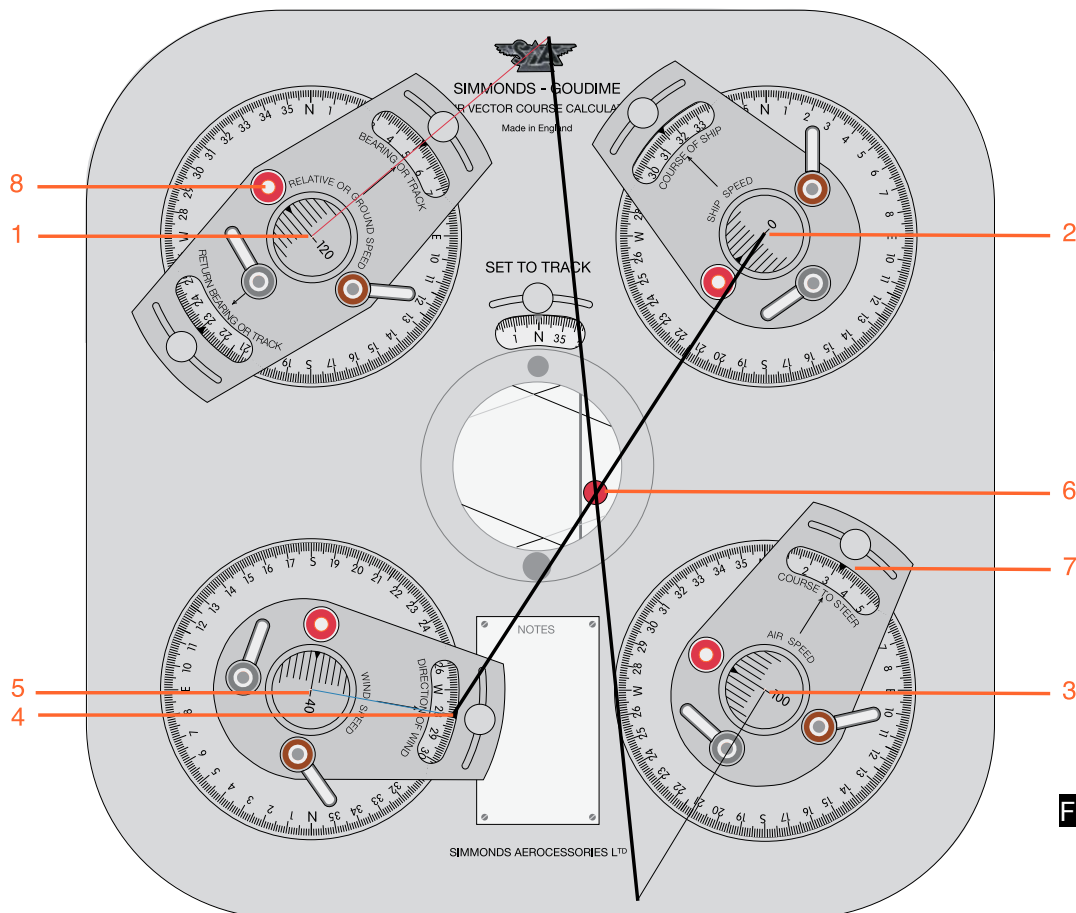


Fig.1.8/10

### 1.8.3.3. Use of the computer for wind determination from drift and ground speed

Example (see Fig.1.8/11):

An airplane departs from point A with constant true airspeed 100 kt and a true heading of 045°. After 15 minutes it reaches point B. Between points A and B the distance of 30 NM and the true track of 060° is taken from the map. This will result in the ground speed 120 kt and the drift angle 15°.

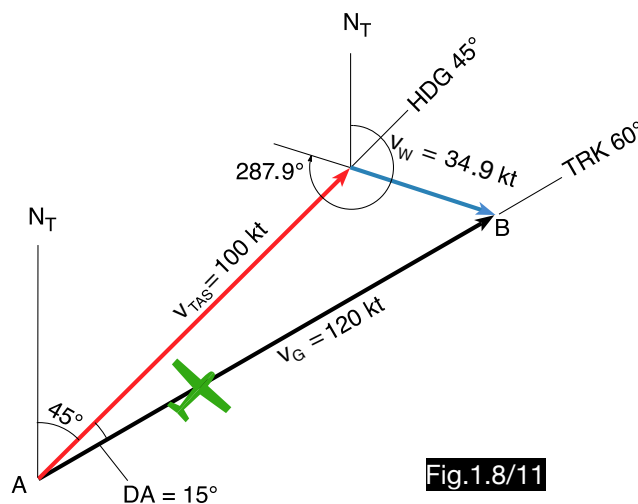


Fig.1.8/11

The following computer settings are necessary (see Fig. 1.8/12):

1. The track 060° and the ground speed 120 kt are set on the track element and both are locked (1).
2. Since a fourth vector does not occur, the ship element is set to zero speed at any course (2).
3. At the airspeed element the true airspeed 100 kt and the heading 045° are set and both are locked (3).  
With this setting one of the two marks 4 in the middle window is already fixed.
4. The wind element 5 must now be brought into a position in which the marks 4 in the window are exactly one above the other. To do this, both the wind direction and the wind speed must be varied using the adjustment knob 6. If both marks are exactly above each other, the wind direction 288° (at position 7) and the wind speed 35 kt (at position 8) can be read.

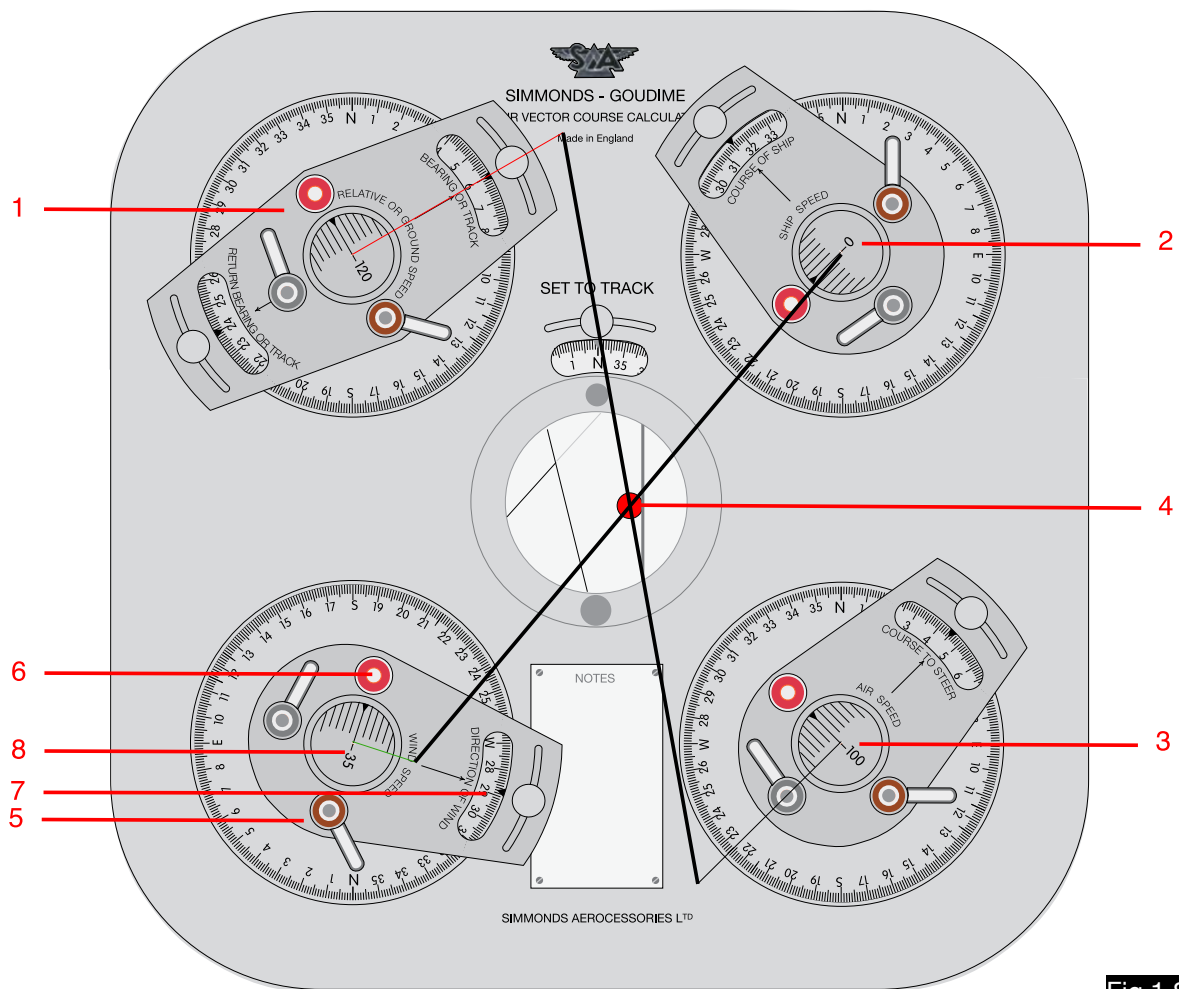


Fig.1.8/12

#### 1.8.3.4. Use of the computer for wind determination from two drift angles

For wind calculations from two drift values (double-drift method) the computer is equipped with an additional compass rose and a pointer in the middle window. The handling is described using the example shown in Fig.1.8/13.

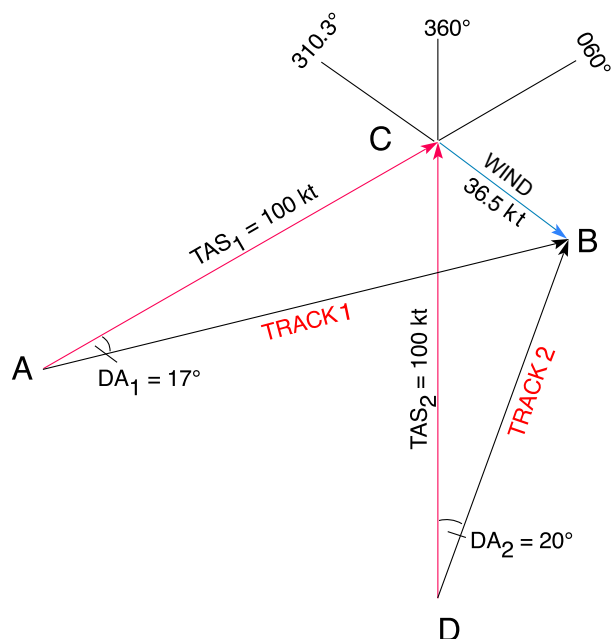


Fig.1.8/13



The two drift triangles ABC and DBC can be completed to the trapezoids ABA'C and DBD'C .

In Fig.1.8/14 correspond

- the line AC to TAS 1,
- the line AB to track 1,
- the line DC to TAS 2,
- the line DB to track 2 and
- the line CB to the wind vector.

The diagonals of the trapezoids intersect at point M, which halves both the diagonals and the wind vector (i.e. MA and MA' have the same length and also MD and MD' have the same length). Thus the direction and size of the wind vector are clearly determined. In the computer, the vectors are simulated by rotating levers of adjustable length. The diagonals are reproduced by pairs of racks, the central part of which has a circular mark always located in the middle between the end points (see Fig.1.8/8).

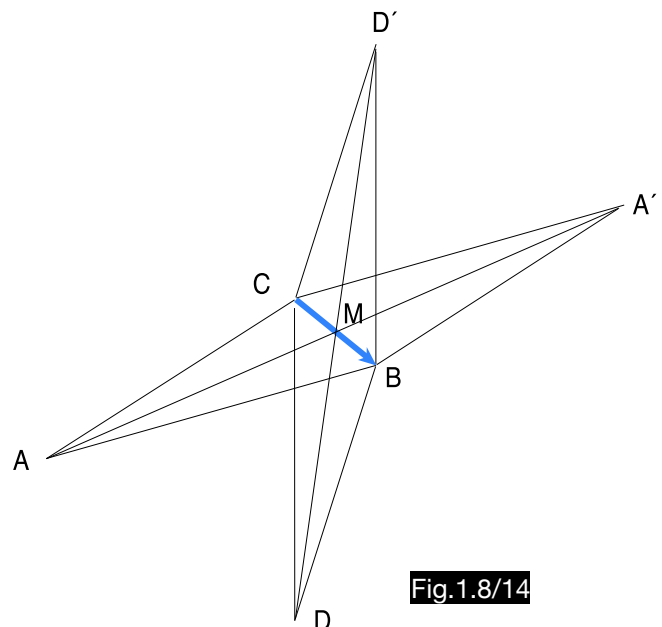


Fig.1.8/14

The following computer settings are necessary:

#### Step 1 (Fig. 1.8/16)

First the air speed arm is positioned on the TAS1 = 100 kt (1) and on the first heading = 060° (2) and both are locked. The track arm is then set to the first track of 077° and locked (7). The position of the middle mark (point M) is not yet determined, because the ground speed 1 is unknown. At Fig.1.8/15 the mechanism of the computer schematically is shown. The ground speed arm is pointing in the direction of track 1 (077°).

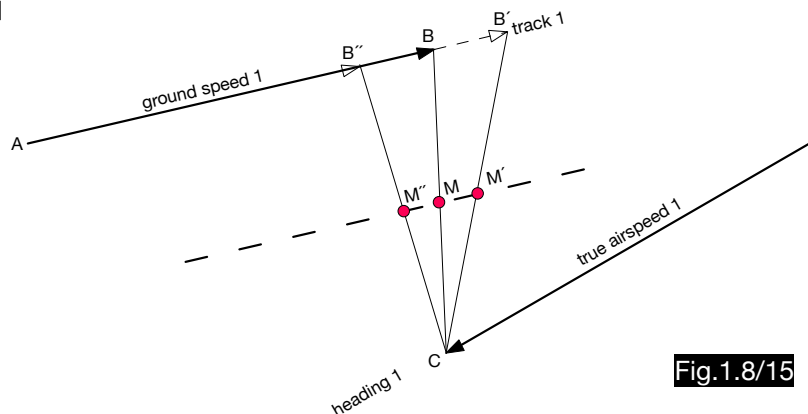


Fig.1.8/15

The true airspeed arm has a length of 100 kt and is pointing in the direction of heading 1 (060°). The line AB represents the unknown ground speed. The line CB corresponds to the rack pair between the two speed arms. The center mark M is located on the rack pair between points C and B. The points B' and B'' stand for arbitrarily assumed ground speeds and the points M' and M'' for the corresponding midpoints. From the intercept theorem follows, that all centers M lie on a parallel to track 1.

#### Step 2 (Fig. 1.8/16)

An arbitrarily selected ground speed (here 100 kt) must be set at the ground speed arm to fix the center point 3 (M in Fig.1.8/15). To set a parallel to the ground speed arm, the pointer 5 must be set to the direction of track 1 by using the scale 6 and over the center point 3 by using the knob 4.

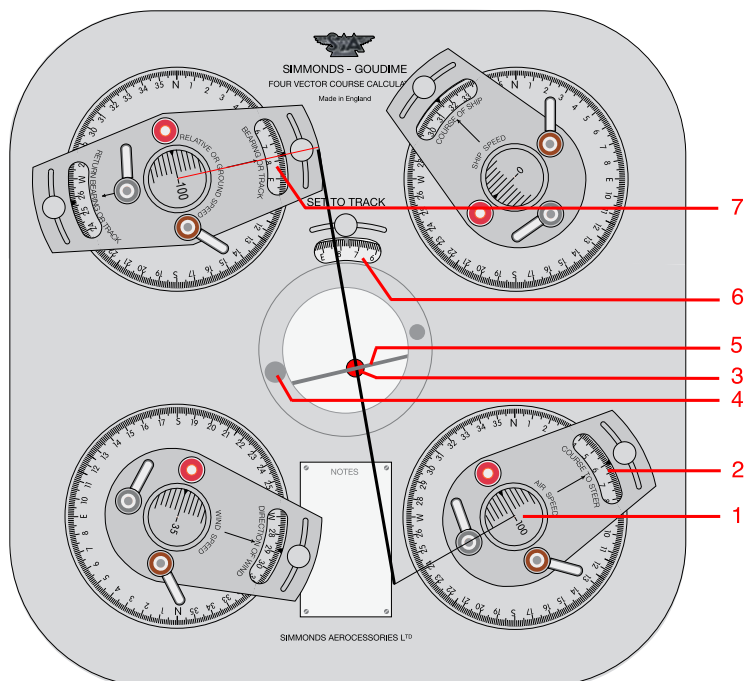


Fig.1.8/16

### Step 3 (Fig. 1.8/17)

The computer, which was constructed for the solution of 4-vector problems, can of course also be used for 3-vector tasks. In this example the element for the course and the speed of the ship is not needed and therefore ship speed zero is used.

The following settings are necessary to consider track 2 for the determination of wind direction and wind speed:

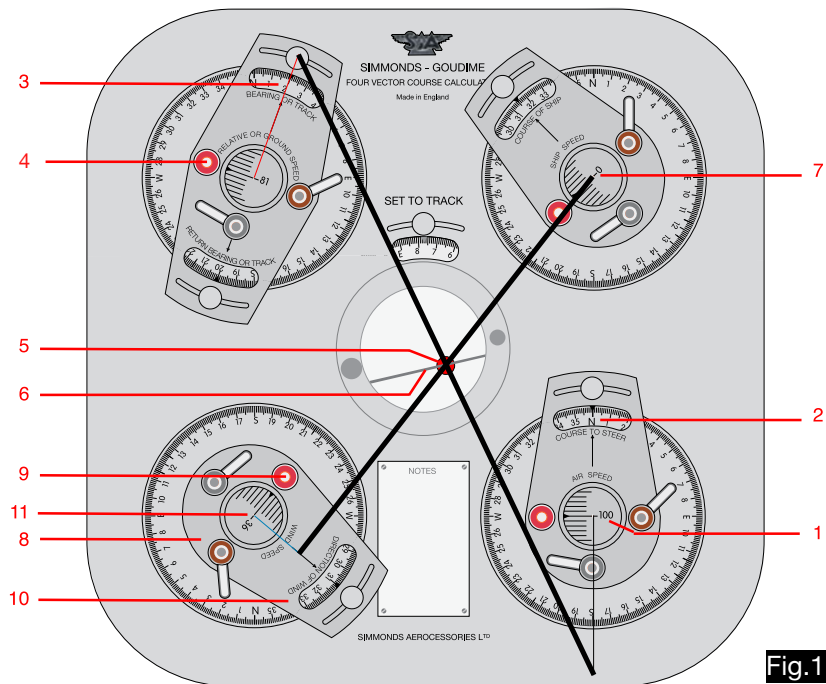


Fig.1.8/17

1. Set the true airspeed 2 (100 kt) and heading 2 (360°) on the airspeed arm and lock both (1 and 2).
2. Set the track arm to track 2 (020°) and lock it (3).
3. Set a ground speed at knob 4, where the circular center mark 5 (that of the rack pair respectively the diagonal track element - air speed element) is below the pointer 6.
4. Set the speed at the ship element (7) to zero.
5. While simultaneously adjusting the arm of the wind direction (8) and the knob for the wind speed (9), an adjustment must be sought at which the second circular middle mark (that of the rack pair respectively the diagonal ship-element - wind-element) is exactly above the other mark 5.
6. The wind direction of 310° (10) and a wind speed of 36 kt (11) can now be read at the wind element.

## 1.9. Calculation of the wind triangle according to Plath

### 1.9.1. Type ATS-4 (Aviator)

#### 1.9.1.1. General

Fig.1.9/1 shows the side of the computer on which the wind triangle mechanically is reproduced. The other side (Fig. 1.9/2) has logarithmic scales for the solution of time-speed-distance-problems as well as a logarithmic sine scale to calculate wind triangles.

#### 1.9.1.2. Example of a mechanical calculation

Fig. 1.9/1 shows the settings for the calculation of this example:

true airspeed	110 kt
true wind	070°/14 m/s
true course	032°

wind correction triangle:

wind corr. angle	+8.8°
ground speed	87.3 kt

drift triangle:

drift angle	- 10.71°
ground speed	90.1 kt

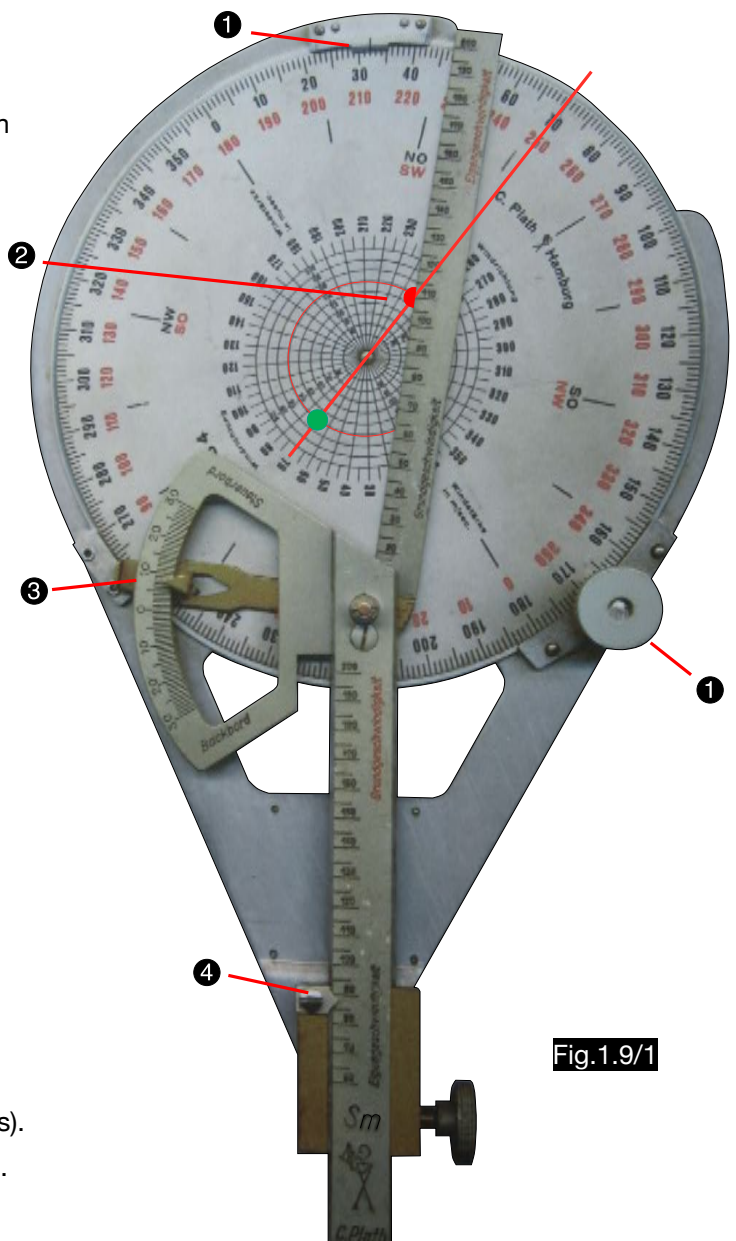


Fig.1.9/1

The calculation of the drift triangle is analogous to the method described above. The calculation also starts with the setting of the course (1). The vertical scale is then set to TAS = 110 kt (4). After the movable arm has been placed against the green wind point, the port drift of 10° can be read at the angle scale. The ground speed of 90 kt can be found at the wind point (green dot in the picture).

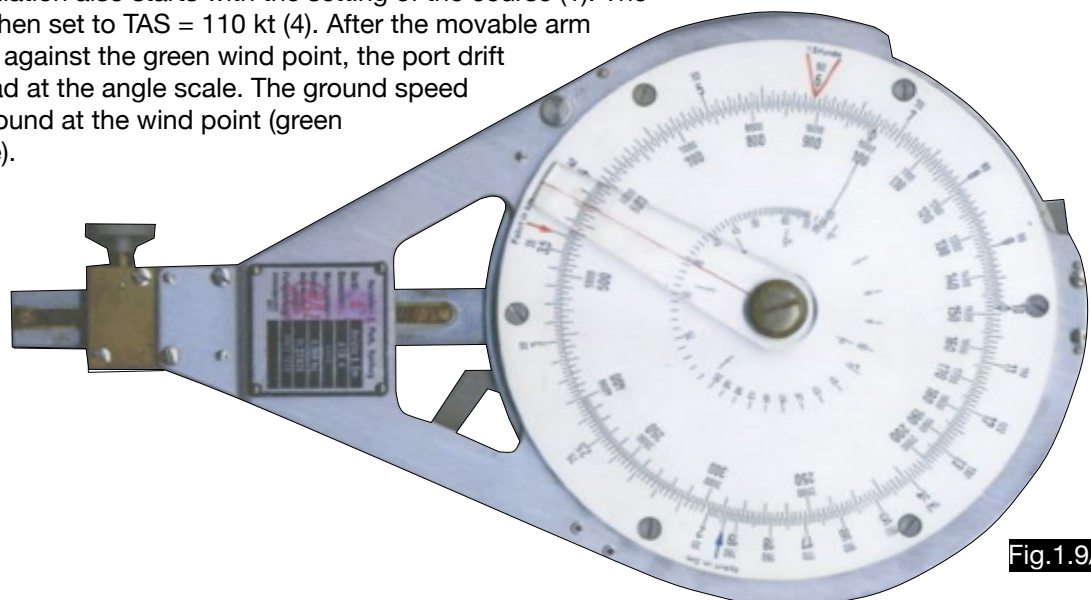


Fig.1.9/2

### 1.9.1.3. Example of a mathematical calculation

In Fig.1.9/3 this example of using the calculator for wind triangle calculations is shown:

wind angle ( $\epsilon$ )	35°
wind speed ( $v_w$ )	200 km/h
true airspeed ( $v_{TAS}$ )	570 km/h
wind corr. angle ( $\alpha_{WCA}$ )	11.6°
ground speed ( $v_G$ )	394.5 km/h

The computer has logarithmic scales in which these settings are made (sine theorem - see equ. 1.1/1 and 2):

$$\frac{v_w}{\sin \alpha_{WCA}} = \frac{v_{TAS}}{\sin \epsilon} \longrightarrow \text{scheme of setting} \longrightarrow \frac{200}{11.6^\circ} \mid \frac{570}{35^\circ}$$

$$\frac{v_w}{\sin \alpha_{WCA}} = \frac{v_G}{\sin (\epsilon - \alpha_{WCA})} \longrightarrow \text{scheme of setting} \longrightarrow \frac{200}{11.6^\circ} \mid \frac{394}{23.4^\circ}$$

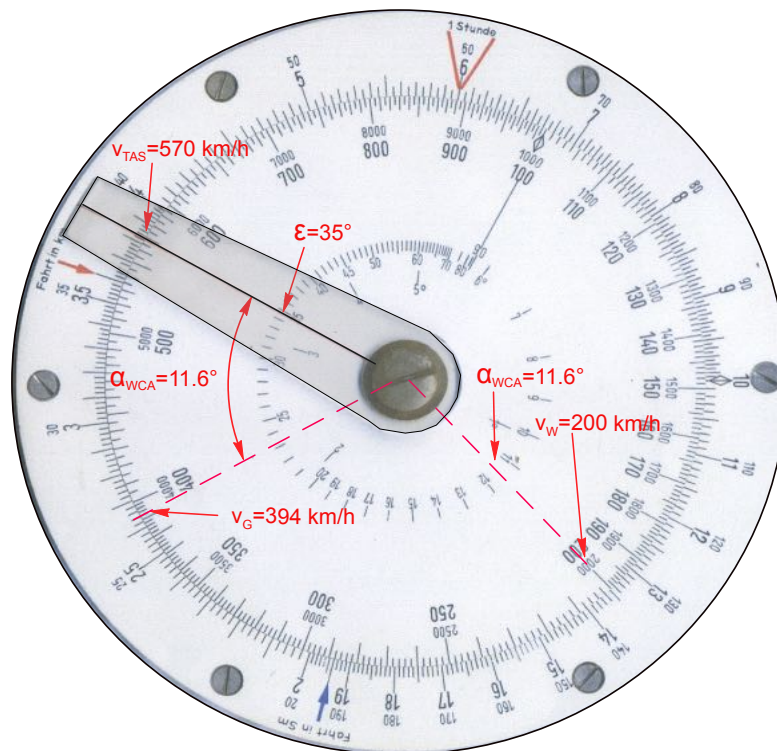


Fig.1.9/3

### 1.9.2. Plotter „Kurskoppler“ Kk7

#### 1.9.2.1. General

The plotter Kk7 (Fig.1.9/4) can be used both for determining courses and distances in the map and for calculating the wind correction angle and ground speed. It is suitable for maps on a scale of 1:1 million, 1:2 million and 1:3 million. The basic body 1 and the compass rose 2, rotatably mounted on it, are made of transparent material. The red arrows 3 support the north orientation. The concentric circles 4 indicate the wind speed. The ruler 5 has 3 scales for the three maps used for distances and speeds.



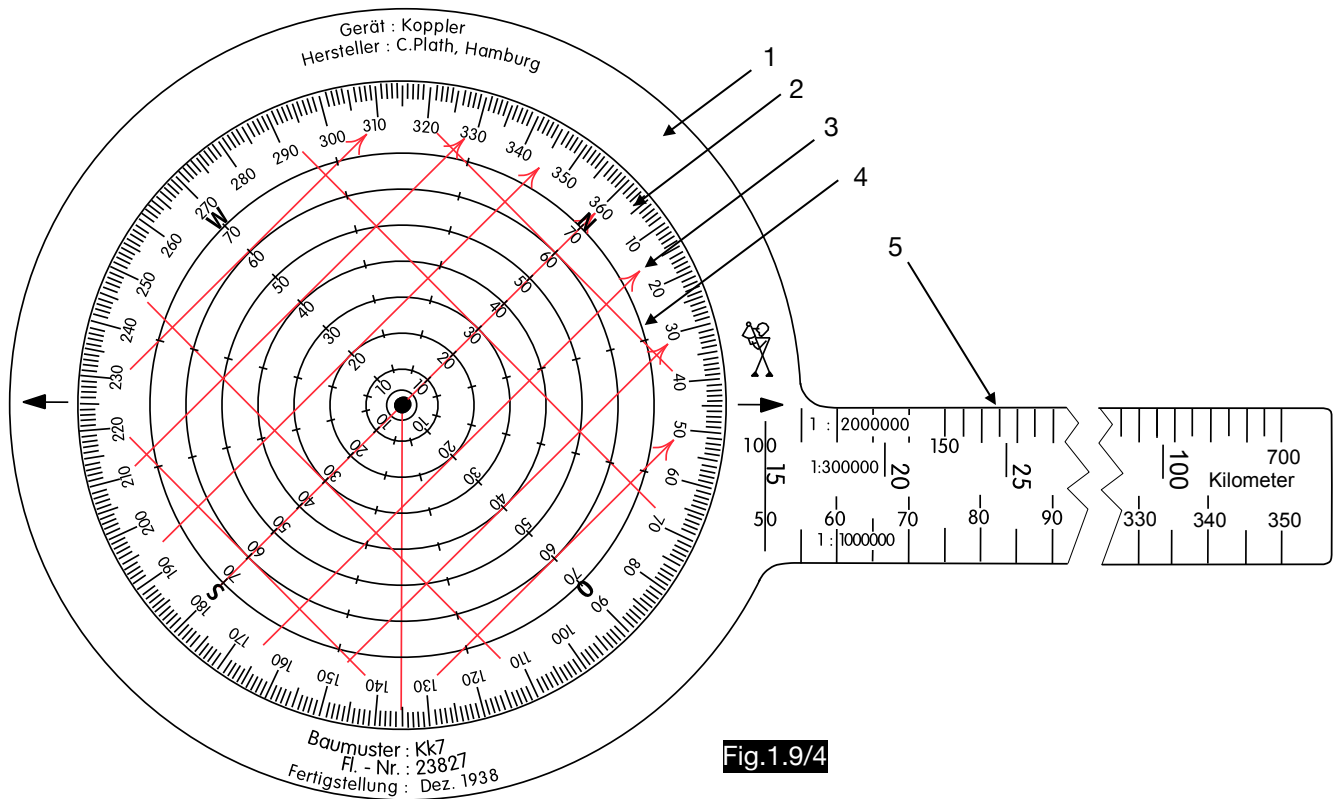


Fig.1.9/4

### 1.9.2.2. Use of the plotter for determining wind correction angle and ground speed

#### Example:

true course	045°		true heading	64.6°
true airspeed	150 km/h		ground speed	141.2 km/h
true wind	135°/50 km/h			

#### Solution (see Fig. 1.9/5 and 6):

1. The route is drawn on the map (line SD).
2. After marking the wind point W (entered in red) on the compass rose and the airspeed mark at 150 on the ruler (red triangle), the wind rose is aligned on the map to true north (red arrows parallel to a meridian).
3. The wind point W is placed over the starting point S.
4. The ruler is rotated until the airspeed mark (red triangle) touches the way line.
5. Point G (opposite the red mark) will be marked at the route in the map (green triangle).
6. GMW and GAB represent the wind triangle, since the distances MG and GA have the length 150 (airspeed), the distances MW and AB have the length 50 (wind speed) and the angles M-W-G and A-B-G correspond to the angle between the course and the wind direction (wind angle). This means that the distances WG and BG must correspond to the ground speed. The angles M-G-W and B-G-A are equal to the wind correction angle, which is not directly readable. The heading can be determined at the intersection of the line MG with the compass rose, because the angle between the lines MC (parallel to the way line) and MG is the wind correction angle. At the arrow on the ruler the true heading of 64° can be read.

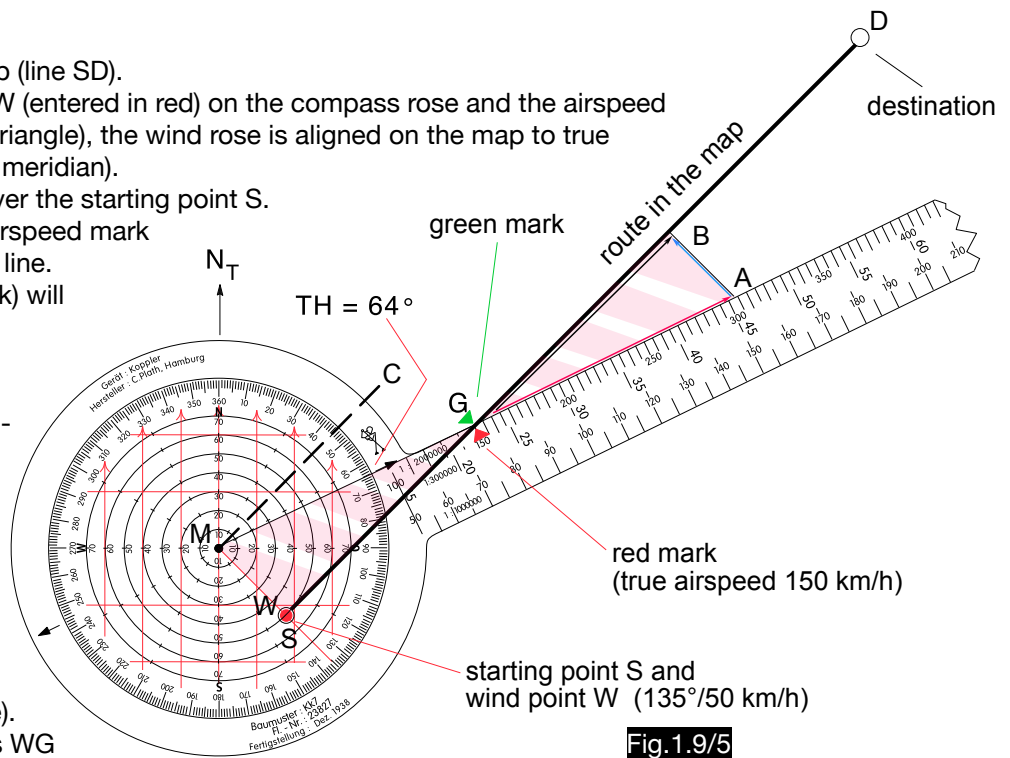


Fig.1.9/5

7. Once the speed ruler has been placed on the route in such a way that the zero point (centre of the compass rose) is on the starting point S, the groundspeed of 141 km/h can be read at mark G (see Fig.1.9/6). The distance SG therefore is completed in a map of the scale 1: 2 million in the time of one hour and in a map of the scale 1:500000 in the time of 15 minutes. The flight time for a 100 km distance can be roughly estimated over the length of the 15-minute section or calculated by measuring the routes S-G (assumed as an example distance 70.6 mm) and S-D (200 mm) in the map using the ratio equation

$$\frac{\overline{SG}}{15} = \frac{\overline{SD}}{T} \rightarrow \frac{70.6}{15} = \frac{200}{T}$$

but also using the speed equation

$$\frac{100 \times 60}{141} = T [\text{min}]$$

In both cases the time is  
T = 42.5 min

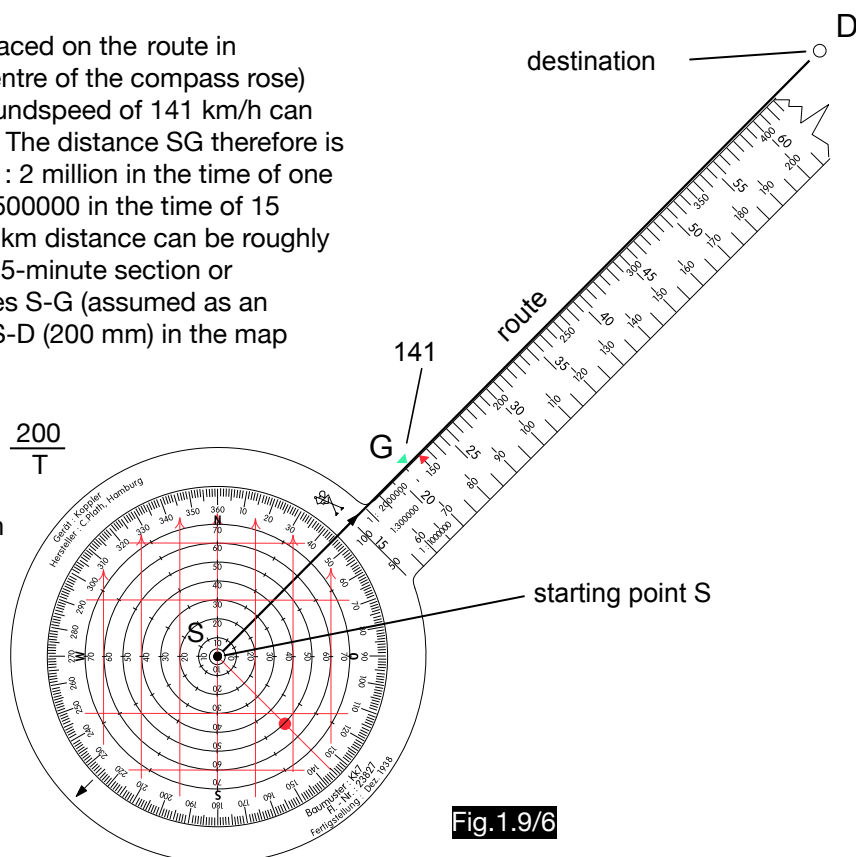
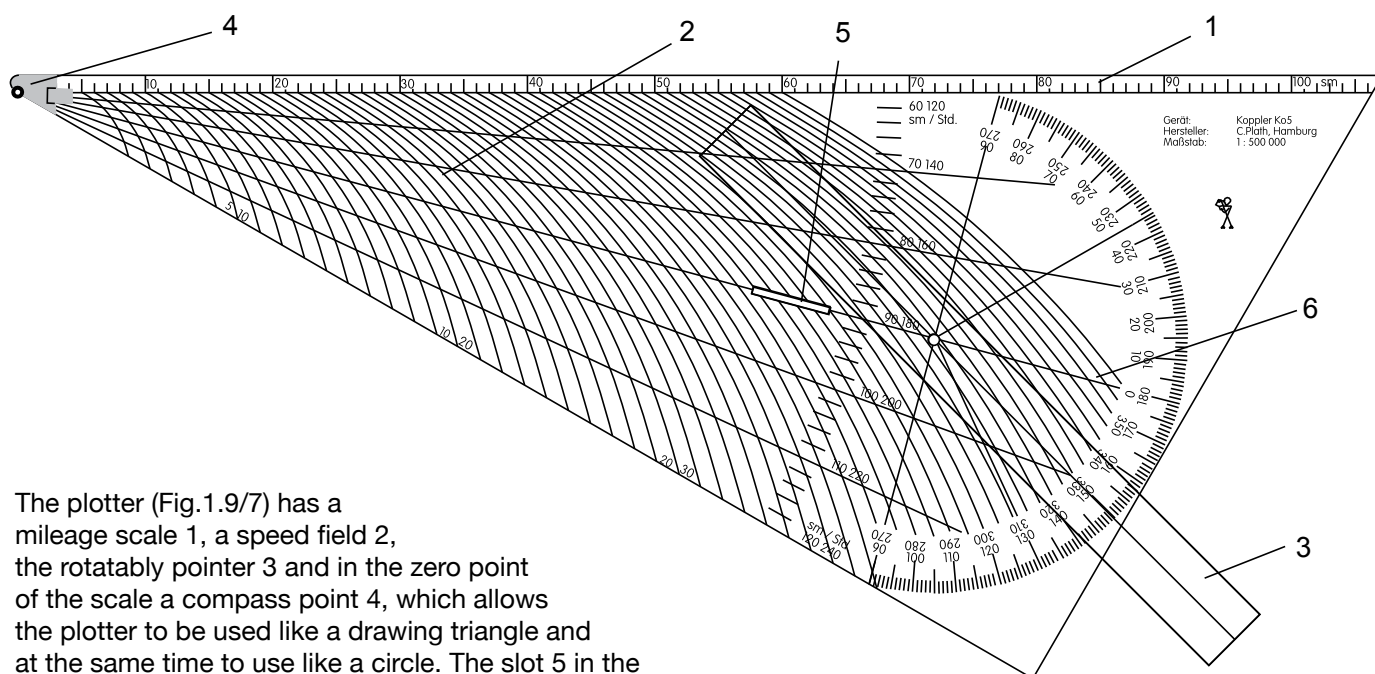


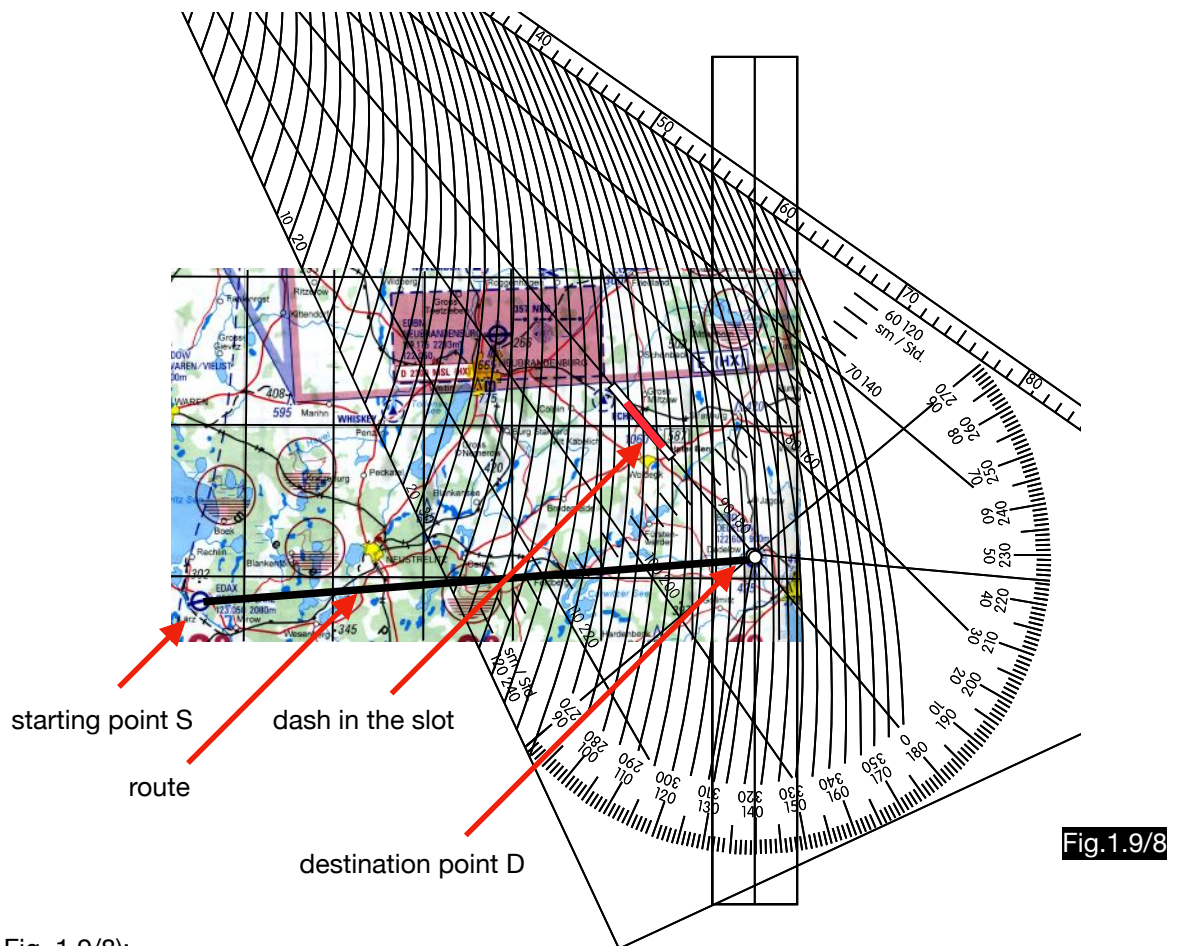
Fig.1.9/6

### 1.9.3. Plotter „Koppler“ Ko 5



The plotter (Fig.1.9/7) has a mileage scale 1, a speed field 2, the rotatable pointer 3 and in the zero point of the scale a compass point 4, which allows the plotter to be used like a drawing triangle and at the same time to use like a circle. The slot 5 in the centre axis 6 serves for the drawing of course marks. The plotter is intended for work with a map of scale 1: 0.5 million. With its help it is possible to graphically determine the data of the wind triangle. An example assumes a true airspeed of 120 kt and a true wind of 320° / 30 kt. The true heading and the flight time are to be calculated. The method is described at the route S-D drawn into the map (black line in Fig.1.9/8).

Fig.1.9/7

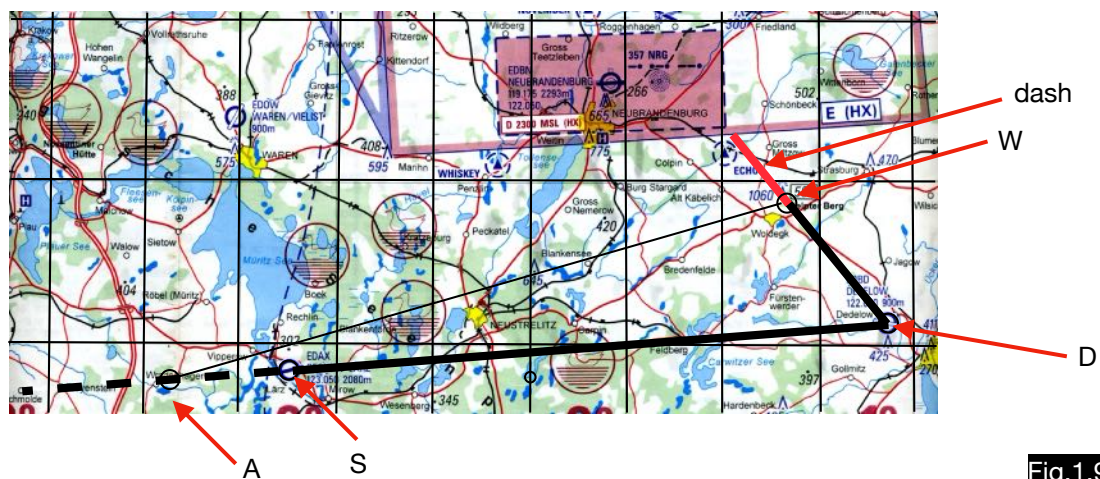


Step 1 (see Fig. 1.9/8):

1. The plotter, whose pointer is set to the wind direction  $320^\circ$ , is placed on the map as follows:  
The center of the compass rose lies on point D (destination) and the pointer is aligned parallel to a meridian.
2. Through the slot a dash is drawn on the map (shown red).

Step 2 (see Fig. 1.9/9):

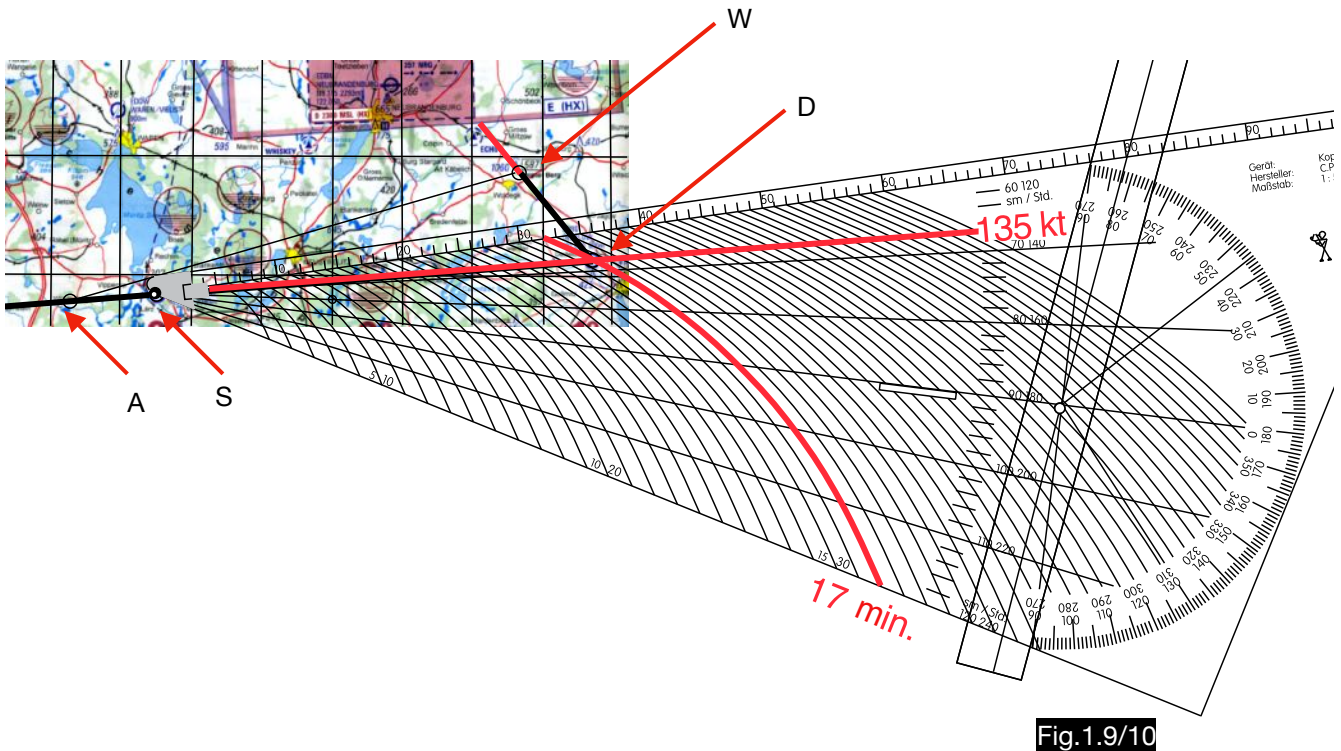
1. The wind vector DW is drawn into the map. For this the mileage scale of the plotter is used. One chooses for the speeds a suitable scale (in this example they were cut in thirds). The point W will be on a straight line in the direction of the red mark (wind from  $320^\circ$ ) at a distance of 10 NM (one third of the wind speed) from point D.
2. Point A is located on the extension of the line SD at a distance of 40 NM (0.3 TAS) from W. When drawing circular arcs, the compass point of the plotter is used. The triangle ADW is now the wind correction triangle in which the distance A-D corresponds to the ground speed vector.





### Step 3:

1. The distance between S and D measured with the mileage scale is 45 NM (multiplied by 3 the ground speed of 135 kt results).
2. The heading is determined by placing the centerline of the plotter on the line AW. After aligning the pointer parallel to a meridian, it shows the true heading of 075°.
3. The flight time is determined in the speed field (see Fig.1.9/10). The ground speed line of 135 kt (highlighted red) must be placed on the route SD in such a way that its zero point is over the start point S. The destination point D then lies on the 17-minute line, which corresponds to the flight time from S to D at a ground speed of 135 kt.



## 1.10. Calculation of the wind triangle according to Bakker

### 1.10.1. General

The Windprotractor (Fig.1.10/1), invented by Olof Bakker is suitable for measuring courses on the map as well as for wind triangle calculations. Although it is suitable for several speeds and map scales, its design is based on the following arbitrary premises:

1. use of maps on a scale of 1: 0.5 million
2. true airspeed 100 kt
3. time interval 8 minutes

In 8 minutes at 100 kt 13.33 NM will be travelled.

This corresponds to a length of 49.38 mm on a map at a scale of 1: 0.5 million. The radius of the transparent disk is therefore 49.38 mm:

$$R = \frac{13.33 \times 1.852 \times 10^6}{500,000} = 49.38 \text{ mm}$$

The scales 1 and 2 are required for map scales other than 1:0.5 million (see 1.10.3)

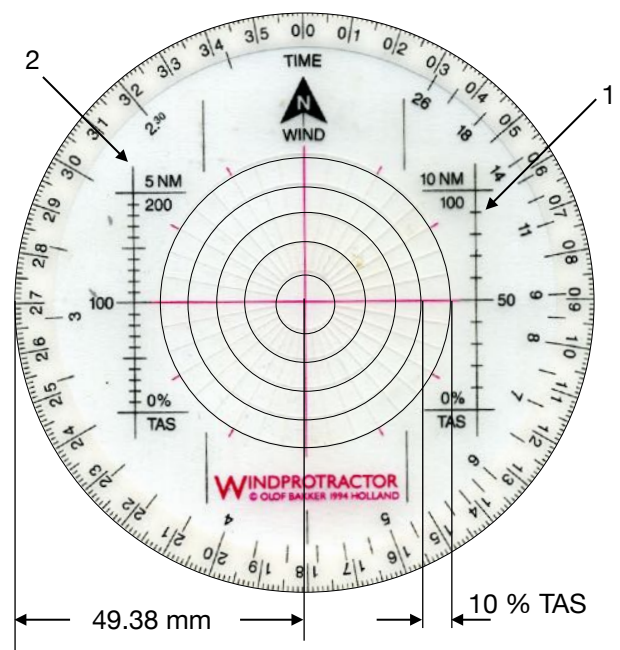


Fig.1.10/1



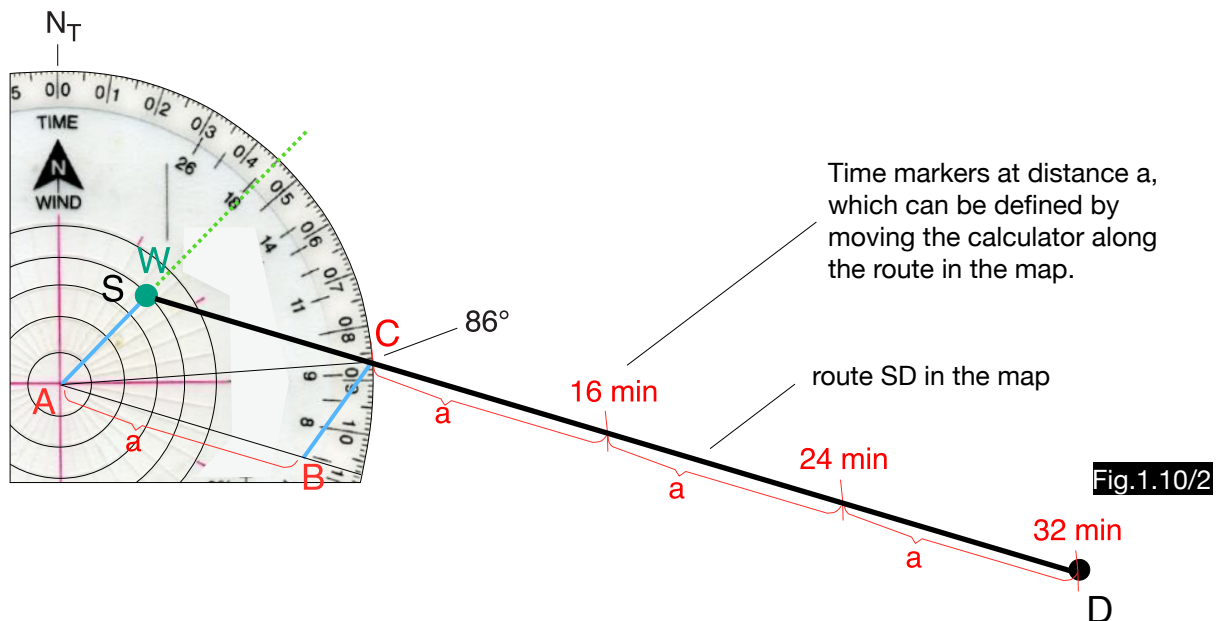
## 1.10.2. Application

### 1.10.2.1. Working with maps of scale 1: 500,000

This example is intended to explain the use of the computer (see Fig.1.10/2):

true course	107°	true heading	86.3°
true airspeed	100 kt	flight time	32 min
true wind	045°/40 kt		
distance	40 NM		

1. Draw the wind point W at 45° on the 4<sup>th</sup> circle (windspeed 40% true airspeed).
2. The computer is then aligned on the map so, that the north arrow is parallel to a meridian and the wind point W is over the starting point S of the flight path SD.  
Since the lines AS and BC correspond to the wind vector and the direction of the lines SC and AB correspond to the course, the triangle ABC is similar to the wind triangle and the angle C-A-B is the wind correction angle. The true heading of 086° (course minus wind correction angle) is therefore directly readable.
3. The distance AC (true airspeed) is of the length  $R = 13.33$  NM and corresponds to the time of 8 minutes. The lines SC and AB correspond to ground speed. Their length  $a$  is therefore the distance that is completed in 8 minutes above ground. In order to determine the flight time, one must determine how often the length  $a$  is included in flight route SD. This can be done by using the disc to enter time marks at intervals from the length  $a$  (8 minutes). In the figure one can see red arcs on the route, labeled with the corresponding flight time. In most cases, the length of the last interval must be estimated.



### 1.10.2.2. Working with maps of a scale other than 1: 500,000

Since the wind speed is calculated in percent of the TAS when drawing the wind point into the computer, the wind triangles are similar and the determination of HEADING or WCA is independent of the airspeed and the map scale. However, this does not apply to the flight time calculation. The distance covered in the time interval of 8 minutes with true airspeed of 100 kt is 13.33 NM. In a map of scale 1: 0.5 million this corresponds to 49.38 mm. Since the calculator has this radius, it cannot be used for determining flight times using other maps and other flight speeds. In the map 1:1 million 13.33 NM correspond only to the half of the length (24.69 mm) and when flying with TAS = 100 kt and using the time interval of 8 minutes one would get only half the flight time. To avoid this, the time interval 4 minutes must be selected for TAS = 100 kt when using the map 1:1 million. If the map 1: 0.25 million is used, the time interval must be doubled (from 8 to 16 minutes). With a true air speed of 200 kt, the map 1 : 0.5 million also shows an incorrect flight time if the time interval of 8 minutes is used, as with doubling the speed a doubled distance is covered or half the flight time is required, so that the 8 minute interval must be halved. The table 1.10/1 shows these relationships:

TAS [kt]		50	100	160	200	240	320
Time[min]	1 : 0,25 Mio.	32	16	10	8	6:40	5
	1 : 0,5 Mio.	16	8	5	4	3:20	2:30
	1 : 1 Mio.	8	4	2:30	2	1:40	1:25

Table 1.10/1

The colored part of the table is displayed on the computer. The 360 degree scale is used simultaneously for the true airspeed and the time interval. The 360-degree rose is used simultaneously to display the assignment of time intervals to speeds. For example, the time interval 8 minutes is displayed beside the 100° mark which also stands for the speed 100 kt. In this way, the calculator is extremely easy to use when using a map 1: 500,000 for all true airspeeds below 340 kt. In order to be able to use other map scales as well, the two additional scales were added to the right and left of the concentric circles (see Fig.1.10/1). For a map with a scale of 1:1 million, the right scale must be placed on a circle of longitude with the angular minutes marked, the distance of which on a great circle corresponds to one mile. Fig.1.10/3 shows that the 10 NM distance (from 50°N to 50°10'N) in a map of the scale 1:1 million in the right scale ends at 50% TAS. This means that at TAS=100 kt the 16-minute time interval must be used (see table). On the protractor the time interval 16 min. is therefore to be found at the number 05. If a map of scale 1: 250,000 is used, the left scale shows that the length of 5 NM leads to the 200 %-TAS mark, so that the time interval can be read at the double TAS. When flying at 160 kt, the time interval to be used is not the 5 minutes listed next to them in the computer, but the 2 minutes 30 seconds opposite the 320 kt (see table). For all other map scales, the procedure shall be applied accordingly.

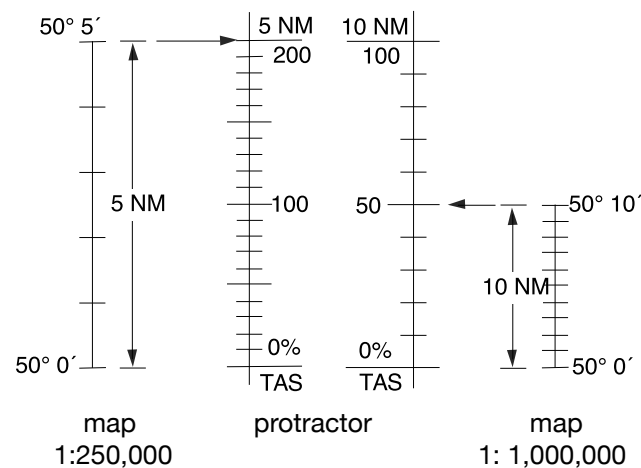


Fig.1.10/3

## 1.11. Calculation of the wind triangle according to Knight

The Knightson computer, (see Fig.1.11/1), works according to the procedure described under 1.10. Each computer is only suitable for one true airspeed (printed at the bottom right) and for use in a map of the scale 1:500,000. The unique time interval of 6 minutes applies to all of them, so that each computer has a different diameter. The computer shown, calculated for 90 kt, has a diameter of 66.66 mm.

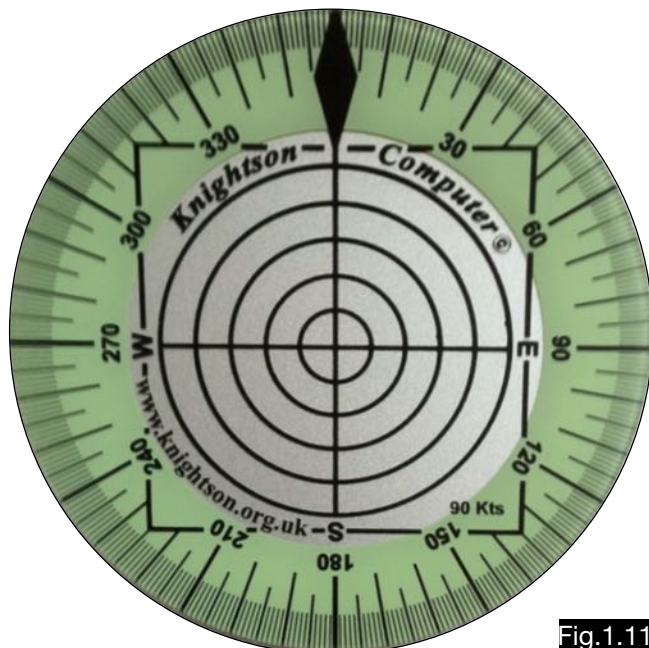


Fig.1.11/1

## 1.12. Calculation of the wind triangle according to Lahr

### 1.12.1. Use of the computer for drift-, wind correction- and ground speed calculations

The calculator, invented by Ray Lahr (see Fig.1.12/2) determines wind correction angle and ground speed using the wind components  $v_{WL}$  and  $v_{WC}$  (see Fig.1.12/1). For smaller wind correction angles ( $<10^\circ$ ) the calculation of the effective true air speed can be omitted in a first approximation. These relations apply:

$\sin \alpha_{WCA} = \frac{v_{WC}}{v_{TAS}}$	$v_G = v_{TAS_{eff}} + v_{WL}$
$v_{TAS_{eff}} = v_{TAS} \cos \alpha_{WCA}$	$v_{TAS_{eff}} \approx v_{TAS}$ valid for $\alpha_{WCA} < 10^\circ$

These equations are calculated in logarithmic scales using these settings:

$\sin \alpha_{WCA}$	$1(\sin 90^\circ)$	$\cos \alpha_{WCA}$	$1(\cos 0^\circ)$
$v_{WC}$	$v_{TAS}$	$v_{TAS_{eff}}$	$v_{TAS}$

The following example explains the handling:

true wind	210°/40 kt	wind corr. angle	- 11.5°
true airspeed	100 kt	ground speed	132.6 kt
true course	360°		

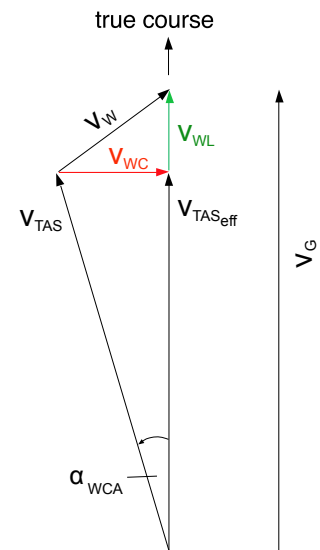
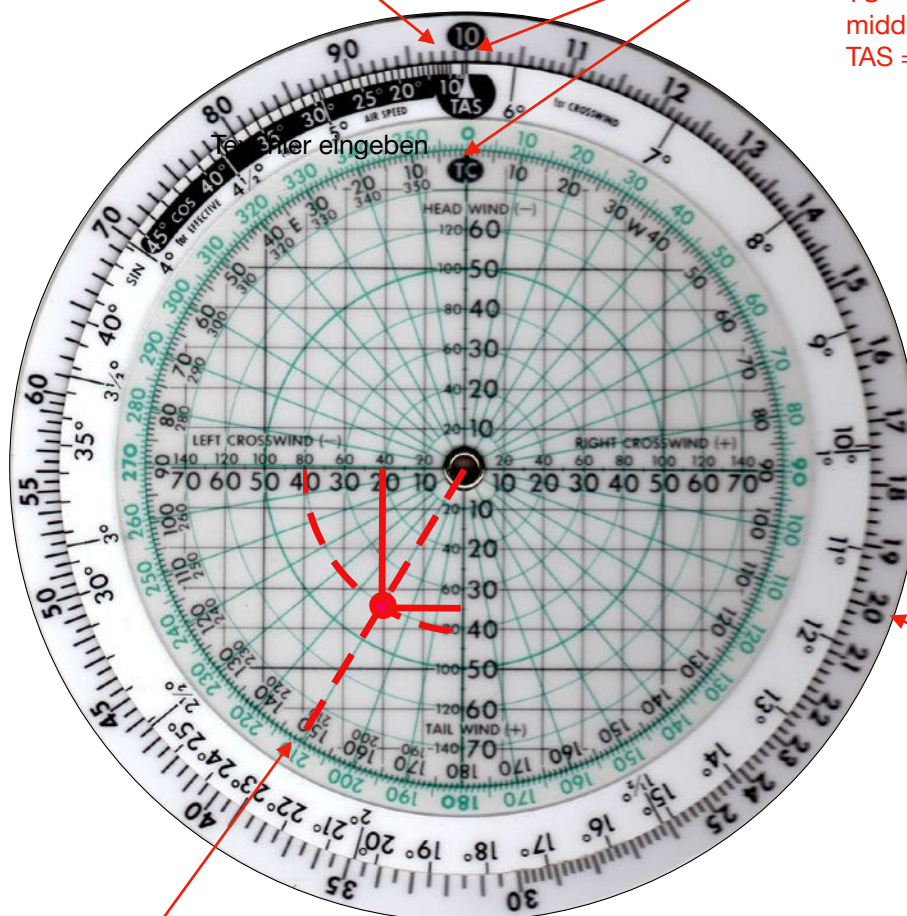


Fig.1.12/1

4. In the cosine scale the effective true airspeed of 98 kt is found opposite to the  $WCA = 11.5^\circ$ . The ground speed is thus  $98 + 35 = 133$  kt.

1. Adjust the inner disc to true course  $TC = 360^\circ$  and the middle disc to  $TAS = 100$  kt.



3. Opposite the cross wind component  $v_{WC} = 20$  kt the wind correction angle  $11.5^\circ$  is found.

2. The wind point is marked at the intersection of the 40 kt-wind-circle with the wind direction  $210^\circ$ . The wind components  $v_{WL}$  of 35 kt and  $v_{WC}$  of 20 kt are determined using the grid.

Fig.1.12/2



## 1.12.2. Use of the computer for wind determination from drift and ground speed

Example (see Fig.1.12/3):

true course	175°
wind correction angle	-15°
true heading	160°
true airspeed	180 kt
ground speed	144 kt

true wind direction	117.7°
wind speed	55.3 kt

To calculate the wind vector BD, its longitudinal component BC (in the direction of true course) and its cross wind component CD (right-angled to it) are first determined. For small wind correction angles, the difference between groundspeed and true airspeed is used as the longitudinal wind component. For larger wind correction angles, as in this case, the longitudinal wind component must be calculated as follows:

$$\overline{BC} = \overline{AC} - \overline{AB}$$

$$\overline{AC} = \overline{AD} \times \cos \alpha_{WCA}$$

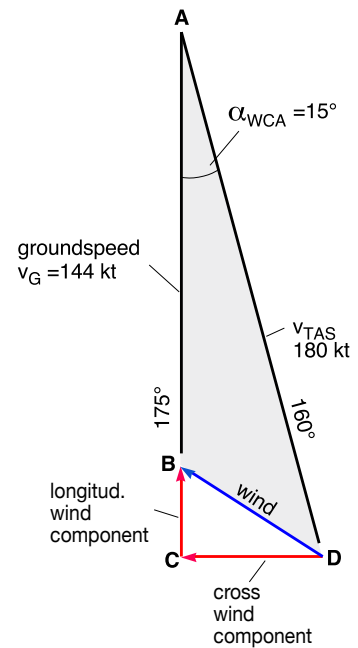


Fig.1.12/3

The following settings and readings on the computer result in a true wind direction of 118° and a wind speed of 55 kt (see Fig.1.12/4):

1. The true airspeed (180 kt) is multiplied by the  $\cos 15^\circ$  and the distance AC is 174 kt. The longitudinal wind component BC is  $174 - 144 = 30$  kt.
2. After setting the TC of  $175^\circ$ , at 30 kt head-wind a horizontal line to the left (wind from the left) is drawn on the compass rose.
3. Opposite to the  $\sin 15^\circ$  (WCA) the cross wind component of 47 kt is determined.
4. The vertical line upwards drawn at 47 kt on the left-crosswind side crosses the headwind component in the wind point, which lies on the 55 kt speed circle (wind speed 55 kt).
5. From the center of the wind rose a line is drawn through the wind point, which marks the true wind direction of  $118^\circ$ .

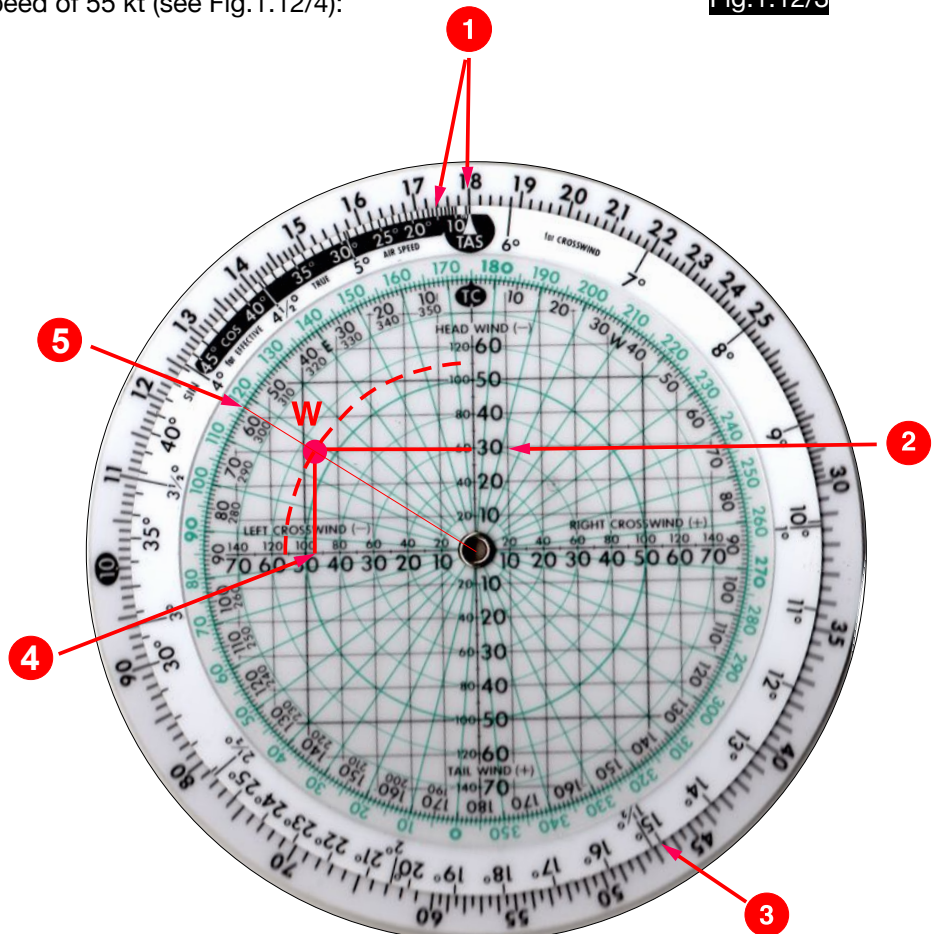


Fig.1.12/4



## 1.13. Calculation of the wind triangle using the computer NR-1

### 1.13.1. General

With the calculator NR-1 (see Fig. 1.13/1 - diameter 25 cm) the splitting of the wind vector into two components is applied, which is already described under 1.12. With this calculator the wind correction angle can be read directly and the calculation of an effective true air speed is not necessary. The computer displays the wind vector in the correct length and position. Its longitudinal component is indicated on the vertical scale. The horizontal cross component is used to determine the wind correction angle according to this formula:

$$\sin \alpha_{WCA} = \frac{V_{WC}}{V_{TAS}}$$

The WCA values can be read in a graph whose vertical scale represents the true airspeed and whose horizontal scale represents the cross component. For better readability and accuracy, the cross wind bar has a smaller scale. Fig. 1.13/2 illustrates the construction of the graph using the example of the line for the wind correction angle 3°. Since the method is not based on the reproduction of the wind triangle, the calculation of the drift triangle is not possible. Note also that wind corrections to the left have positive signs on this calculator.

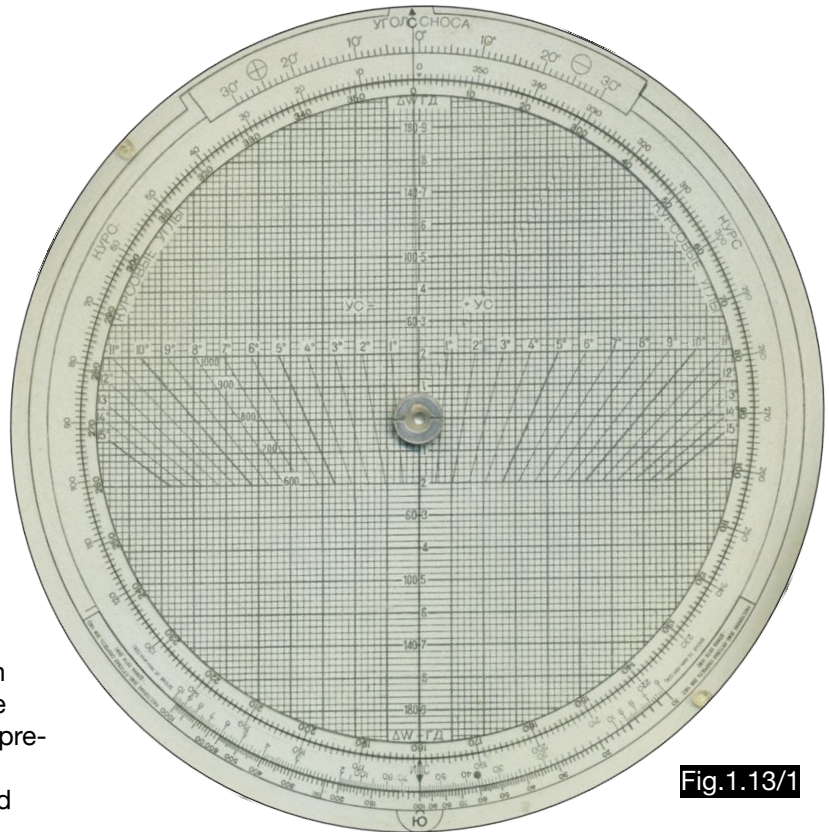


Fig.1.13/1

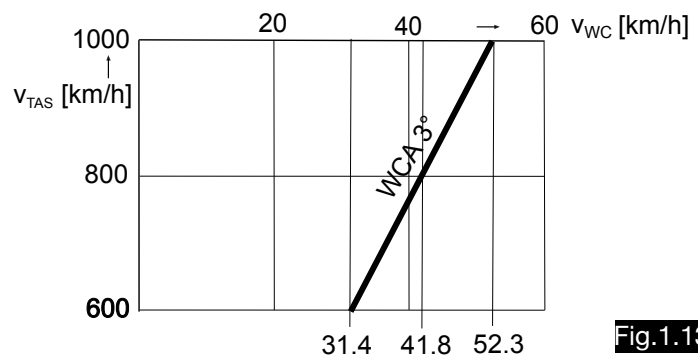


Fig.1.13/2

### 1.13.2. Use of the computer for wind correction- and ground speed calculations

The following example explains the application:

true course	360°
true air. speed	800 km/h
true wind	225°/120 km/h

wind corr. angle	- 6.1°
ground speed	880.2 km/h

#### Step 1 (Fig. 1.13/3):

The wind point W is marked on the transparent disc. To do this, set course 225° and mark a point at wind speed 120 km/h on the downward-facing wind scale.

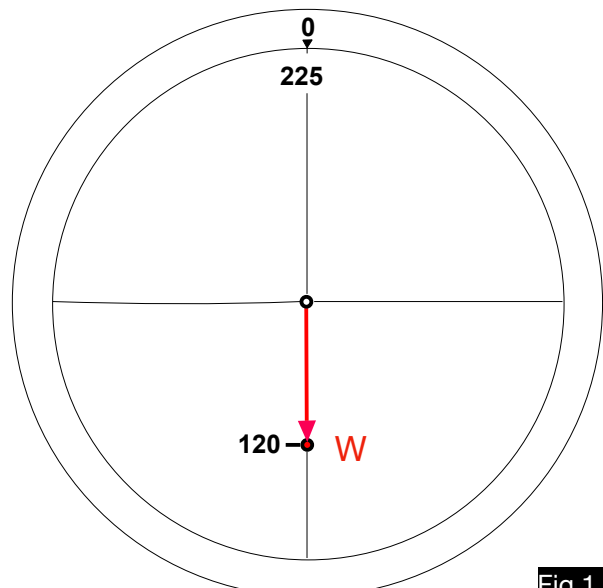
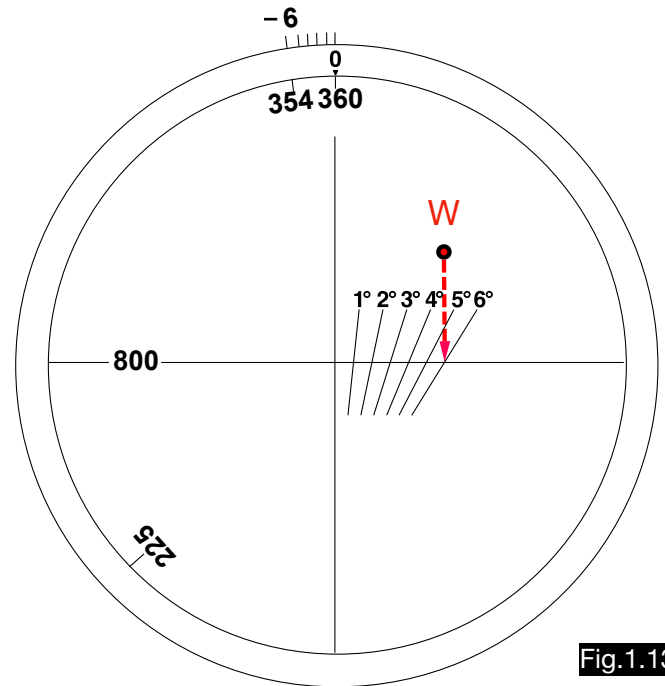


Fig.1.13/3

**Step 2 (Fig. 1.13/4):**

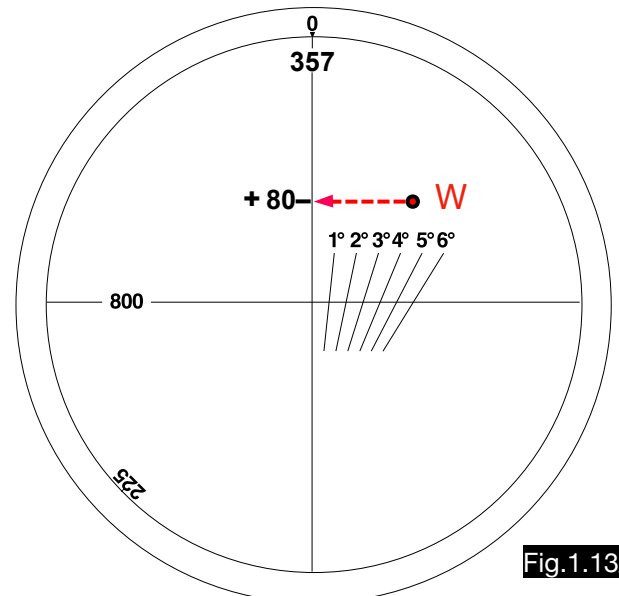
The disc is set to the course 360°. At the intersection of the line TAS = 800 km/h and the vertical line through the wind point one finds the WCA = - 6°. At the upper scale the required heading 354° can be determined.



**Fig.1.13/4**

**Step 3 (Fig.1.13/5):**

The value 357° (course 360° minus 0.5 WCA) is set on the transparent disc. The value of + 80 km/h (ground speed minus true airspeed ) can be seen on the wind speed scale by means of a horizontal line through the wind point. Therefore, 880 km/h are determined as groundspeed.



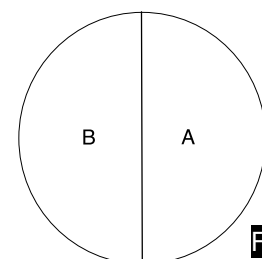
**Fig.1.13/5**

The rotation of the disc by half the amount of the WCA is a good approximation to increase or decrease the longitudinal component of the wind vector so that the correct ground speed is obtained after addition to the true airspeed. Table 1.13/1 and Fig. 1.13/6 show the necessary direction of rotation.

Analytical calculations demonstrate that the resulting errors are negligible (see Table 1.13/2).

wind point in sector A	rotation of the disc by 0.5 WCA	clockwise
wind point in sector B		counter- clockwise

**Table 1.13/1**



**Fig.1.13/6**

relative windangle	ground speed calculated according to the sine theorem	ground speed analytically calcu- lated over the longitudinal com- ponent using the NR-1 method
10°	119.635	119.632
45°	113.14	113.10
90°	97.98	97.98

**Table 1.13/2**

### 1.13.3. Use of the computer for wind determination from two drift values

The calculation of wind direction and wind speed from two drifts is explained by the following example:

true heading 1	360°
true heading 2	090°
drift angle 1	+7°
drift angle 2	+10°
true airspeed	660 km/h
true wind direction	333.0°
wind speed	143.6 km/h

#### Step 1 (Fig.1.13/7):

At the transparent disk the first track 007° (HDG 1 + DA 1 = 360 + 7) is set opposite zero. A vertical line is drawn from the intersection of the line 660 km/h with the wind correction angle 7° (shown in red).

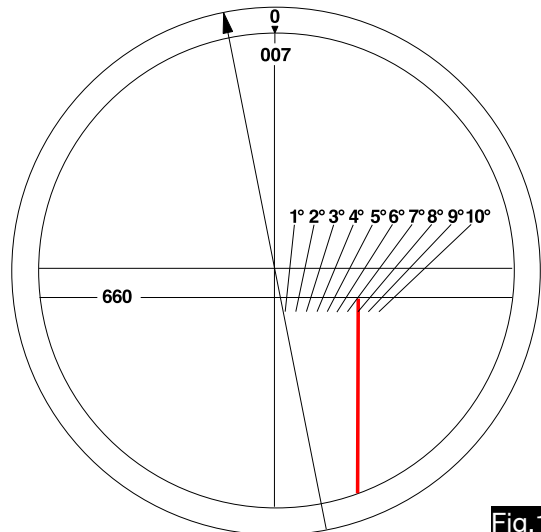


Fig.1.13/7

#### Step 2 (Fig.1.13/8):

At the transparent disc, the second track 100° (HDG 2 + DA 2 = 90 + 10) is set opposite to zero. A further vertical line is drawn from the intersection of the line 660 km/h with the WCA 10° (shown as a dashed red line). The wind point lies at the intersection of the two red lines.

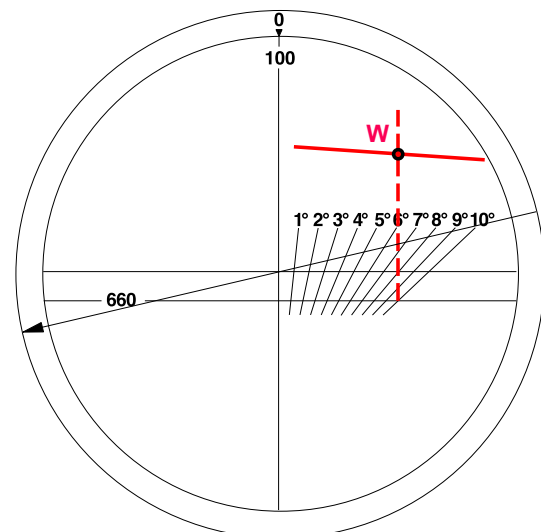


Fig.1.13/8

#### Step 3 (Fig.1.13/9):

The wind point is rotated into the vertical wind speed scale, from which a wind speed of 140 km/h can be read. The wind direction is found at the lower arrow of the scale at 332°.

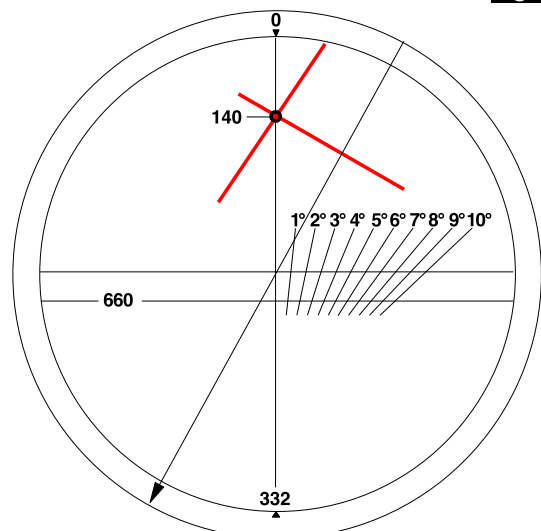


Fig.1.13/9

#### 1.13.4. Use of the computer for wind determination from drift and ground speed

The calculation of wind direction and wind speed from drift angle and ground speed is explained by the following example:

true track	352°
drift angle (DA)	- 5°
true airspeed (TAS)	800 km/h
ground speed (GS)	750 km/h
true wind direction	048.0°
wind speed	84.1 km/h

Step 1 (Fig.1.13/10):

On the transparent disk the track 352° is set opposite to zero. From the intersection of the line 800 km/h (TAS) with the DA - 5° a vertical line (red in the drawing) is drawn upwards (headwind).

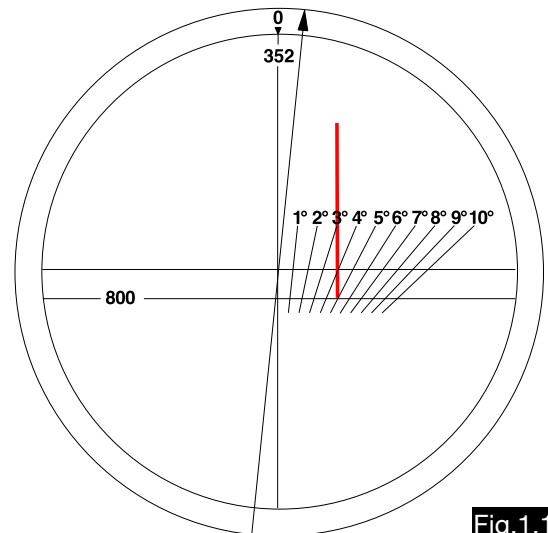


Fig.1.13/10

Step 2 (Fig.1.13/11):

The transparent disc is rotated by the value  $0.5 \text{ DA} = 2.5^\circ$  (counterclockwise - see table 1.13/1). That means course 354.5° is to set opposite zero. The drawn line intersects with the horizontal 50 km/h - line (TAS - GS) at the wind point.

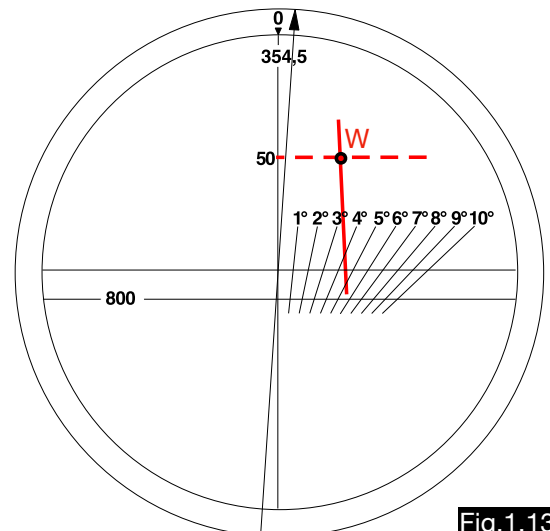


Fig.1.13/11

Step 3 (Fig.1.13/12):

The wind point W is rotated into the vertical scale, where a wind speed of 84 km/h can be read. The wind direction is found opposite the zero at 048°.

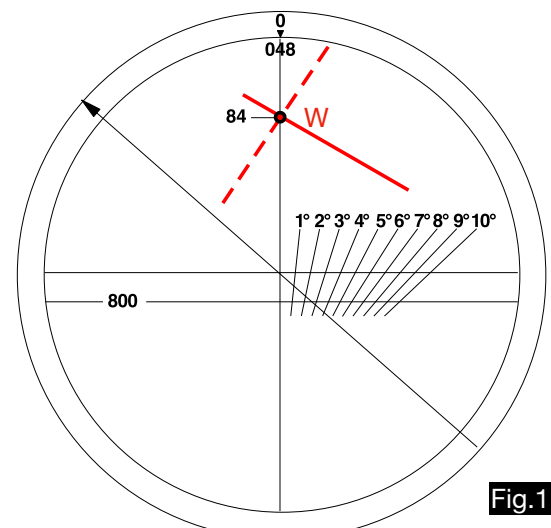


Fig.1.13/12



## 1.14. Calculation of the wind triangle using the AVIONAUT computer

### 1.14.1. General

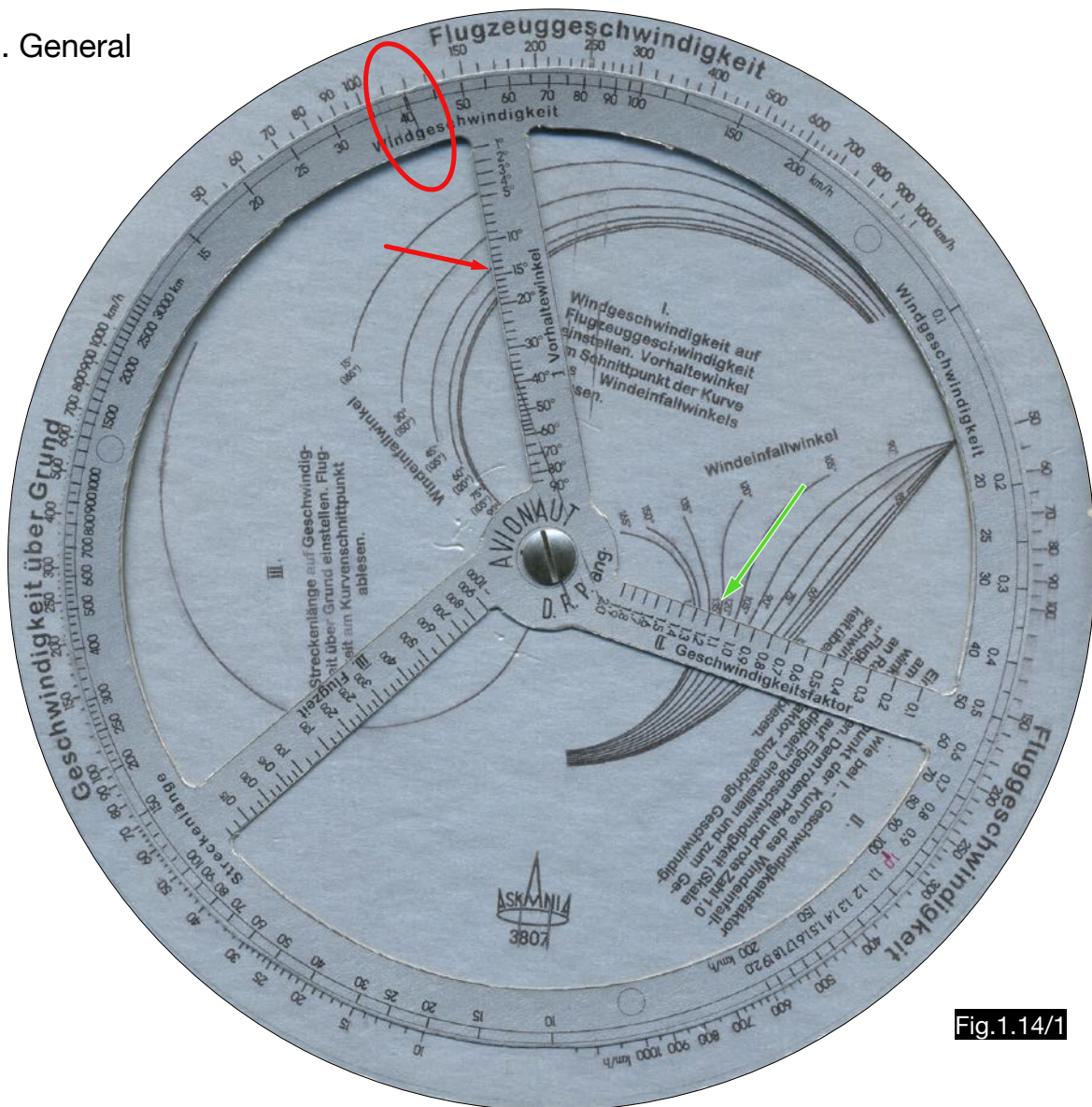


Fig.1.14/1

The AVIONAUT computer (Fig.1.14/1) consists of two disks that have three logarithmic scales at their perimeter. The upper disc additionally has three radial scales which work together with sets of curves on the lower disc. For design details, see 1.14.3.

### 1.14.2. Calculation of wind correction angle and ground speed

The following example explains the application (see Fig.1.14/2):

wind angle	135°
true airspeed	120 km/h
wind speed	40 km/h
wind corr. angle	- 13.6°
ground speed	144.9 km/h

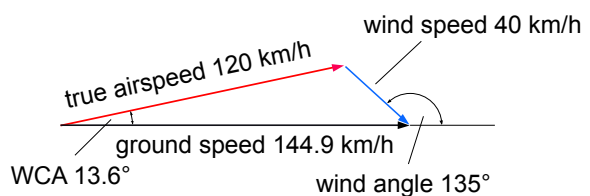


Fig.1.14/2

Step 1 (red highlighted in Fig.1.14/1):

The value 40 (wind speed 40 km/h) is placed below the value 120 (aircraft speed 120 km/h) on the outer logarithmic scale. Then the value 14° is read off from the radial scale „Vorhaltewinkel" (WCA) at the point of intersection with the wind angle curve (135° / 45°).

Step 2:

With the same setting of the computer, a value of approx. 1.2 (green arrow in Fig.1.14/1) is read from the radial scale „Geschwindigkeitsfaktor" (speed factor) at the intersection with the wind angle curve (135°).

Step 3:

On the logarithmic scale „Fluggeschwindigkeit" (airspeed) the multiplication  $120 \times 1.2$  is carried out by setting the red arrow opposite the value 120, so that opposite the value 1.2 a ground speed of approx. 145 km/h can be read (red arrows in Fig.1.14/3).

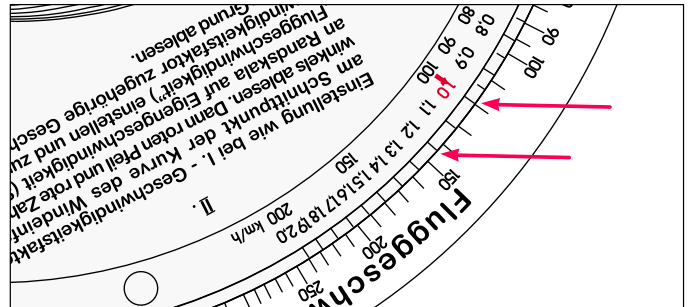


Fig.1.14/3

Step 4:

In the logarithmic scale „Geschwindigkeit über Grund" (ground speed) the flight time is determined by setting the flight distance opposite the speed. The flight time is read on the radial scale at the point of intersection with the curve. The example in Fig.1.14/4 was chosen so that the distance is equal to the speed and the flight time is one hour.

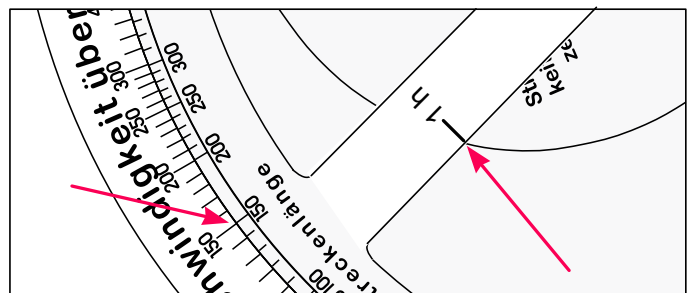


Fig.1.14/4

### 1.14.3. Design details

Explanations to Fig.1.14/5:

- $\alpha$  wind correction angle
- $\epsilon$  wind angle
- $v$  true airspeed
- $u$  wind speed
- $w$  ground speed
- $k$  factor

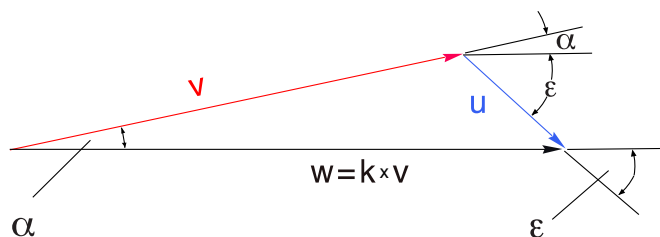
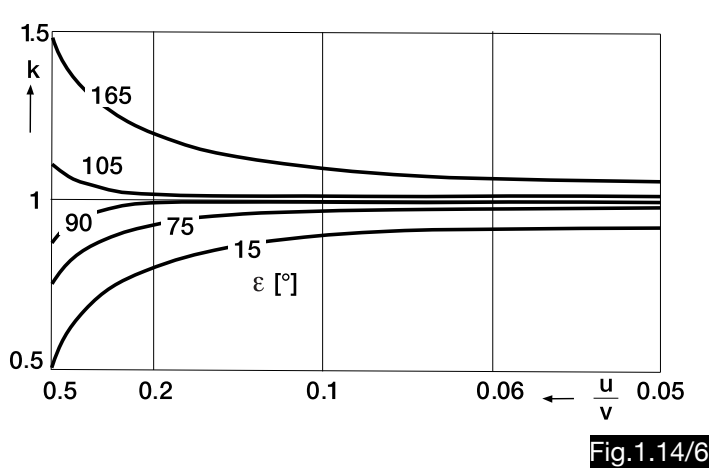


Fig.1.14/5

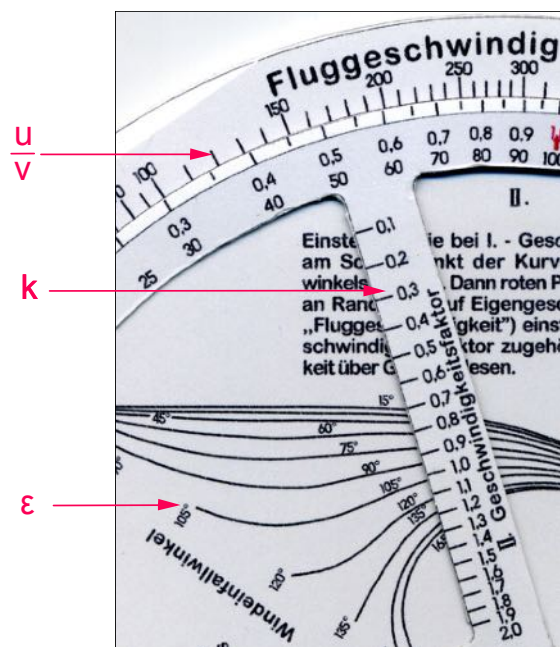
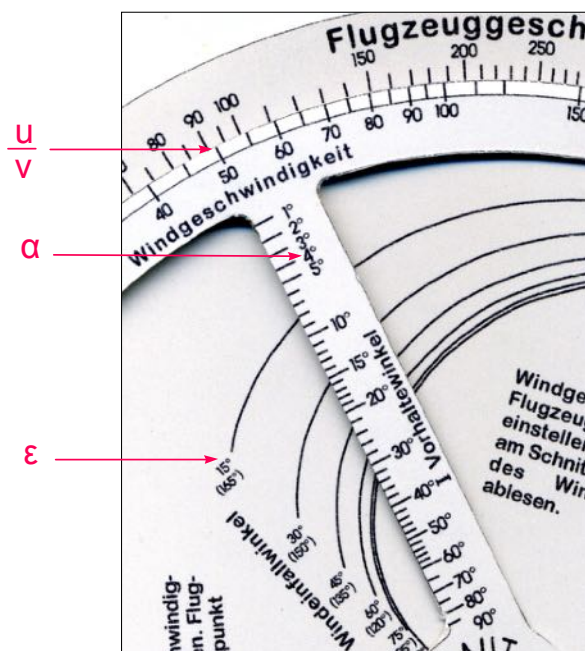
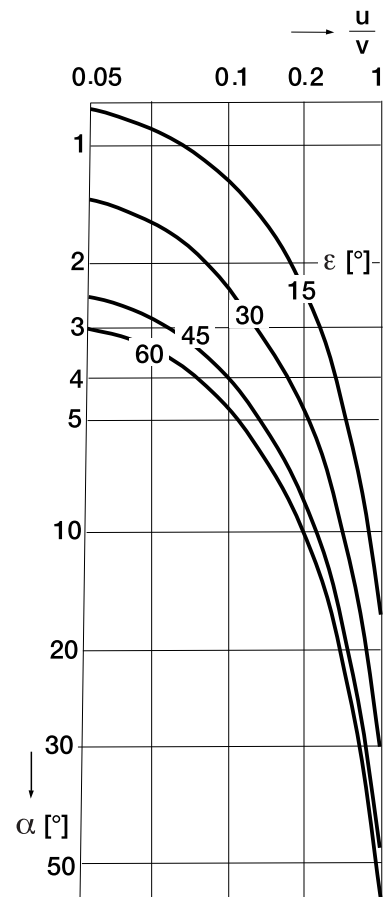
In this wind triangle, the sine theorem applies:

$\frac{\sin \alpha}{u} = \frac{\sin \epsilon}{v} \rightarrow \frac{u}{v} = \frac{\sin \alpha}{\sin \epsilon}$	$\frac{\sin \alpha}{u} = \frac{\sin (\epsilon + \alpha)}{w}$	$w = k v \rightarrow k = \frac{u \sin (\epsilon + \alpha)}{v \sin \alpha}$
---	--	--

Consequently,  $\alpha$  and  $k$  are functions of the angle  $\epsilon$  and the quotient  $u/v$ , which can be represented in the diagrams Fig.1.14/6 and 7, in which the angle  $\epsilon$  occurs as a parameter.



These diagrams for determining the variables  $k$  and  $\alpha$  are displayed in circular form in the computer, where the ordinates are the radii and the abscissa are circular arcs. In the figures 1.14/8 and 9 the speed  $v$  is described as „Flugzeuggeschwindigkeit“ (aircraft speed) or „Fluggeschwindigkeit“ (flight speed), the speed  $u$  as „Windgeschwindigkeit“ (wind speed),  $\epsilon$  as „Windeinfallswinkel“ (relative wind angle),  $\alpha$  as „Vorhalte-  
winkel“ (wind correction angle) and  $k$  as „Geschwindigkeits-  
faktor“ (speed factor). The values  $u$  and  $v$  are opposed to each other on the logarithmic outer scales, so that the radial scales for „ $\alpha$ “ and „ $k$ “ stand at the correct ratio  $u/v$  over the curves. The outer scale (see Fig. 1.14/3) also serves to determine the ground speed by multiplying the speed  $v$  by the factor  $k$ .



The third radial scale is available for flight time calculations. At this the flight time  $T$  can be read, after the ground speed  $w$  ("Grundgeschwindigkeit") was opposed to the distance  $D$  ("Streckenlänge") at the logarithmic outer scales. The time could also be determined in this way. Following the two other methods, however, a curve was constructed which is shown in Fig.1.14/10. In this figure, the horizontal axis corresponds to the logarithmic outer scale. The curve was chosen so that the vertical scale (which corresponds to the radial scale of the computer) allows a more accurate reading, especially for shorter flight times.

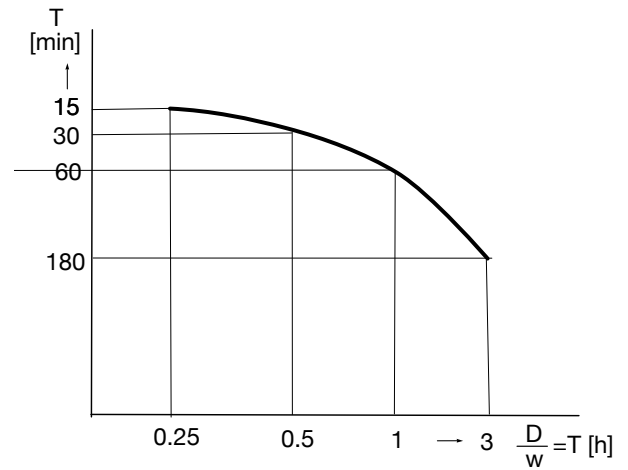


Fig.1.14/10

## 1.15. Calculation of the wind triangle according to Campbell-Harrison

### 1.15.1. Calculation of wind correction angle and ground speed

Fig.1.15/1 shows a schematic sketch of the COURSE AND DRIFT CALCULATOR (CDC) invented by Campbell-Harrison. It is a simplified version of the COURSE AND DISTANCE INDICATOR invented by Battenberg for naval applications. The mode of operation consists in the mechanical reproduction of the wind correction triangle.

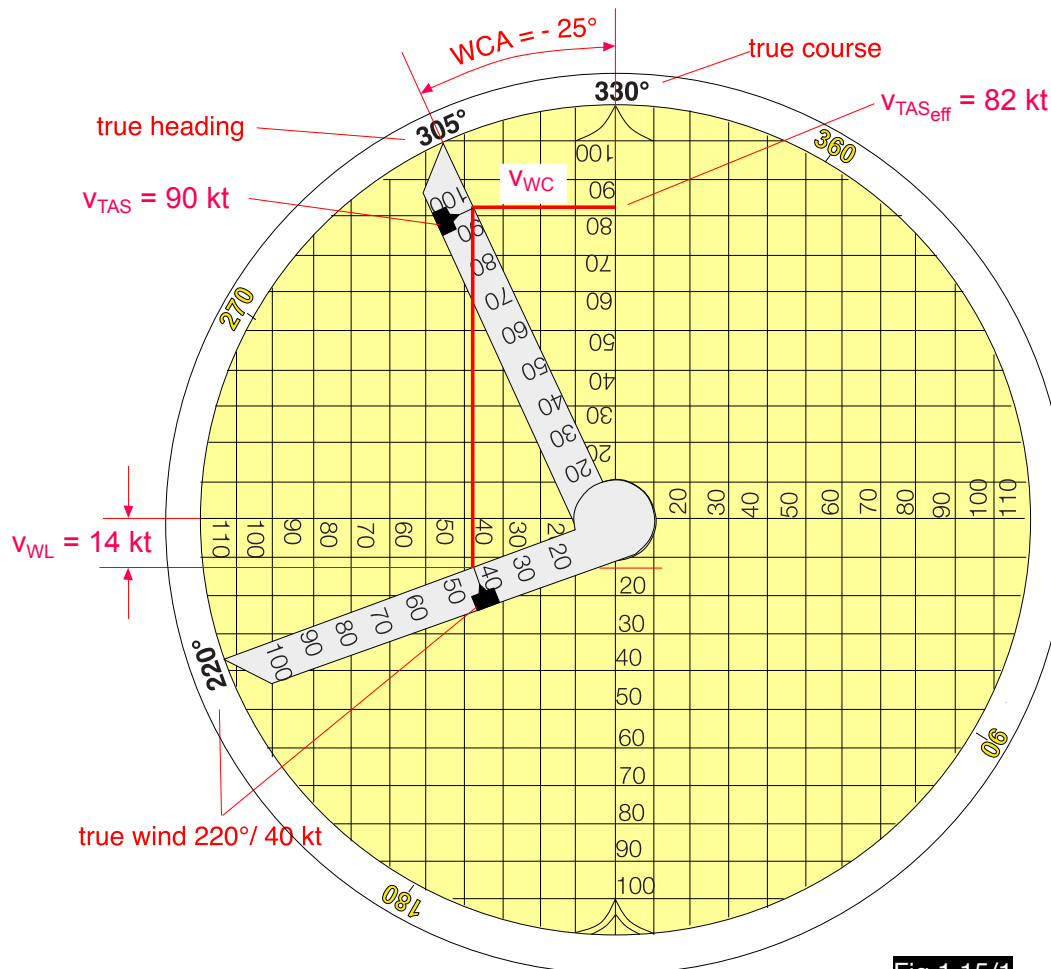


Fig.1.15/1



The use of the CDC is explained by the example shown in Fig.1.15/2.

Solution (see Fig.1.15/1 and 2):

1. The compass rose is set to the true course  $330^\circ$ .
2. The wind direction shall be adjusted to  $220^\circ$  on a speed arm and the wind speed shall be adjusted by means of an adjustable mark at 40 kt.
3. A horizontal line from the 40 kt mark to the vertical scale leads to a longitudinal component of  $v_{WL} = 14$  kt.
4. The second speed arm (TAS arm) shall be set so that the mark 90 kt is vertically above the wind speed mark.
5. The TAS arm now shows the true heading (between true heading  $305^\circ$  and true course  $330^\circ$  the wind correction angle is  $-25^\circ$ ).
6. A horizontal line through the 90 kt mark cuts the vertical scale at  $TAS_{eff} = 82$  kt.
7. The sum of the  $TAS_{eff}$  and the longitudinal component gives the ground speed 96 kt ( $82 + 14$ ).

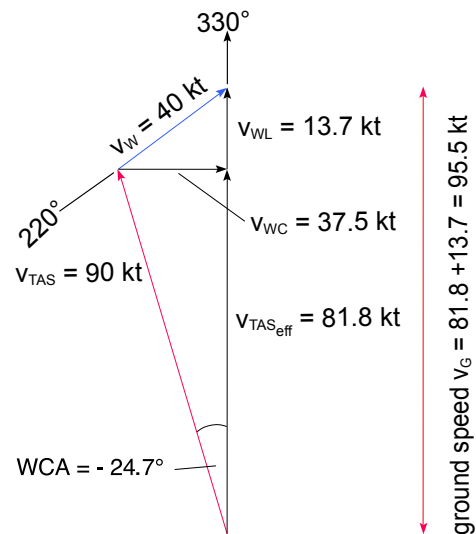


Fig.1.15/2

### 1.15.2. Use of the computer for wind determination from drift and ground speed

Due to the graphical simulation of the wind triangle in the computer CDC, wind determinations from drift angle and groundspeed are possible quickly and easily, but only for air speeds below 110 (kt, m.p.h., km / h, m/s). Fig.1.15/3 contains the data of an example whose calculation is shown in Fig.1.15/4.

Solution:

1. One speed arm is set to true heading  $45^\circ$  and its mark to  $TAS = 80$  kt.
2. The second speed arm is set to track =  $30^\circ$  (heading minus DA) and its mark is set to the ground speed 100 kt.
3. The disc with the speed scales shall be aligned so that the grid lines are parallel to the assumed connecting line (virtual wind vector) between the marks on the speed arms.
4. The wind direction  $168^\circ$  is indicated at the end of the arrow on the centerline.
5. In the grid the wind speed is determined by counting (here between 45 and 76) to 31 kt.

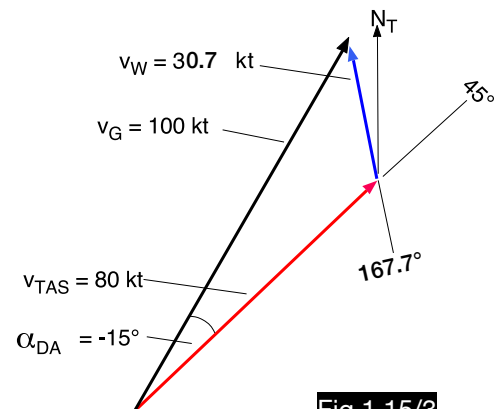
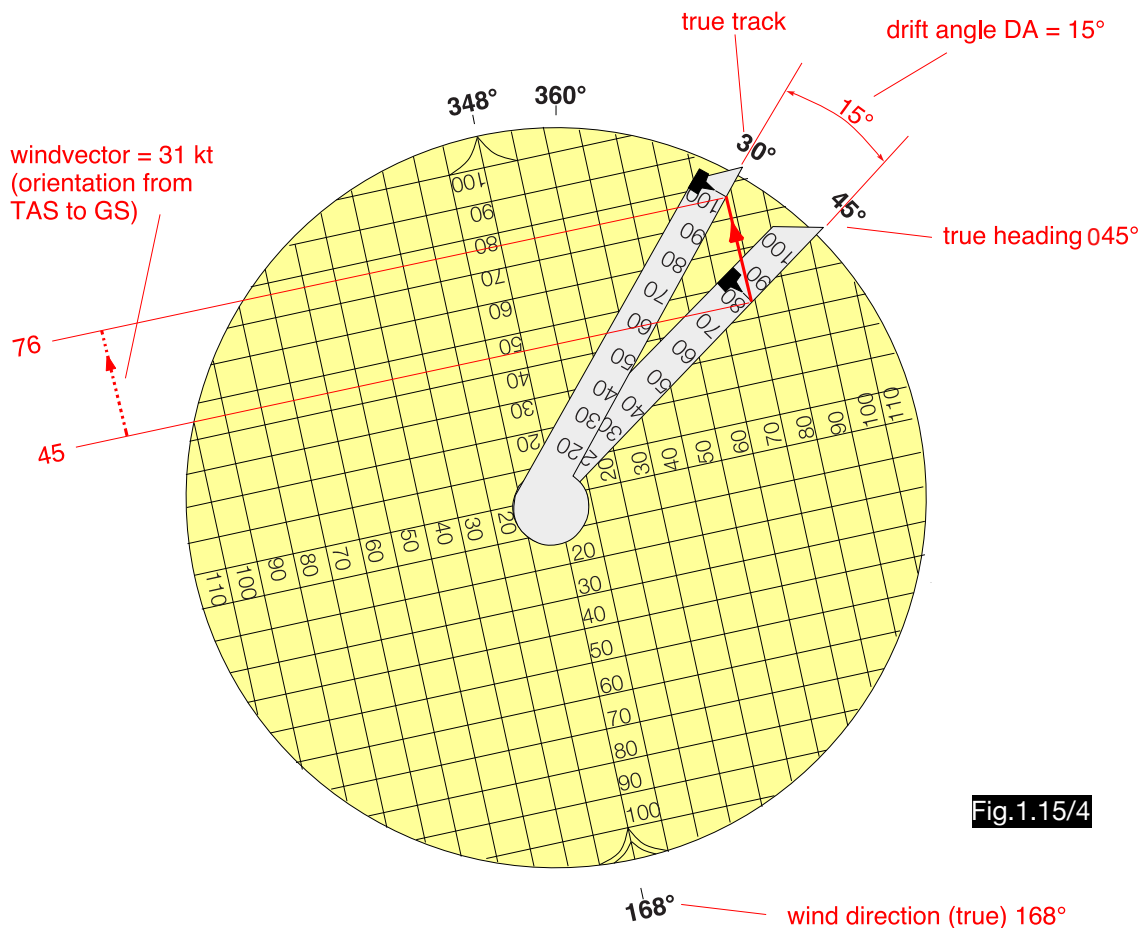


Fig.1.15/3



#### Note:

The CDC calculator has been designed for airspeeds below 110 (kt, m.p.h. or km/h). For TAS > 110 the computer is nevertheless suitable, if relative speeds are used (TAS = 100 % and wind speed in percent of TAS).

## 1.16. Calculation of the wind triangle according to Lyon

### 1.16.1. General

The PRACTICAL AIR NAVIGATION COMPUTER (see Fig. 1.16/1), invented by Thoburn C. Lyon reproduces the wind triangle without using a diagram slide. As a result, the airspeed vector cannot be shifted vertically, so that the wind vector must be found in a right-angled grid aligned to the wind direction and positioned between the speed arc and the vertical axis (direction of the true course). The inscription "FOR TAS 90 - 110 MPH" on this variant of the Lyon calculator indicates that it was intended for aircraft flying at a cruising speed of about 100 (m.p.h., knots or km/h). The errors that occur with true airspeeds of  $100 \pm 10$  are in the area of 12 - 15 %.

### 1.16.2. Calculation of wind correction angle, drift angle and ground speed

The following example explains the application:

true course	270°
true airspeed	100 kt
true wind	240°/25 kt
wind correction angle	- 7.2°
ground speed 1	77.6 kt
drift angle	+ 9.1°
ground speed 2	79.3 kt

Solution for the wind correction triangle  
(shown green in Fig.1.16/1):

1. Set the triangle mark TC (true course) to 270° and the red wind mark to 240°.
2. The wind vector, which shall have a length of 5 intervals (equivalent to 25 kt wind speed), shall be fitted so that it begins at the TAS circle of 100 kt (arc through the center point) and ends on the ground speed axis (centerline).
3. The ground speed is about 22 kt lower than the true airspeed and one gets  $GS = 100 - 22 = 78$  kt.
4. In the drift field the WCA is approx.  $-7^\circ$ .

Solution for the drift triangle (shown magenta in Fig.1.16/1):

1. Set the triangle mark TC (true course) to 270° and the red wind mark to 240°.
2. The wind vector, which must have a length of 5 intervals (corresponds to 25 kt wind speed), begins at the centre and ends at the -20 kt speed line, so that a ground speed of  $GS = 100 - 20 = 80$  kt results. In the drift field a DA of approx.  $+9^\circ$  is determined.

Note:

Another variant of the Lyon calculator is labeled in the centre axis with speeds from 60 to 140 (see Fig.1.16/2).

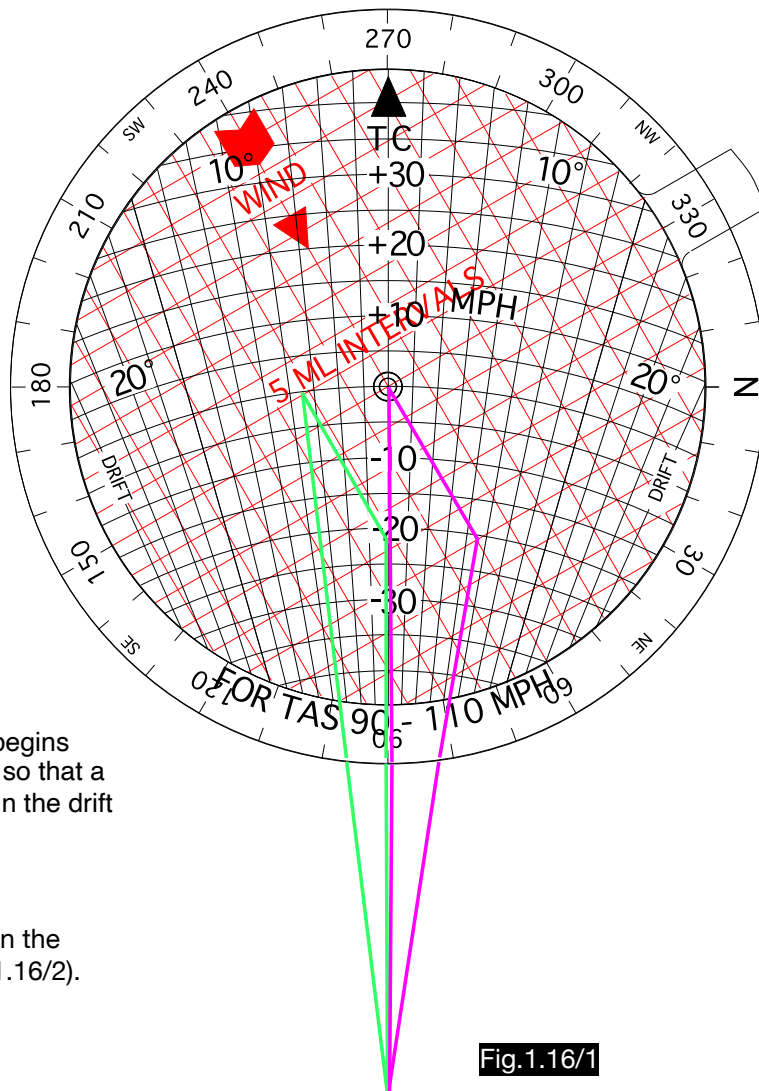


Fig.1.16/1

### 1.16.3. Use of the computer for wind determination from drift and ground speed

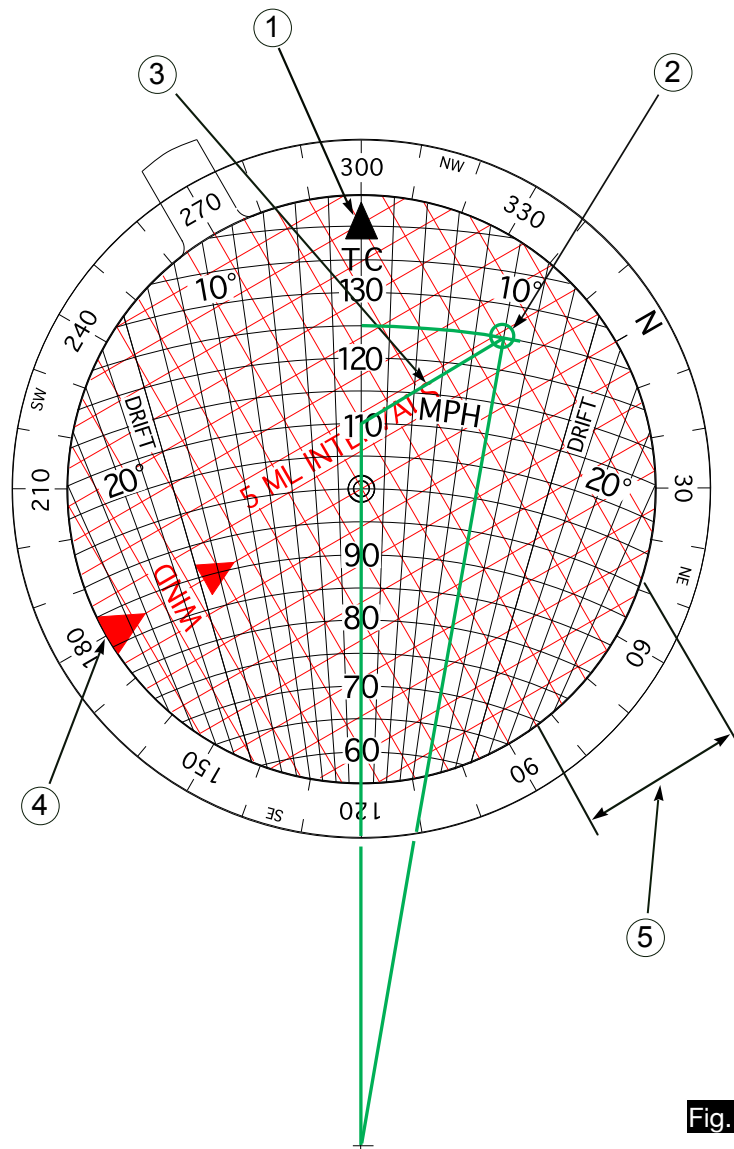
The following example explains the application:

true heading	300°
true airspeed	110 kt
drift angle	+10°
ground speed	125 kt

true wind direction	179,0°
wind speed	25.4 kt

Solution (see Fig.1.16/2):

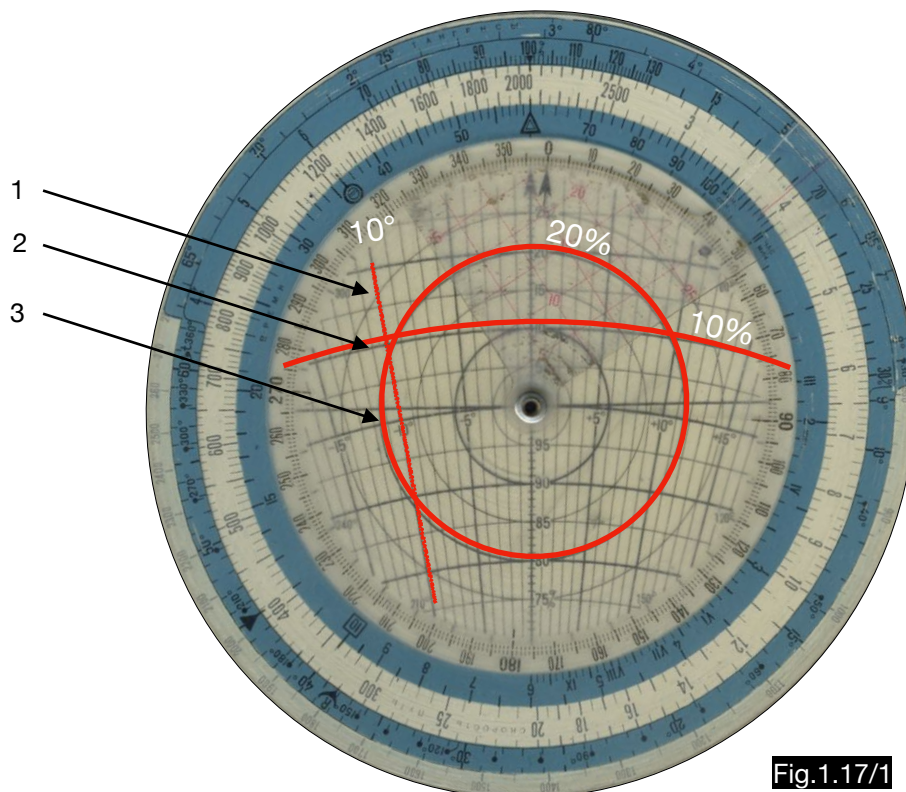
1. The triangle mark TC (true course) is set to 300°.
2. The wind point is found and marked at the intersection of the  $+10^\circ$  drift line and the 125 kt ground speed arc.
3. A connecting line shall be marked between the wind point and the point TAS 110 on the centre line.
4. The disc with the wind speed grid shall be rotated so that the wind direction lines are parallel to the line drawn. Then the wind direction of  $180^\circ$  can be read from the red triangle.
5. The length of the wind vector can be determined with the help of the intervals of the grid, which have a distance of 5 kt. The wind speed is therefore  $5 \times 5 = 25$  kt.



### 1.17. Calculation of the wind triangle according to Kalaschnikow

### 1.17.1. General

The computer NRK-2 (see Fig.1.17/1) reproduces the drift triangle. For this purpose the compass rose contains the concentric circles 3, the radius of which corresponds to the length of the wind vector in percent of the TAS (highlighted red the 20% circle). The air speed vector ends at the center of the compass rose and has the length 100%. Its starting point is outside the instrument. It is the center of the ground speed arcs 2, marked 80, 90, 10, 20 and 30% (highlighted red the 10% arc). In addition, lines 1 of the drift angle start from this point (highlighted red the line -10°).





## 1.17.2. Calculation of wind correction angle and ground speed

The operating principle of the computer is explained using the following example:

true course	360°	wind corr. angle	- 12.2°
true airspeed	100 kt	ground speed	119.0 kt
true wind	225°/ 30 kt ( $\approx 30\%$ TAS)		

Solution (see Fig.1.17/2):

After drawing the wind point W at the intersection of the wind angle line 45° and the wind speed circle 30%, course 360° is set. The drift triangle shown in this way provides a ground speed of 123 kt and a drift angle of 10°. In contrast, the analytically calculated (green shown) wind correction triangle has a ground speed of 119.0 kt and a WCA of 12.2°. The differences between the two wind triangles are due to the different wind angles. This error can be reduced by turning the air speed vector by 10° to the left (this means the numerical equation of DA and WCA). So one gets a wind triangle, (in the sketch with red drift lines) which resembles the wind correction triangle, because its ground speed is 119.8 kt and its wind correction angle is 11.8°.

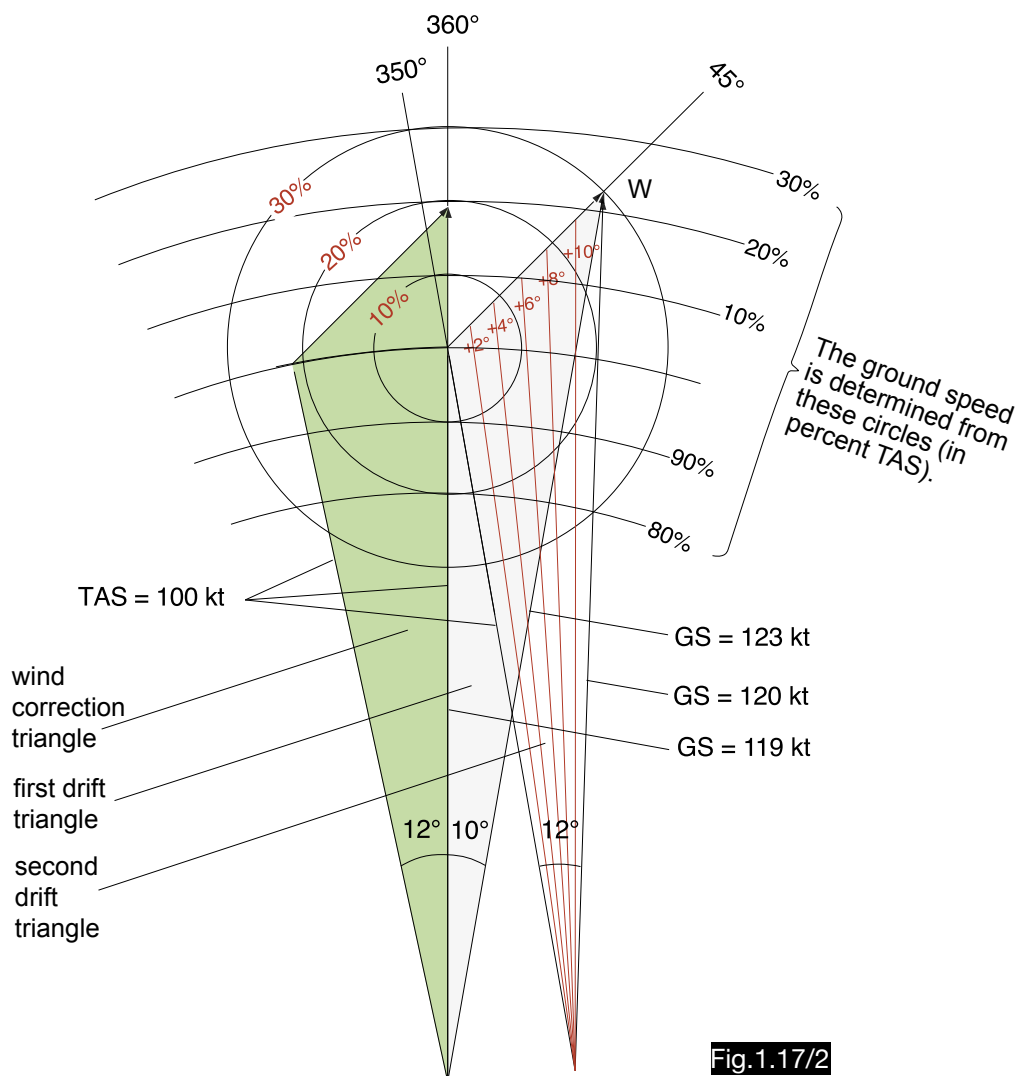


Fig.1.17/2

### 1.17.3. Use of the computer for wind determination from drift and ground speed

Fig.1.17/3 shows how the wind point and thus the direction and size of the wind vector can be determined if heading (here  $360^\circ$ ), drift angle (here  $-12^\circ$ ), true airspeed and ground speed (here  $120\%$  TAS) are known. For this example one gets a wind speed of  $30\%$  TAS (exactly  $30.4$ ) and a wind direction of  $125^\circ$  (exactly  $124.8^\circ$ ).

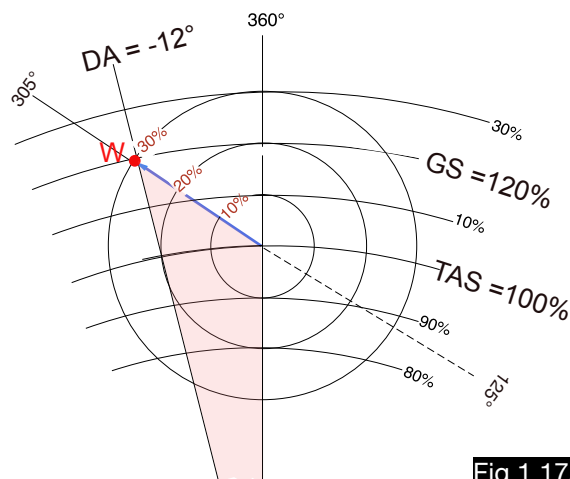


Fig.1.17/3

### 1.17.4. Use of the computer for wind determination from two drift angles

The following example is shown in Fig.1.17/4: On the true heading  $060^\circ$  a drift angle of  $+6^\circ$  was observed. Therefore a pencil line is drawn on the computer over the drift line  $+6^\circ$ . The drift line  $10^\circ$  on the heading  $360^\circ$  had already been drawn before. The two lines intersect at the wind point W.

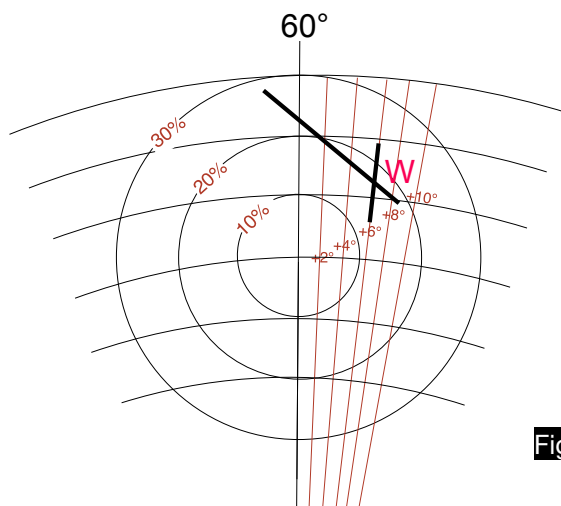


Fig.1.17/4

### 1.17.5. Use of the computer for wind determination from two ground speeds

Fig.1.17/5 shows the following example: On the true track  $060^\circ$  a ground speed of  $110\%$  TAS was observed. On the computer the  $10\%$ -arc was marked with a pencil. Before that, a pencil line had already been drawn over the arc  $20\%$  on the track  $360^\circ$ . The two curves intersect at the wind point W.

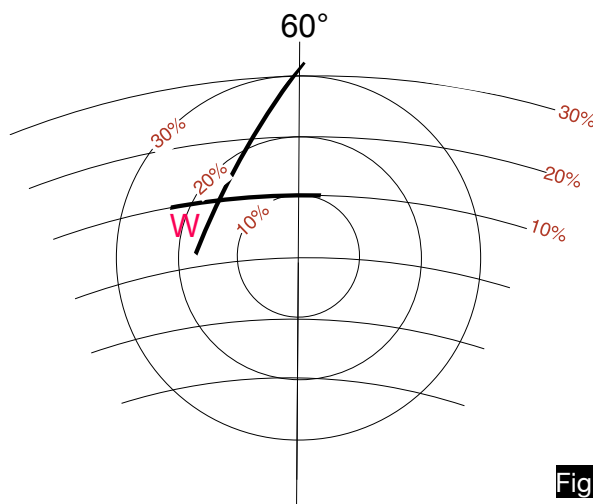


Fig.1.17/5

## 1.18. Calculation of the wind triangle according to Sterligow

### 1.18.1. General

The computer Wetrochet, invented by B. W. Sterligow (see Fig.1.18/1), reproduces the wind triangle. The rotatable compass rose can be adjusted to the airspeed (here 100) by moving it along the longitudinal axis. To mark the wind point, the rose must also be set to the wind direction (here 90°). For the calculation of the drift triangle the wind point must be drawn from the center to the bottom (here the red point on the 30 km/h circle). If the wind correction triangle is to be calculated, the wind vector must point upwards (green point).

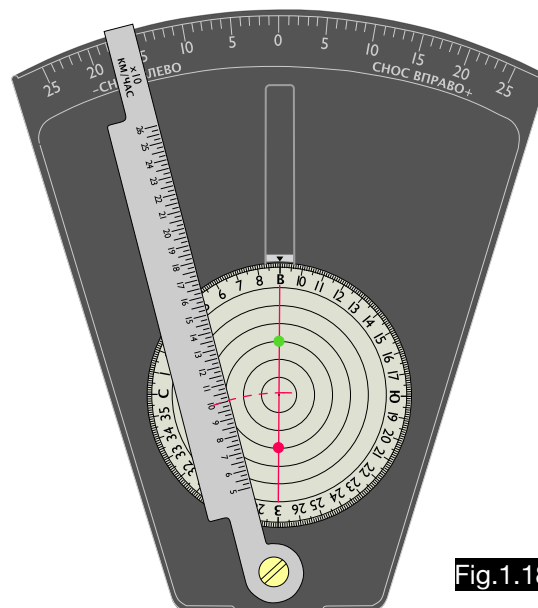


Fig.1.18/1

### 1.18.2. Calculation of the drift triangle

Fig.1.18/2 shows this drift triangle:

true airspeed	100 km/h
true heading	225°
true wind	90°/ 30 km/h
drift angle	+ 9.9°
ground speed	123.1 km/h

#### Explanation:

The compass rose is set to 225° and its center to 100 (TAS = 100 km/h). The pointer is then placed at the wind point. The drift angle (+10°) is readable at the upper scale and the ground speed (123 km/h) at the pointer.

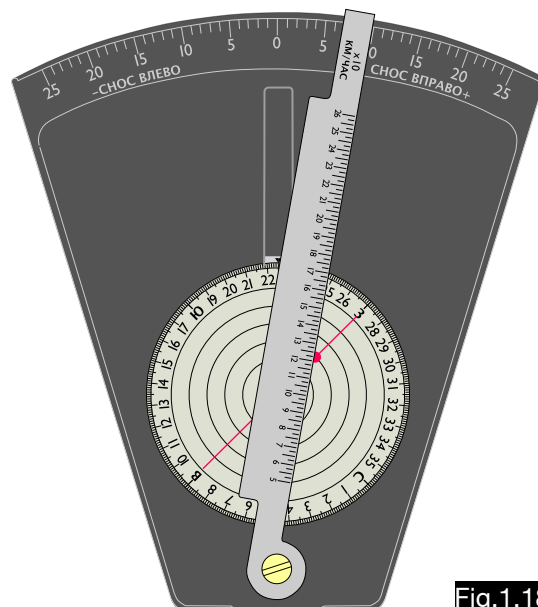


Fig.1.18/2

### 1.18.3. Calculation of the wind correction triangle

Fig.1.18/3 shows this example of a wind correction triangle:

true airspeed	100 km/h
true course	225°
true wind	090°/ 30 km/h
wind corr angle	- 12.2°
ground speed	119.0 km/h

#### Explanation:

Compass rose and pointer are adjusted so that the wind point is at 100 (TAS) on the pointer and the course setting is 225°. The wind correction angle ( - 12°) is indicated by the upper scale and the ground speed (120) by the pointer after turning it to the center of the rose. The procedure is relatively time-consuming, so that it is preferable to correct the drift triangle according to the method described under 1.17.2. This means that first the drift triangle is calculated and then the course is corrected by the drift angle. Consequently, the compass rose must be set to 215° (225 - 10 ). As shown in Fig.1.18/4, these values can then be read:

WCA  $\approx$  - 12,5° (true value: - 11.84°)  
ground speed  $\approx$  120 km/h  
(true value: 119.75 km/h)

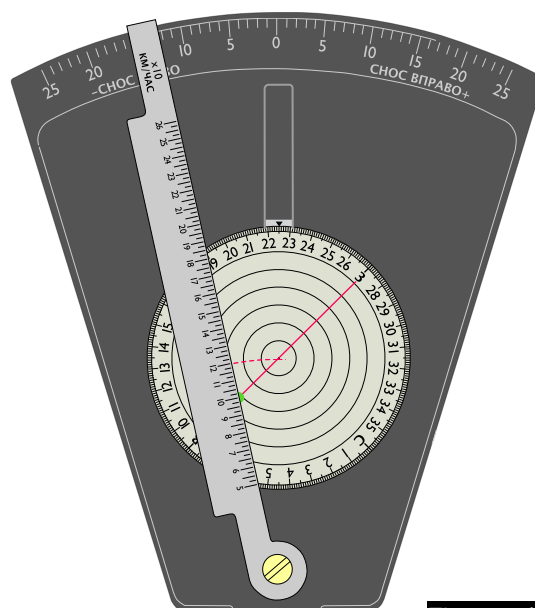


Fig.1.18/3

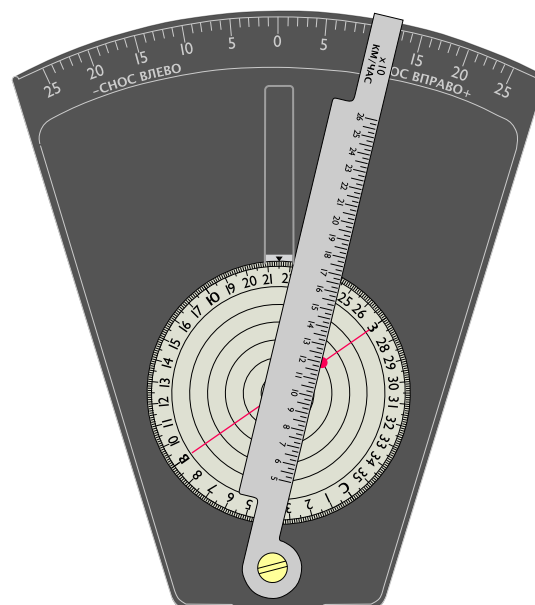


Fig.1.18/4

### 1.18.4. Use of the computer for wind determination from drift angle and ground speed and from two drift angles

The calculation of wind speed and wind direction from the data drift angle, air- and ground speed, using the Wetrochet is carried out in analogy to the procedures described under 1.17.3. and 1.17.4. so that their description is not given here.



## 1.19. Calculation of the wind triangle according to Immler

A copy of the "Universal diagram for the solution of the wind triangle" introduced by Werner Immler shows Fig.1.19/1. It is used to calculate both the drift triangle and the wind correction triangle. The simplified schematic representations below are multicolored for better understandability. Since this possibility was omitted in the original, only the parallel arcs are represented by extended lines (drawn red in Fig.1.19/3 and 4) while the concentric arcs (drawn green in Fig.1.19/3 and 4) are described by crosses lined up next to each other. Although this wind triangle calculation is rather based on a graphical method, it is inserted here, not least because numerous mechanical computers make use of this procedure.

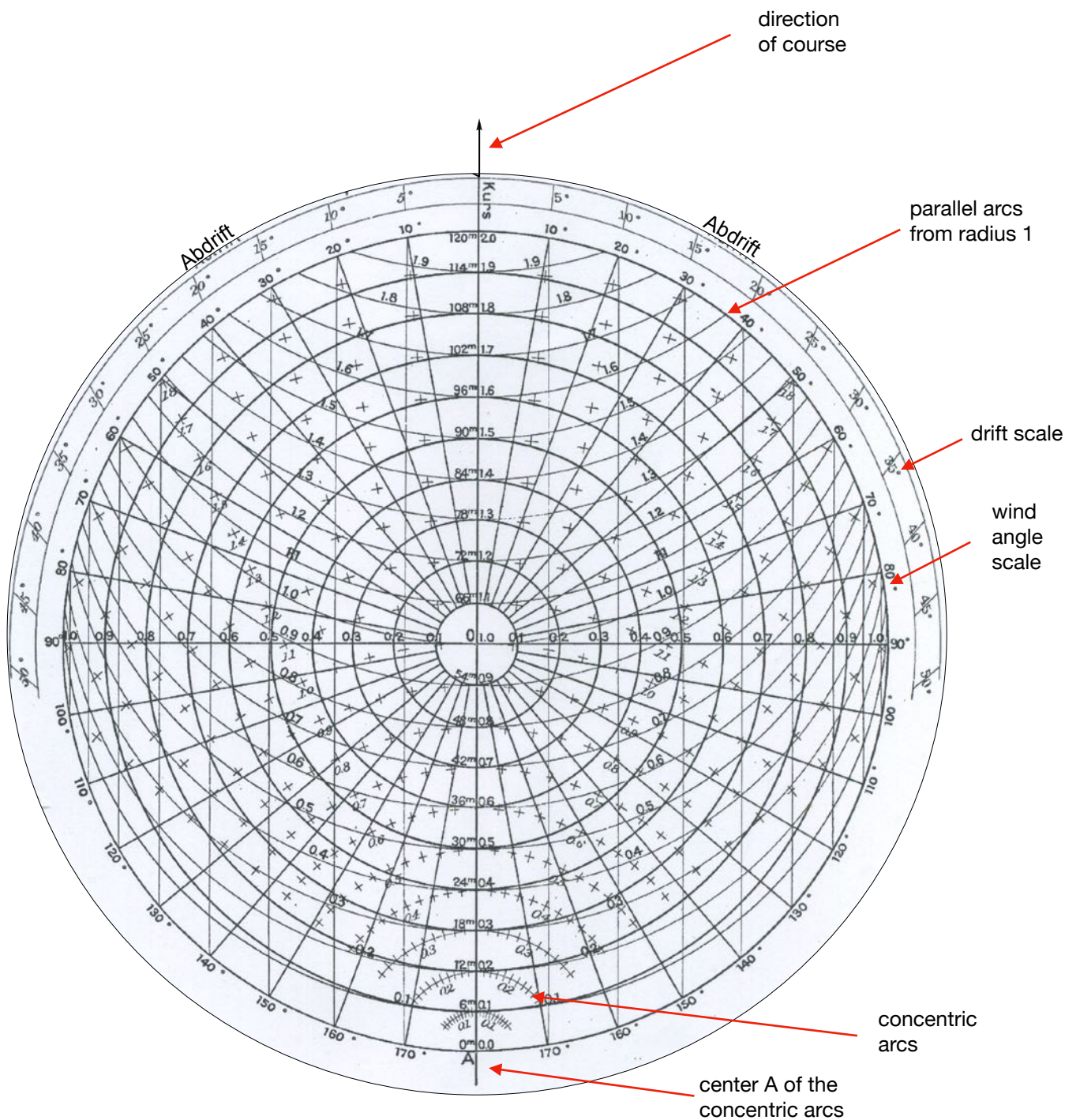


Fig.1.19/1

The calculation of the wind triangle with the aid of Immler's "Universal Diagram" is described using the following example chosen by the inventor himself (Fig.1.19/2):

true heading / course	054°
true airspeed	130 km/h
true wind	WNW/12 m/s
	293° / 43.2 km/h / 0.33 TAS
wind angle	59°
drift angle	+ 13.6°
ground speed 1	156.9 km/h
wind correction angle	- 16.5°
ground speed 2	146.8 km/h

In Fig.1.19/3 the right half of the diagram is shown, in which first the wind point W is drawn, namely at the crossing point of the wind angle 59° (line O-4) with the speed circle 1.33 (wind speed 0.33 TAS).

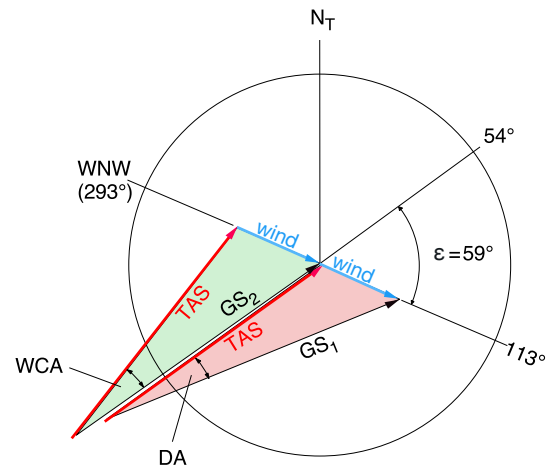


Fig.1.19/2

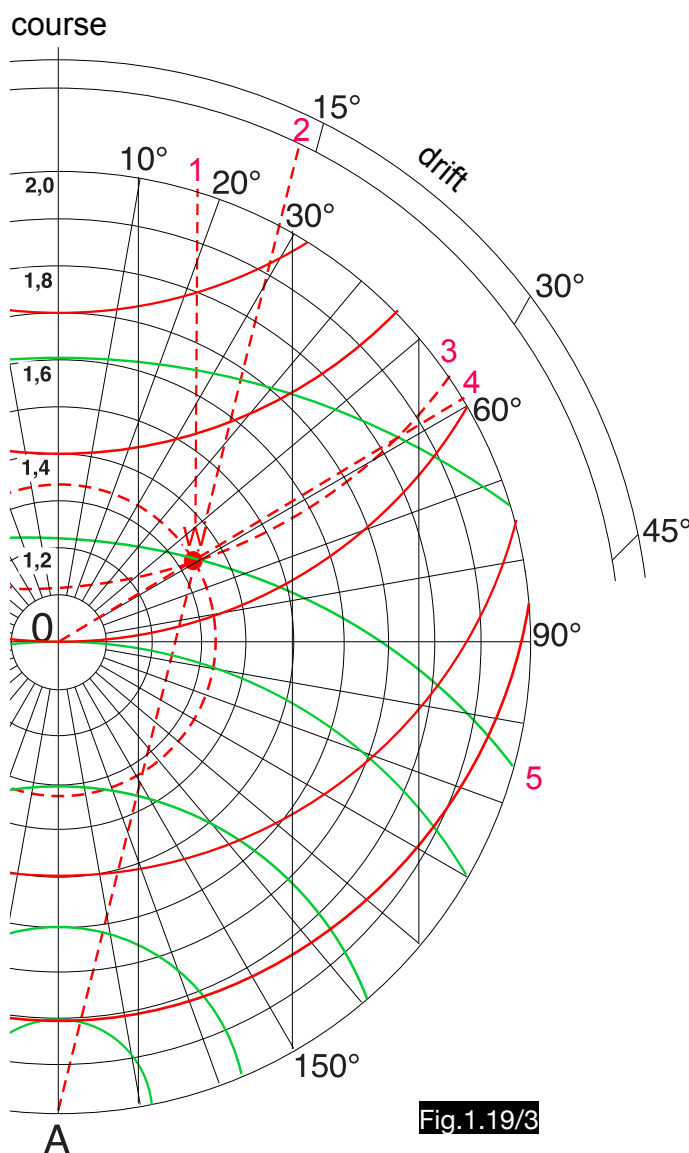


Fig.1.19/3

#### Description of handling:

A vertical line from wind point W upwards intersects the wind angle scale at  $WCA = 16.5^\circ$  (1). A line from point A through wind point W intersects the outer scale at drift angle  $DA = 14^\circ$  (2). The parallel (red) arc of the circle (3) crossing the wind point has a value of 1.13. The ground speed in the wind correction triangle is therefore  $GS_2 = 1.13 \times 130 = 146.9$  km/h. The concentric (green) arc around point A through the wind point has the radius 1.21 (5). This gives a ground speed in the drift triangle of  $GS_1 = 1.21 \times 130 = 157.3$  km/h.

The lines MW and MW' correspond to the wind vector and the line MA to the airspeed. Therefore the triangle AWM represents the drift triangle and the angle M-A-W is the drift angle, which can be read at point D. The airspeed vector AM is of the length of the circle radius (  $1 \approx 100\%$  ). The distance AW corresponds to the ground speed in the drift triangle and its length can be determined at the concentric (green) arcs around point A (here at 1.2, i.e., GS = 1.2 TAS).

[illegible]



## 1.20. Calculation of the wind triangle according to Coutinho

### 1.20.1. General

Fig.1.20/1 shows a remade of the „Corrector de Abatimento“ of Gago Coutinho. The device was originally intended for the determination of the wind from drift angles measured on two headings which differ by  $45^\circ$ . The scales of the two pointers are therefore offset by this angle.

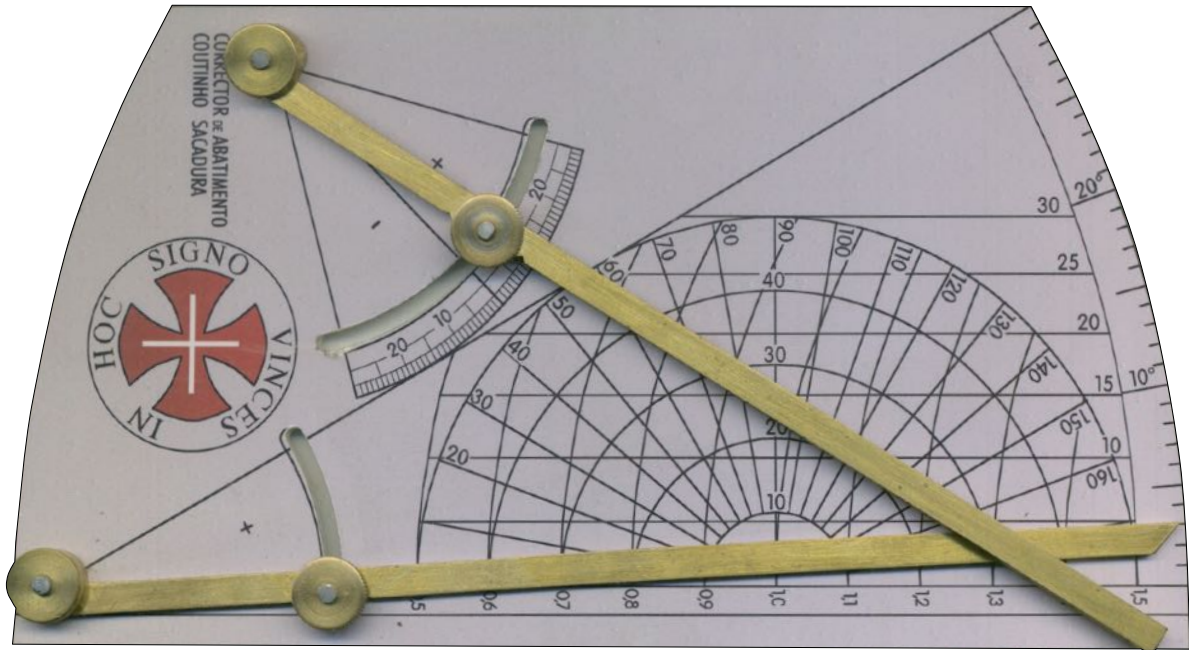


Fig.1.20/1

### 1.20.2 Application for the calculation of drift- and wind correction angle and ground speed

In principle, the calculator is based on the wind diagram described under 1.19., which is why no further explanation is given here.

### 1.20.3. Application in the calculation of the wind from two drift angles

The calculation of wind direction and wind speed from two drifts is explained by the following example:

true heading 1	$090^\circ$
true heading 2	$135^\circ$
drift angle 1	$-6^\circ$
drift angle 2	$-13^\circ$
true airspeed	100 kt

true wind direction	$239.7^\circ$
wind speed	25.4 kt

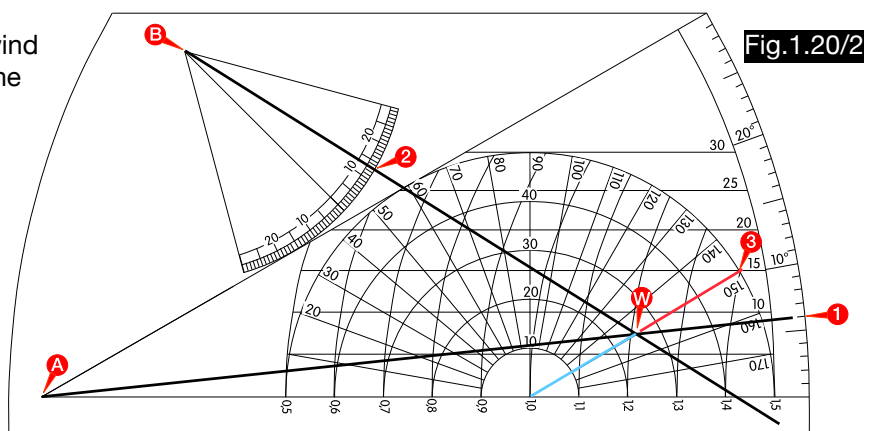


Fig.1.20/2

Fig.1.20/2 shows the necessary settings on the computer. The pointers rotating around points A and B cross each other at wind point W after being adjusted to the two drift angles (1 and 2). At 3 the wind angle of  $150^\circ$  can be read. For the calculation of the wind direction see Fig.1.20/3. If the wind falls from the left during the first drift measurement (i.e. drift angle positive), the second drift measurement must be made on a course which is  $45^\circ$  lesser. The situation is then inverted on the computer.



At Fig.1.20/3 lines AM and BM correspond to airspeed vector (marked red). Its length was chosen in the interest of a higher accuracy of the length  $2R$  ( $R$  = radius of the outer circle). The inner circles serve to determine the wind or ground speed and are therefore labelled with relative speed specifications (1.1 means 10 % above or below the true airspeed). The drift angle  $DA_1 = -6^\circ$  is measured on the heading  $TH_1 = 90^\circ$  and the drift angle  $DA_2 = -13^\circ$  is measured on the heading  $TH_2 = 135^\circ$  which is larger than  $TH_1$  by  $45^\circ$ . In the two drift triangles A-M-W and B-M-W, the lines AW and BW are the tracks that intersect at the wind point W. Thus the wind vector M W is well defined. The wind direction  $\delta$  results from the wind angle  $\alpha = 150^\circ$  and the  $TH_1 = 90^\circ$  to  $240^\circ$  ( $90 + 150$ ). The wind speed is determined along the circular arcs. In this example, about 1.25 is found, i.e. the wind speed is  $0.25 \times TAS = 0.25 \times 100 \text{ kt} = 25 \text{ kt}$ .

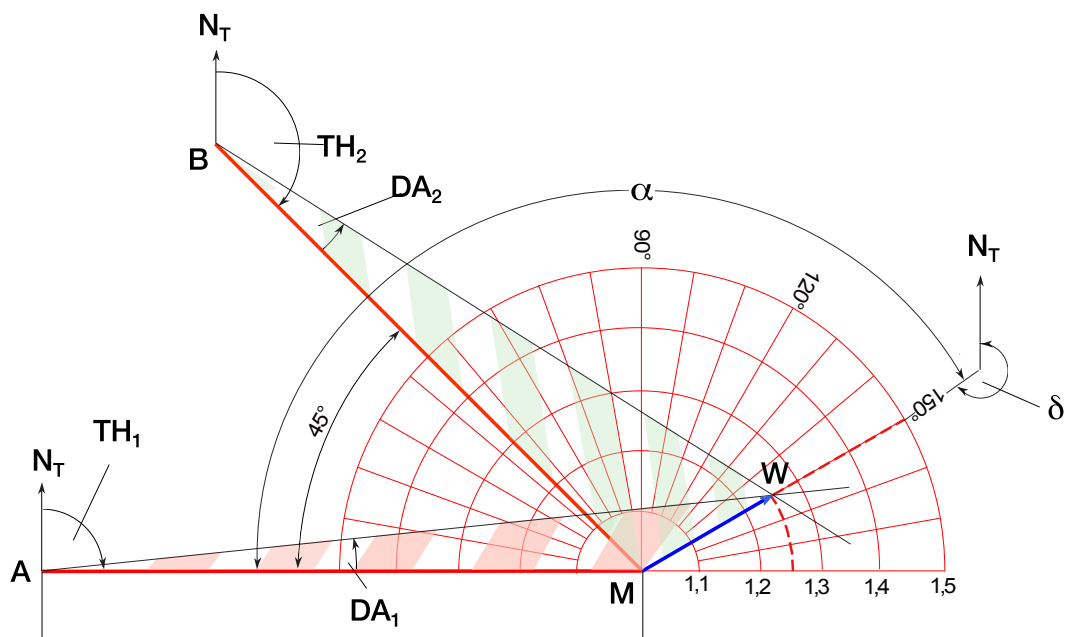


Fig.1.20/3

## 1.21. Calculation of the wind triangle according to Mailloux

### 1.21.1. General

Fig.1.21/1 shows the „Cercle Calculateur“ invented by Louis-Joseph Mailloux. The computer has 360 cuts around the perimeter through which threads can be fastened. In this way, points or lines can be defined without the need to make any entries.

### 1.21.2. Use of the computer for drift-, wind correction- and ground speed calculations

Fig. 1.21/1 illustrates the handling of the device when calculating the wind triangles using the following example:

true airspeed	120 kt
true heading/course	360°
true wind direction	050° (wind angle 50°)
wind speed	40 kt (0.33 TAS)
wind correction angle	+14.79°
ground speed 1	90.31 kt
drift angle	- 18.0°
ground speed 2	99.1 kt

1. To mark the direction of the wind vector, first thread (1 - 1) is fastened between the points 050° and 230° of the outer scale. On this line the wind point W is on the position 0.33 TAS.
2. A second thread (2 - 2) is fastened between the zero point of the outer scale and the wind point W. This thread ends in the lower outer scale at the drift angle - 18°.
3. The dashed green line along the green (concentric) arcs through the wind point ends on the speed scale at 0.83, so that in the drift triangle the ground speed is  $0.83 \times 120 \text{ kt} = 99.6 \text{ kt}$ .
4. The dashed red line along the red (parallel) arcs through the wind point ends at 0.74 on the speed scale, so that in the wind correction triangle a ground speed of  $0.74 \times 120 \text{ kt} = 90 \text{ kt}$  results.
5. From the wind point along the vertical lines upwards one finds the wind correction angle of 15°.

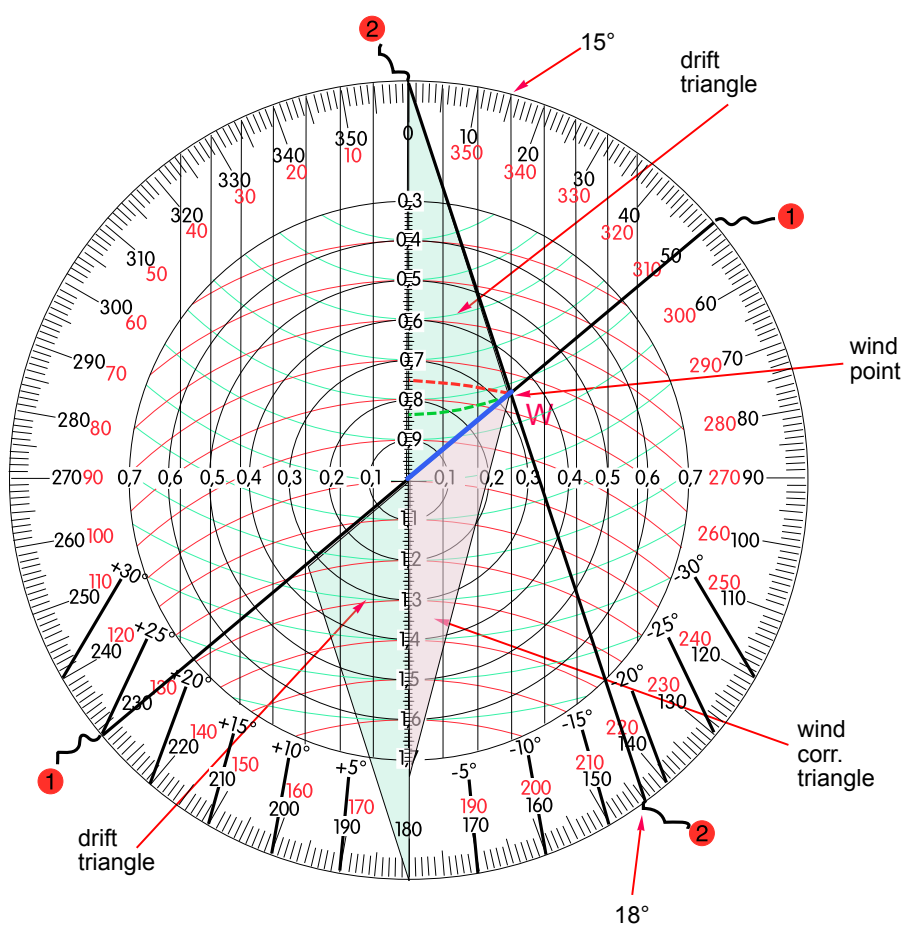


Fig.1.21/1

### 1.21.3. Use of the computer for wind determination from drift and ground speed

Fig.1.21/1 is also used for the description of the wind calculation from a drift angle ( $-18^\circ$ ) and a ground speed (99 kt). First a thread is fastened between the zero point of the outer scale and the point DA =  $18^\circ$  on the drift scale, on which the wind point must lie at the position  $GS : TAS = 99 : 120 = 0.83$ . Consequently, the wind point is found at the intersection of the green speed circle 0.83 with the drift line. For the determination of the wind direction a second thread is fastened through the wind point and the center, which cuts the outer scale at the wind direction  $050^\circ$ . The wind point lies on a circle of the radius 0.33, so that a wind speed of  $0.33 \times 120 \text{ kt} = 40 \text{ kt}$  results.

### 1.21.4. Use of the computer for wind determination from two drift angles

Fig. 1.21/2 illustrates the handling of the device when calculating the wind from two drifts using the following example:

true heading 1	$360^\circ$
true heading 2	$090^\circ$
drift angle 1	$+14^\circ$
drift angle 2	$+10^\circ$
true airspeed	100 kt
true wind direction	$295.0^\circ$
wind speed	24.5 kt

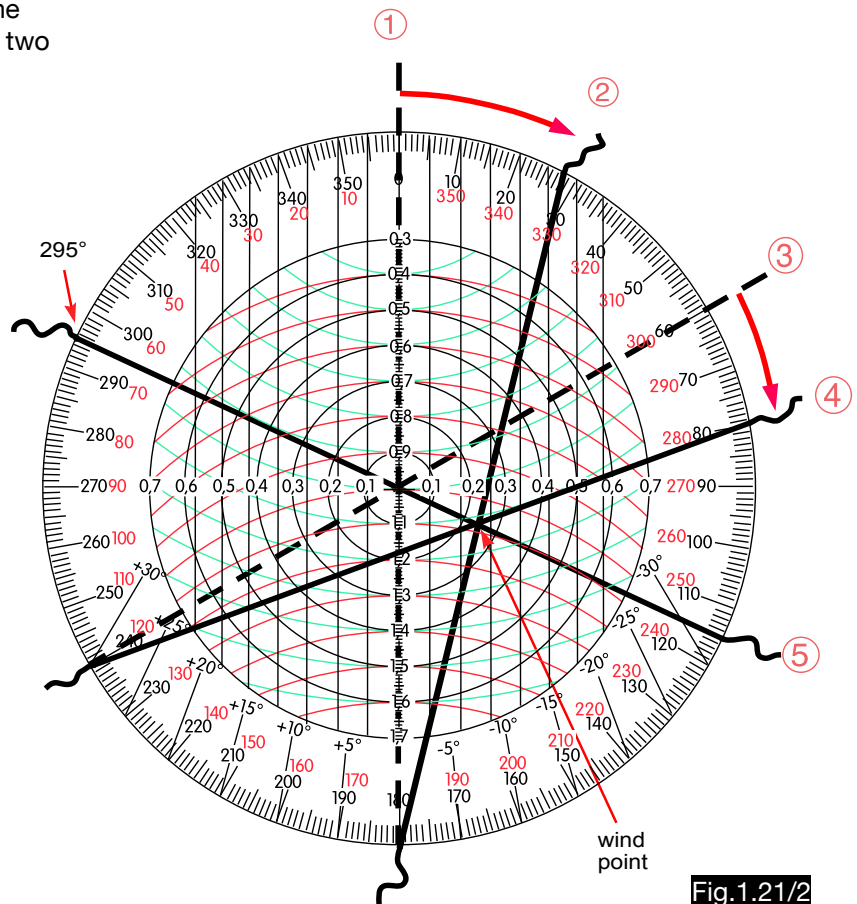


Fig.1.21/2

#### Solution:

First a thread is fastened which corresponds to the 1st course of  $360^\circ$  (dashed line 1), but it is fixed in the notch  $28^\circ (2 \times 14)$ . Because of the double radius, this corresponds in good approximation to a rotation around the 1st drift angle ( $14^\circ$ ). Line 2 represents the first track. In the same way, a thread is fastened in the direction of the second course ( $060^\circ$ ) (dashed line 3). After rotation by  $20^\circ$  (doubled second drift angle of  $10^\circ$ ), this represents track 2. It must therefore be fixed at  $80^\circ (60^\circ + 2 \times 10^\circ)$ . At the intersection of the threads 2 and 4 one will find the wind point. A third thread (5) is fastened so that it lies above the wind vector running from the centre to the wind point. The wind direction thus results in  $295^\circ$ . The wind speed is determined by concentric circles. Since the wind point lies between the circles 0.2 and 0.3 (about 0.25), the wind speed is about 25 kt ( $0.25 \times 100 \text{ kt}$ ).

## 1.22. Calculation of the wind triangle according to Peters

### 1.22.1. General

The PORTLAND DEAD RECKONING CALCULATOR (see Fig. 1.22/1), invented by M. W. Peters mechanically reproduces the wind triangle.

### 1.22.2. Calculation of the wind correction triangle

Fig.1.22/1 shows the device in the setting necessary for the calculation of the wind correction triangle in the example shown in Fig.1.22/2 (use the red markings and inscriptions). The discs 2, 3 and 4 are rotatably arranged on the base plate 1. The cursor 5 with its speed scale can be moved horizontally and vertically. First, the compass rose (disc 2) is set so that the course  $0^\circ$  is opposite the DATUM mark of plate 1. Then the red mark on disk 3, which has a wind speed scale (called TIDE), must be set to the wind direction  $045^\circ$ . The cursor is then moved so that the zero point of its scale is above 5 (wind speed 50 kt). After turning part 4 with its speed scale so that the mark 10 (TAS = 100 kt - here marked by the red inscription SHIP SPEED) intersects the vertical scale of the cursor 5, the wind correction triangle is reproduced (hatched red in Fig.1.22/1). The red inscription COURSE TO STEER on part 4 refers to a red triangle on which the HEADING of  $021^\circ$  can be read. The ground speed of 58 kt (marked by the red inscription SPEED MADE GOOD) is determined at the intersection of the scales of disc 4 and cursor 5.

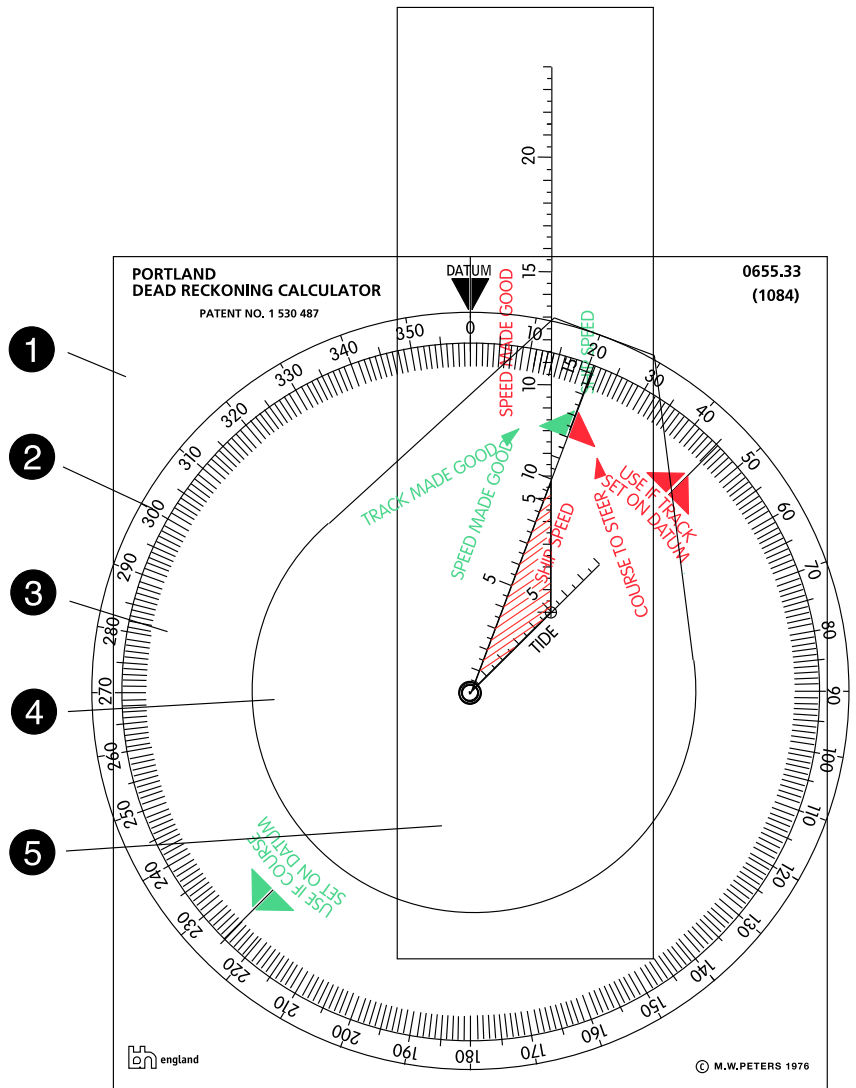


Fig.1.22/1

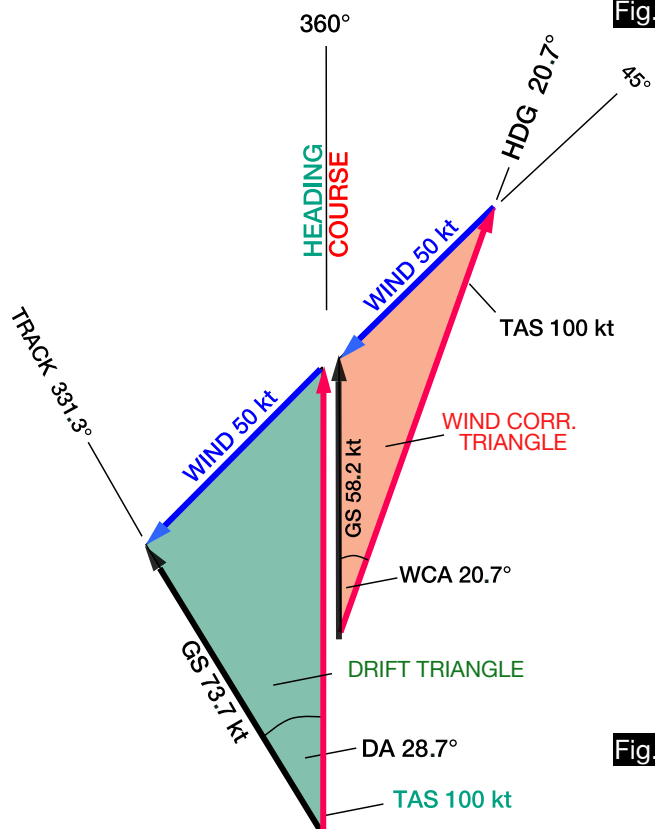


Fig.1.22/2



### 1.22.3. Calculation of the drift triangle

Fig.1.22/3 shows the setting necessary for the calculation of the drift triangle in the example of Fig.1.22/2 (the green markings and inscriptions must be applied). Analogous to the wind correction triangle, after setting the compass rose to 360°, the green mark on the TIDE scale is set to 045° and then the zero of the cursor scale is set to the 5 (50 kt wind) of the TIDE scale. Turn disc 4 to the left until it crosses the vertical scale SHIP SPEED at 10 (TAS = 100 kt). Thus the drift triangle, hatched green in the drawing, is reproduced. Now the track of 331° is read from the green triangle on disc 4 (next to the green inscription TRACK MADE GOOD) and the ground speed of 74 kt is read on the speed scale (marked by the green inscription SPEED MADE GOOD).

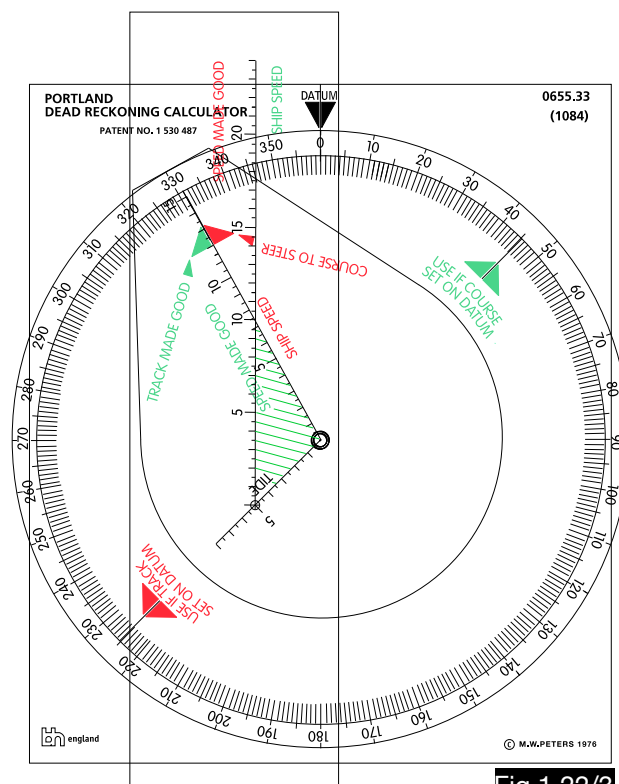


Fig.1.22/3

### 1.22.4. Use of the computer for wind determination from drift and ground speed

The following example explains the application (see Fig.1.22/4):

true heading	100°	true wind direction	251.4°
true airspeed	100 kt	wind speed	54.4 kt
wind corr. angle	-10°		
ground speed	150 kt		

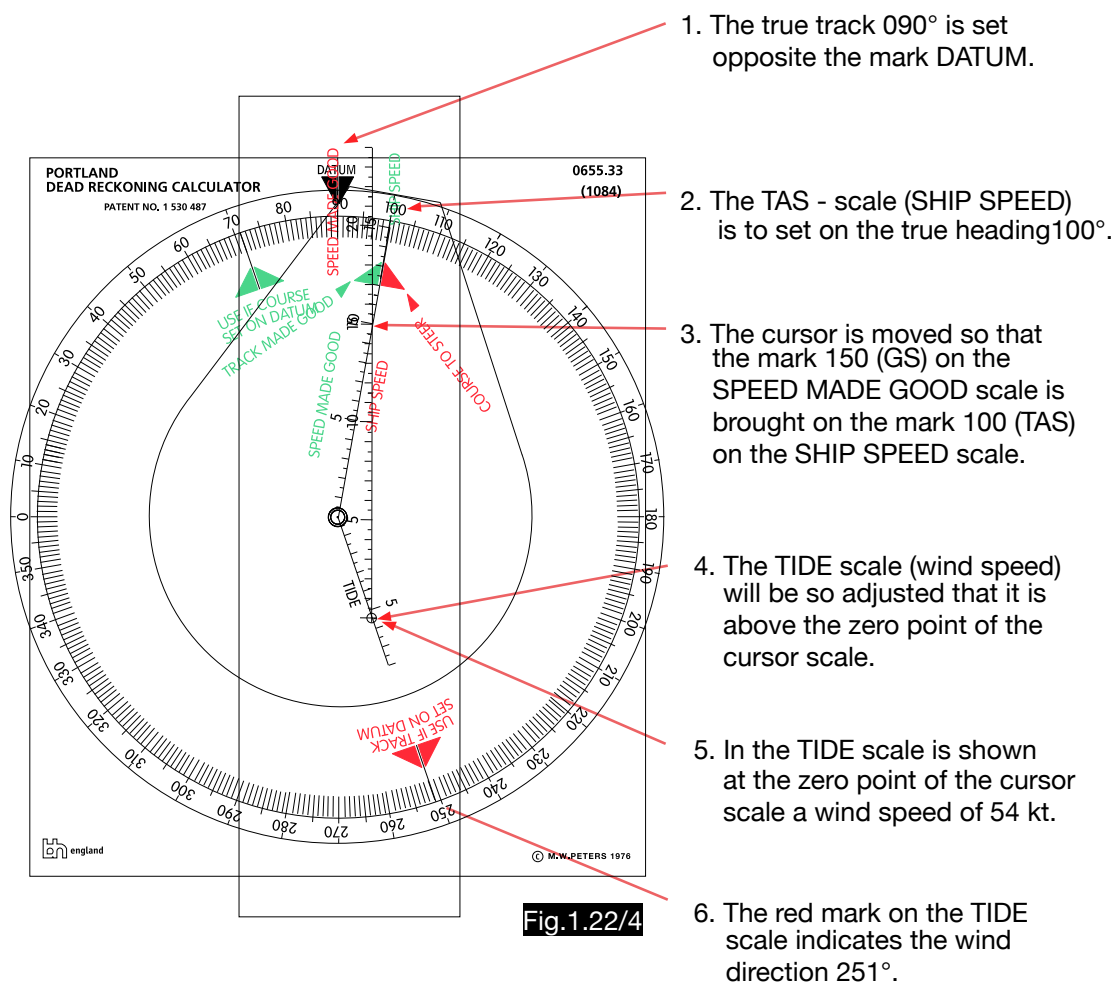


Fig.1.22/4

## 1.23. Calculation of the wind triangle according to Batori

### 1.23.1. General

With the computers B-24 and B-26 invented by Oscar Batori, the wind triangle is calculated logarithmically with the help of the longitudinal and the cross components of the wind. Fig.1.23/1 shows an enlarged view of the type B-24. The two computers are of identical construction and differ only in size. The diameter of the type B-24 is 76 mm. The variant B-26 has a diameter of 100 mm and can be mounted on the cockpit sideboard. In the logarithmic outer scales the arbitrarily chosen quotient 62:70 was set. The result 0.88 is shown opposite the figure 10 (green arrows). The corresponding angles are 42.5° for the tangent function, 62.3° for the sine function and 27.6° for the cosine function. How these values are displayed in the drift window can be seen in the zoomed illustration (Fig. 1.23/2).

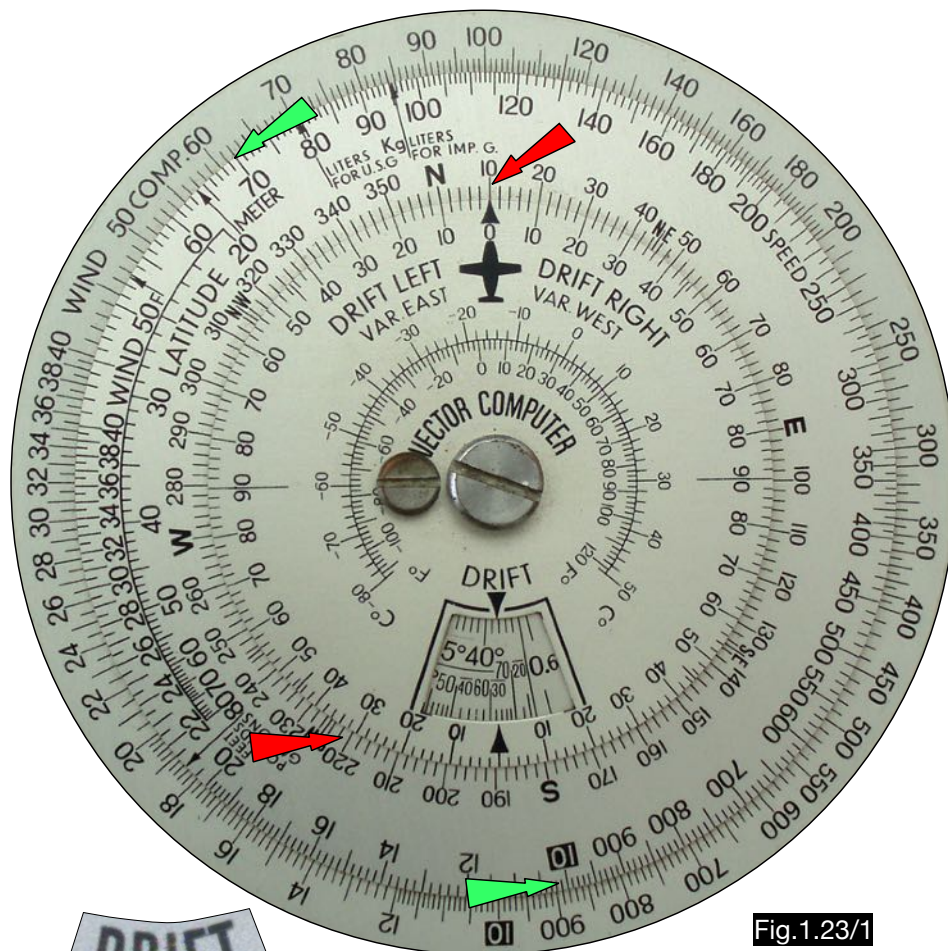


Fig.1.23/1



Fig.1.23/2

### 1.23.2. Use of the computer for wind correction- and ground speed calculations

The application is described in the following example (see Fig.1.23/3):

true wind direction	220°
wind speed	60 kt
true airspeed	120 kt
true course	010°
$\alpha_{WCA}$	14.5°
ground speed	168.2 kt
cross component of the wind $v_{WC}$	30 kt
longitudinal component of the wind $v_{WL}$	52.0 kt

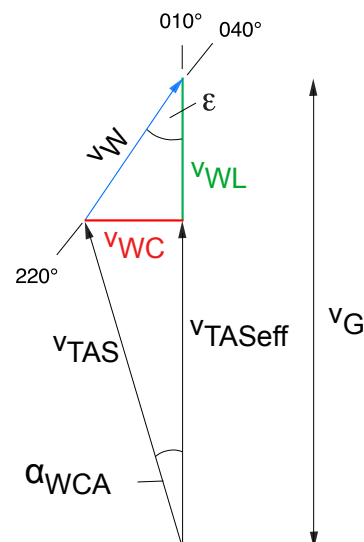


Fig.1.23/3

Step 1: To determine the wind angle the aircraft symbol on the compass rose is placed on the course (010°) and the wind angle  $\epsilon = 30^\circ$  is read opposite the wind direction 220° (see red arrows in Fig.1.23/1).

Step 2: After setting  $\sin 30^\circ$  in the drift window, the cross component 30 can be read in the logarithmic outer scale opposite the  $v_W = 60$  kt (multiplication in logarithmic scales:  $v_{WC} = v_W \times \sin \epsilon = 60 \text{ kt} \times 0.5 = 30 \text{ kt}$ ).

Step 3: After setting  $\cos 30^\circ$  in the drift window, the longitudinal component 52 kt can be read in the logarithmic outer scale opposite the figure  $v_W = 60$  kt ( $v_{WL} = v_W \times \cos \epsilon = 60 \text{ kt} \times 0.866 = 52.0 \text{ kt}$ ).

Step 4: In Fig.1.23/3 the following applies:  $\sin \alpha_{WCA} = \frac{v_{WC}}{120} = \frac{30}{120} \longrightarrow \alpha_{WCA} = 14.5^\circ$

After the quotient 30/120 has been set in the logarithmic outer scale, a wind correction angle of approx.  $15^\circ$  can be read in the sine scale of the drift window (division in logarithmic scales).

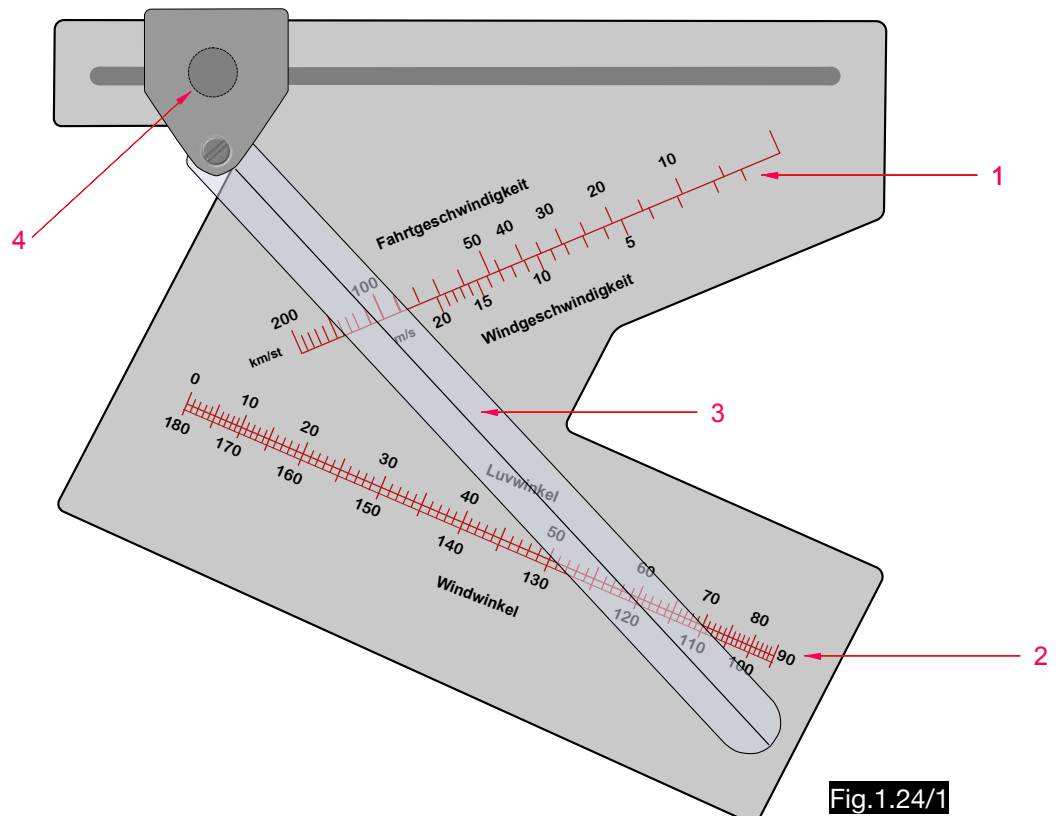
Step 5: In Fig.1.23/3 the following applies:  $v_{TASeff} = v_{TAS} \times \cos \alpha_{WCA} = 120 \text{ kt} \times 0.97 = 116.2 \text{ kt}$   
After setting  $\cos 14.5^\circ$  in the drift window, the  $v_{TASeff} = 116$  kt can be found in the logarithmic outer scale opposite the 120 kt mark ( $v_{TAS}$ ).

Step 6: The following applies to  $v_G$ :  $v_G = v_{TASeff} + v_{WL} = 116 \text{ kt} + 52 \text{ kt} = 168 \text{ kt}$ .

## 1.24. Calculation of the wind triangle according to Goerz

### 1.24.1. General

With the "Goerz wind and speed calculator" (see Fig.1.24/1) the wind triangle is calculated via an alignment table which has the angle scale 2 and the speed scale 1. The lengthwise movable and rotatable pointer 3 can be locked by means of the adjusting screw 4.



## 1.24.2. Use of the computer for wind correction- and ground speed calculations

The following example explains the application (see Fig. 1.24/2) :

wind angle  $\epsilon'$                        $45^\circ$   
 true airspeed                      100 km/h  
 wind speed                        10 m/s (36 km/h)

wind corr. angle                 $-14.7^\circ$   
 ground speed                    122.2 km/h

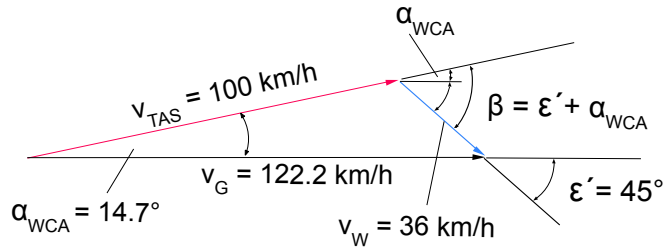


Fig.1.24/2

Solution (Fig.1.24/3):

1. The bearing of the pointer is set so that its centre line (drawn in red) runs through the wind speed 36 km/h and the wind angle  $45^\circ$ . The pivot point A of the pointer is then fixed. If the pointer is now adjusted so that it intersects the speed scale at the value 100 (TAS = 100 km/h), a WCA of approx.  $15^\circ$  (true value  $14.7^\circ$ ) can be read at the angle scale below the pointer (dashed red line).

2. The bearing pointer is set so that its centerline runs through the wind speed 36 km/h and the wind angle  $45^\circ + 15^\circ = 60^\circ$  (drawn in green). The pivot point B of the pointer is then fixed. If the pointer is now adjusted so that it intersects the angle scale at the value  $15^\circ$ , the ground speed of approx. 120 km/h (true value 122.2) can be read off the speed scale below the dashed green line.

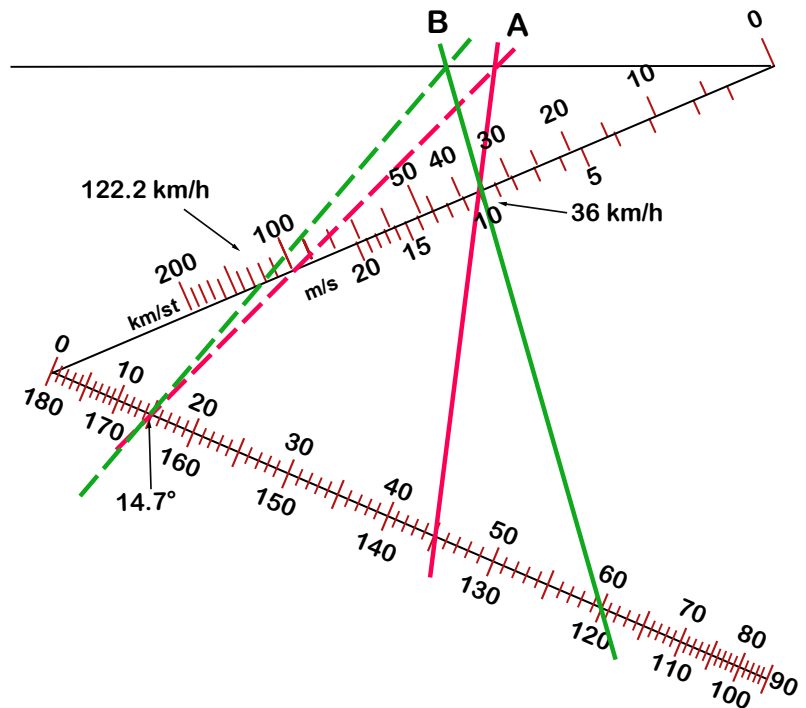


Fig.1.24/3

### Explanation:

The length of an unknown triangular side is calculable if the other sides and an angle or two angles and one side are known. In the first solution step, the unknown angle  $\alpha_{WCA}$  is calculated using the known speeds  $v_W$  and  $v_{TAS}$  as well as the known wind angle  $\epsilon'$  (red alignment lines from point A). In the second solution step, the unknown ground speed  $v_G$  is calculated using the known wind speed  $v_W$  and the angles  $\alpha_{WCA}$  and  $\beta$  (green alignment lines from point B). The angle  $\beta$  results to  $\beta = 45^\circ + \alpha_{WCA} = 45^\circ + 15^\circ = 60^\circ$  (see Fig.1.24/2). A triangle of two sides and the included angle cannot be reproduced in this diagram. However, since this problem occurs when calculating the wind from airspeed, groundspeed and wind correction angle, such a wind calculation is not possible.



### 1.24.3. Construction of the alignment line diagram for triangle calculations

Fig.1.24/5 shows the diagram schematically. It is based on this arbitrary choice:

- length of the angle scale AB
- length of the distance scale AC
- the angles  $\alpha$  and  $\beta$
- the position of point E (here chosen 50) on the distance scale.

The division of the angle scale was constructed over a linear division of the line DC. The connecting lines of the points  $F'$  and  $F''$ , e.g., through the point E lead to the points  $40^\circ$  and  $35^\circ$  on the angular scale.

The calculation of the position of the other line points is explained by an arbitrarily chosen triangle (see Fig.1.24/4).

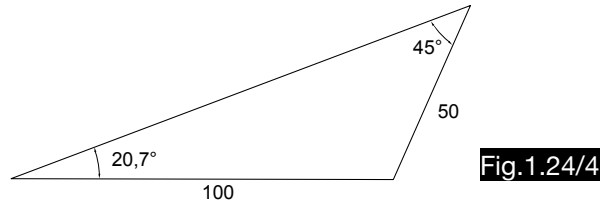


Fig.1.24/4

The line from the point  $45^\circ$  on the angle scale over the point E (on position 50 of the speed scale) leads to the point F on the line CD (guide rail of the pointer). The connecting line from point  $\alpha = 20.7^\circ$  on the angle scale to point F intersects the speed scale at point 100.

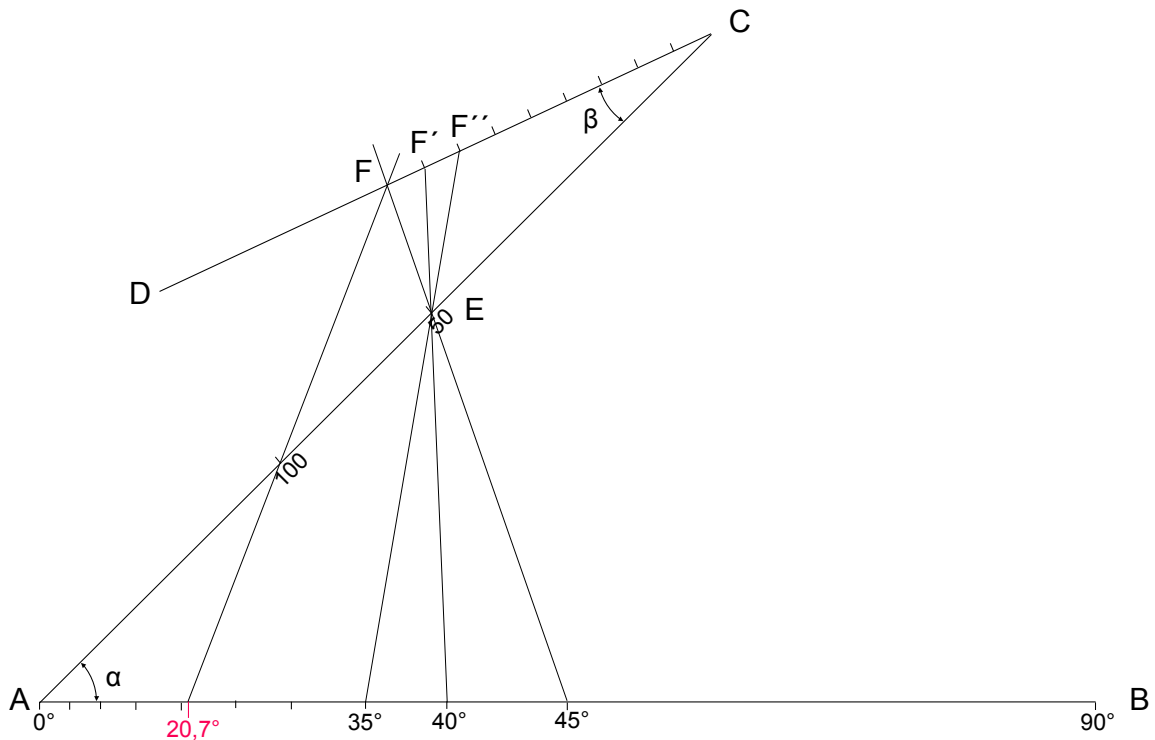


Fig.1.24/5

## 1.25. Calculation of the wind triangle according to Jensen

### 1.25.1. General

The NAVIGATIONAL COMPUTER invented by H. M. Jensen calculates the wind triangle graphically. Fig.1.25/4 shows the calculator (diameter of the original device 105 mm).

### 1.25.2. Use of the computer for wind correction- and ground speed calculations

The following example explains the application (see Fig. 1.25/1) :

true wind direction	315°
wind speed	40 kt
true airspeed	120 kt
true course	090°
$\alpha_{WCA}$	- 13.6°
ground speed	144.9 kt
true heading	076.4°

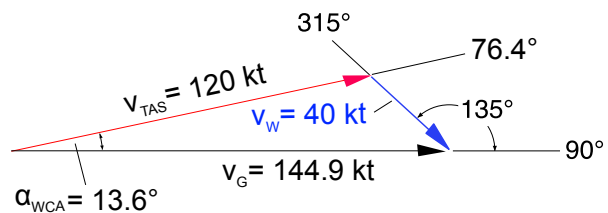


Fig.1.25/1

#### 1. Calculation of the wind angle (see Fig.1.25/2):

The red GS arrow is set to true course 90° on the black compass rose and opposite to the black wind direction 315° is shown on the red outer scale the wind angle 135°. At the wind angle the drift direction can be determined. In this case a right drift is created and the heading is lower by the wind correction angle than the course.

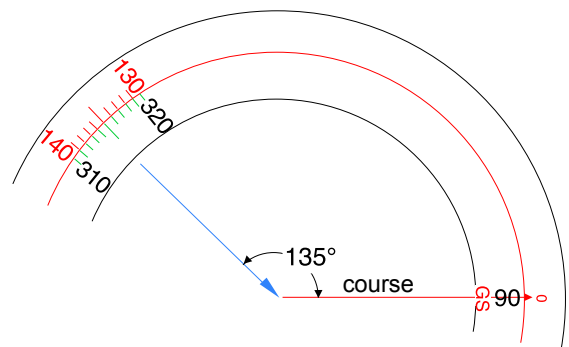


Fig.1.25/2

#### 2. Entry of the wind point (see Fig.1.25/3):

After the red arrow (GS) was shifted to the north, the wind point is marked with a pencil in the intersection of the 40 kt -line (wind speed) and the line of the wind angle (135°) .

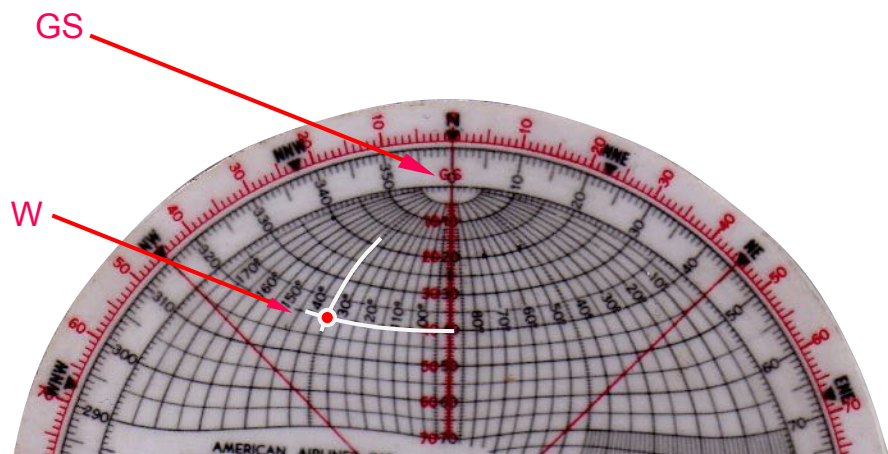
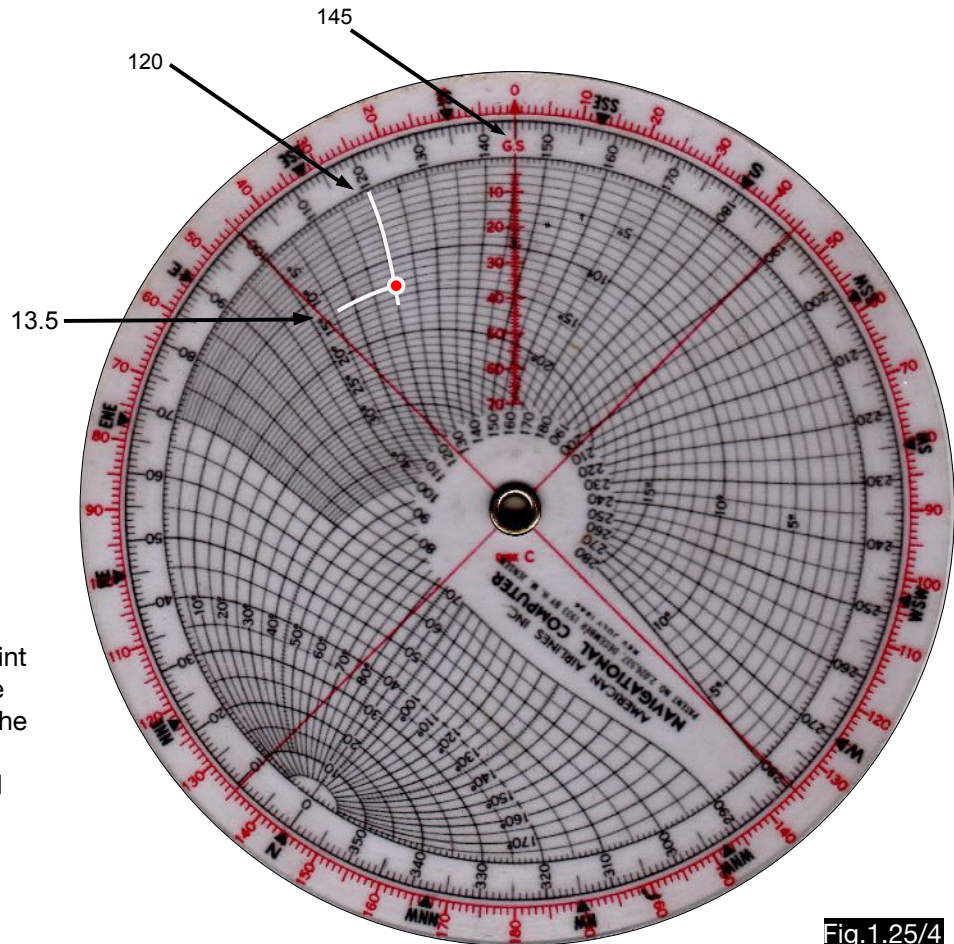


Fig.1.25/3

### 3. Calculation of the of the ground speed and the wind correction angle (see Fig.1.25/4):



The transparent disc is rotated until the wind point above the line TAS = 120 kt is standing. Under the wind point a wind correction angle of 13.5° can be read. The ground speed 145 kt is indicated under the red arrow (GS mark).

Fig.1.25/4

### 1.25.3. Use of the computer for wind determination from drift and ground speed

The following example explains the application:

magnetic heading	270°	wind speed	69.1 km/h
drift angle	+10°	wind angle	141.1°
variation	10°W	magnetic wind direction	138.9°
true airspeed	250 km/h	true wind direction	128.9°
ground speed	300 km/h		

Solution:

1. Determination of the wind point (see Fig.1.25/5):

- 1a. The ground speed arrow is set to 300 km/h.
- 1b. At the intersection of the lines TAS = 250 km/h and DA = +10°, the wind point is marked.

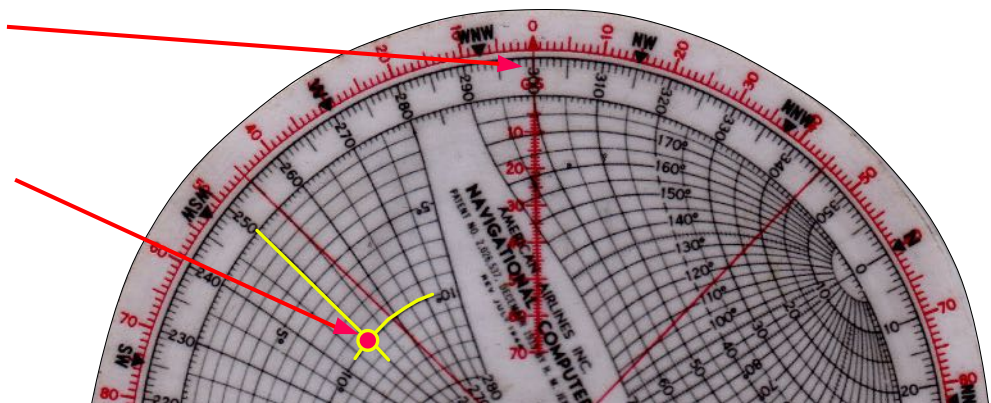


Fig.1.25/5



2. Determination of the wind angle and the wind speed (see Fig.1.25/6):

2a. The ground speed arrow will be set to North.

2b. Under the wind point the wind speed (69 km/h) and the wind angle (141°) are found.

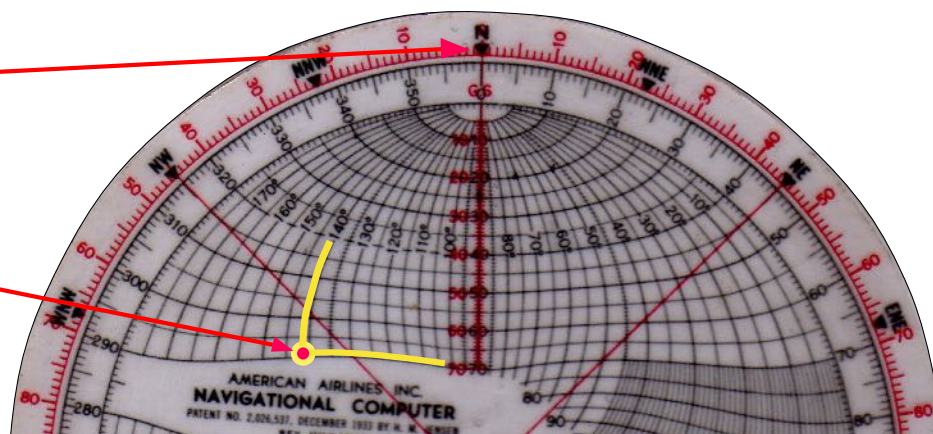


Fig.1.25/6

3. Determination of the wind direction (see Fig.1.25/7):

3a. The ground speed arrow is positioned above the magnetic track 280°.

3b. Opposite the magnetic wind angle of 141° (red outer scale) the magnetic wind direction 139° is found (black inner scale). The true wind direction results from the variation of 10° West to 129°.

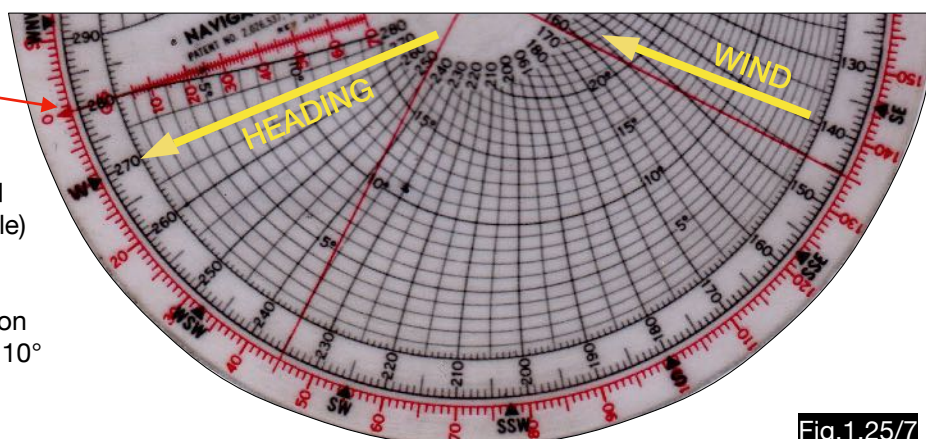


Fig.1.25/7

Note: The wind angle of 141° occurs in the red scale twice. Select the angle with the relevant drift direction (right in this case).

#### 1.25.4. Description of the process and the design

In the Jensen calculator the ground speed is represented as ordinate. The airspeeds are applied to them at a distance of one degree WCA. In order to complete the wind triangle, the wind vector, which can assume different directions (wind angles) and different lengths (speeds), must be drawn into this system. This is only possible in one place. If, for example, the angle between groundspeed and wind vector is 45° and the wind speed 20 kt, the wind vector drawn in red in Fig.1.25/8 is defined to scale and direction. At an assumed air speed of 100 kt, a wind triangle is obtained in which the WCA is approx. 8° and the ground speed approx. 113 kt.

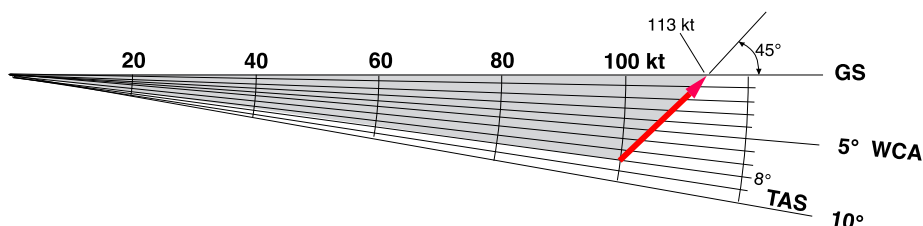


Fig.1.25/8

In the Jensen calculator, the wind triangles were arranged circularly for better handling, with the ordinate (ground speed) representing the outer circle. The airspeeds applied to these at different angles (WCA) are thus formed in a spiral shape. This means that all sides of the wind triangle including the wind vector to be fitted are curved lines (see Fig.1.25/9).

The wind triangle shown in Fig.1.25/8 is highlighted in white in Fig.1.25/9.



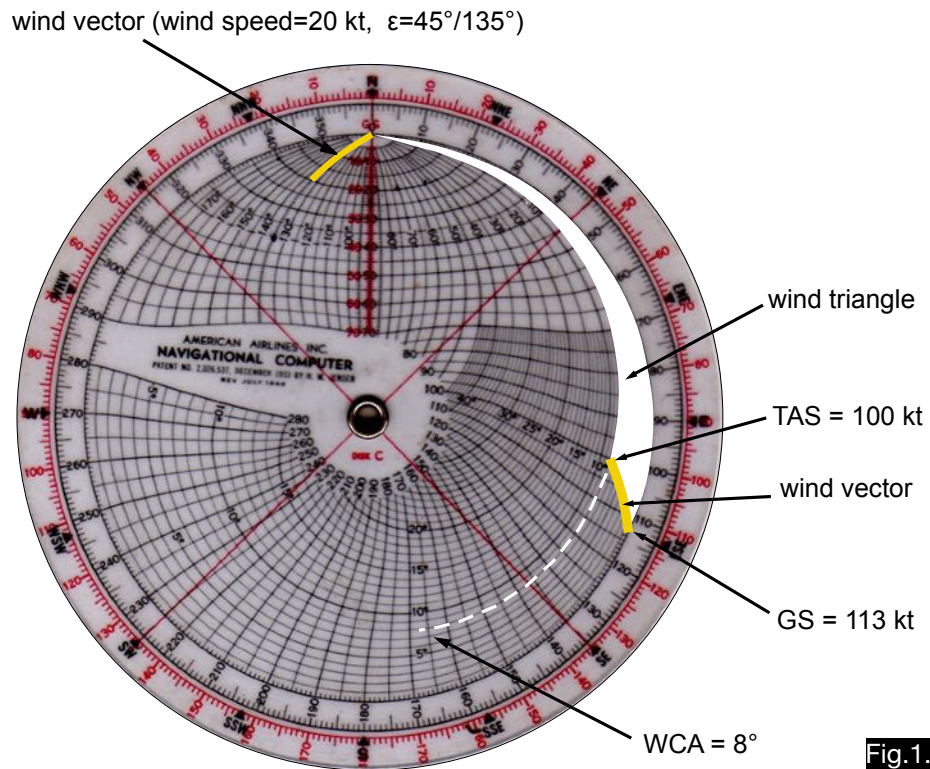


Fig.1.25/9

## 1.26. Calculation of the wind triangle according to Gillmer

### 1.26.1. General

The GILLMER COMPUTER invented by T. C. Gillmer calculates the wind triangle mechanically using two diagrams. Fig.1.26/1 shows its part one with the compass rose diagram and Fig.1.26/2 the second transparent part with the drift angle diagram. With this method, the triangle calculation is performed in principle in the same way as with the diagram slide computer (see 1.2).

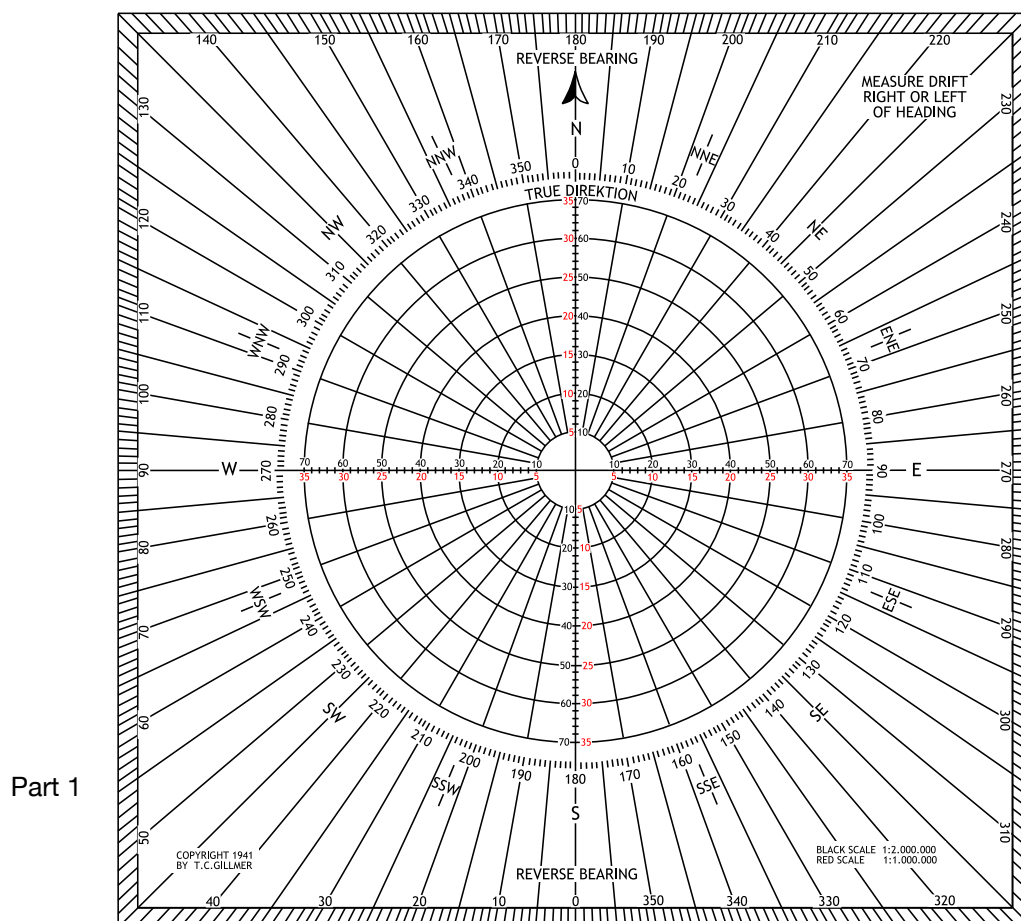


Fig.1.26/1

Part 2

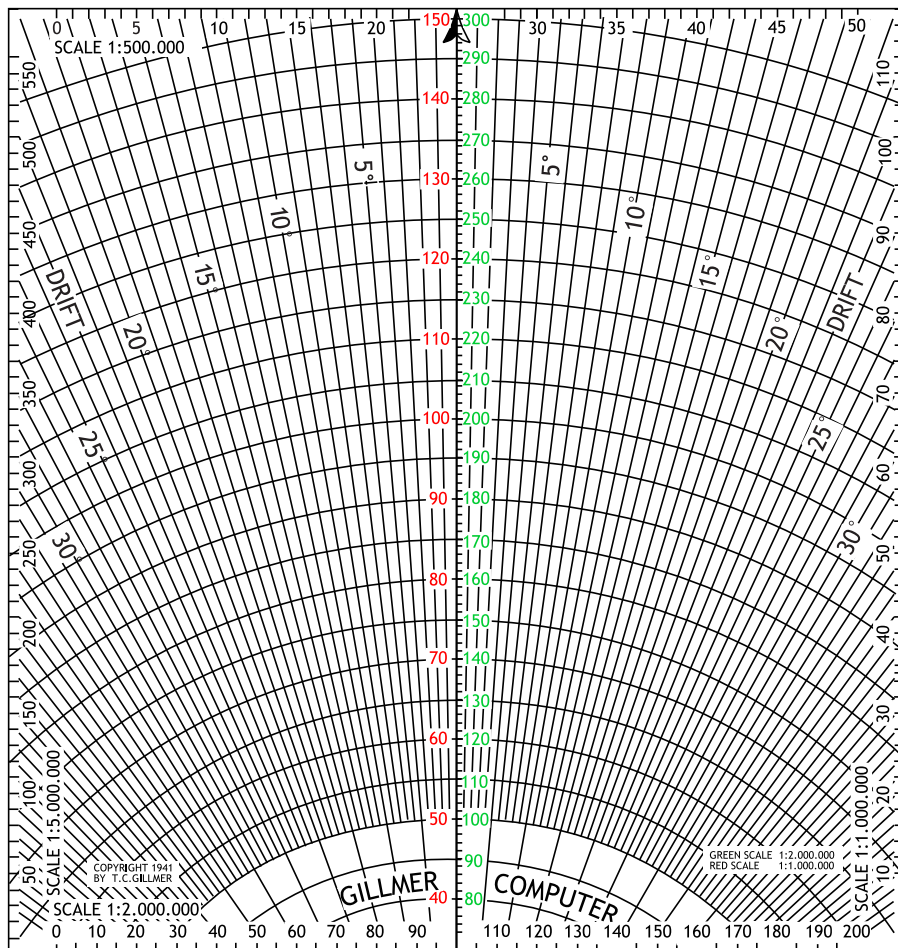


Fig.1.26/2

## 1.26.2. Use of the computer for wind correction- and ground speed calculations

Both parts are placed one over the other to build a complete wind triangle. Then wind correction angle and ground speed can be read directly (see Fig. 1.26/3).

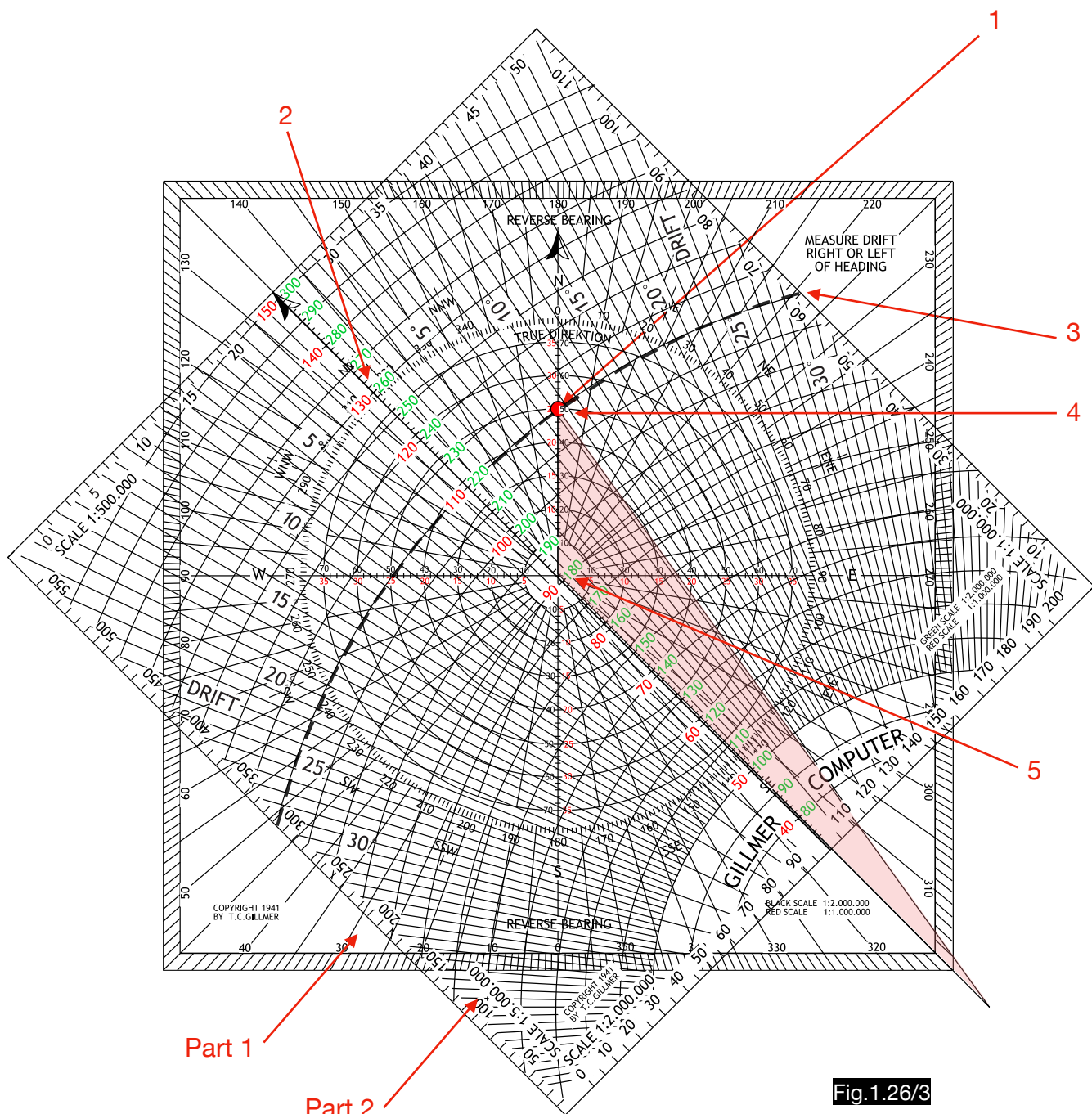
The following example explains the application:

true wind	360°/50 kt
true airspeed	220 kt
true course	315°

$\alpha_{WCA}$	+ 9.3°
ground speed	181.8 kt

Solution (see Fig. 1.26/3):

1. On part 1 the wind point is marked at 50 kt (black scale) and 360°.
2. Part 2 is thus aligned with part 1, that the axis of symmetry (arrow) is above the course line 315°.
3. Part 2 will be shifted along the course 315° so that the highlighted dashed TAS-circle marked 220 (green digits) is going through the wind point.
4. A wind correction angle of +9° is determined at the wind point.
5. At the center of Part 1 at the green scale of the Part 2, the ground speed of 182 kt is found.





## 1.27. Calculation of the wind triangle according to Plumly

### 1.27.1. General

The PLUMLY COMPUTER calculates the wind triangle mechanically using the sine theorem and logarithmic scales. Only wind angles below  $90^\circ$  are used. The definition of the sign of the WCA and the sign of the difference  $GS - TAS$  is facilitated by inscriptions in the four quadrants (e.g. TAIL WIND R.H. or HEAD WIND L.H. and + HEADING or - SPEED). Fig.1.27/1 shows the side with the sine scale (diameter of the calculator 95 mm). The sine scale is divided into two sections ( $1^\circ$ - $10^\circ$  and  $10^\circ$ - $90^\circ$ ), placed opposite each other. The sine values whose angles are opposite to each other differ by a factor of 10, which is irrelevant for logarithmic calculations.. One example is highlighted by red arrows ( $\sin 3^\circ = 0.0523$ ;  $\sin 31,53^\circ = 0.523$ ). This trick made it possible to shorten the sine scale to half its length.

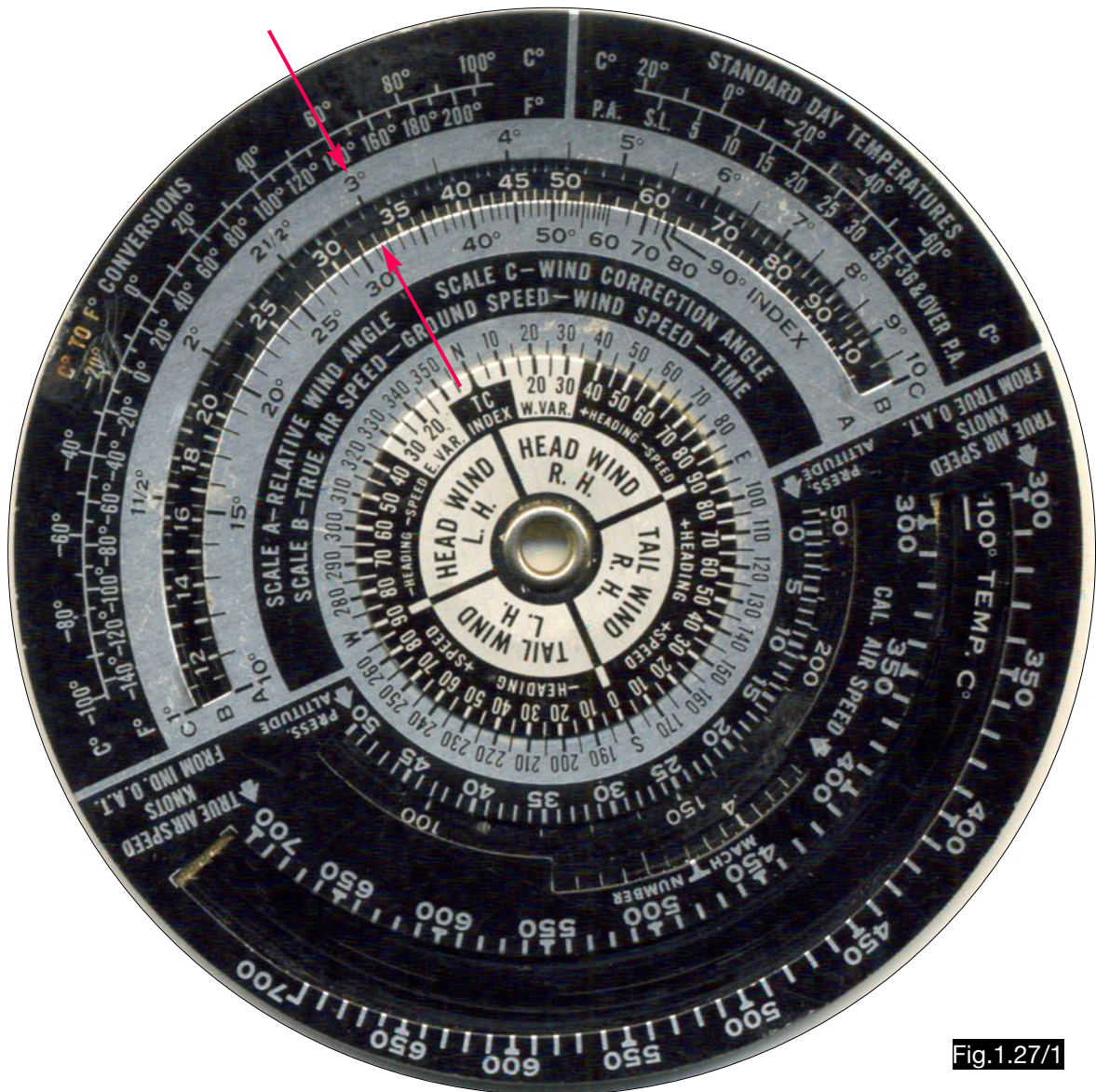


Fig.1.27/1



### 1.27.2. Calculation of wind correction angle and ground speed

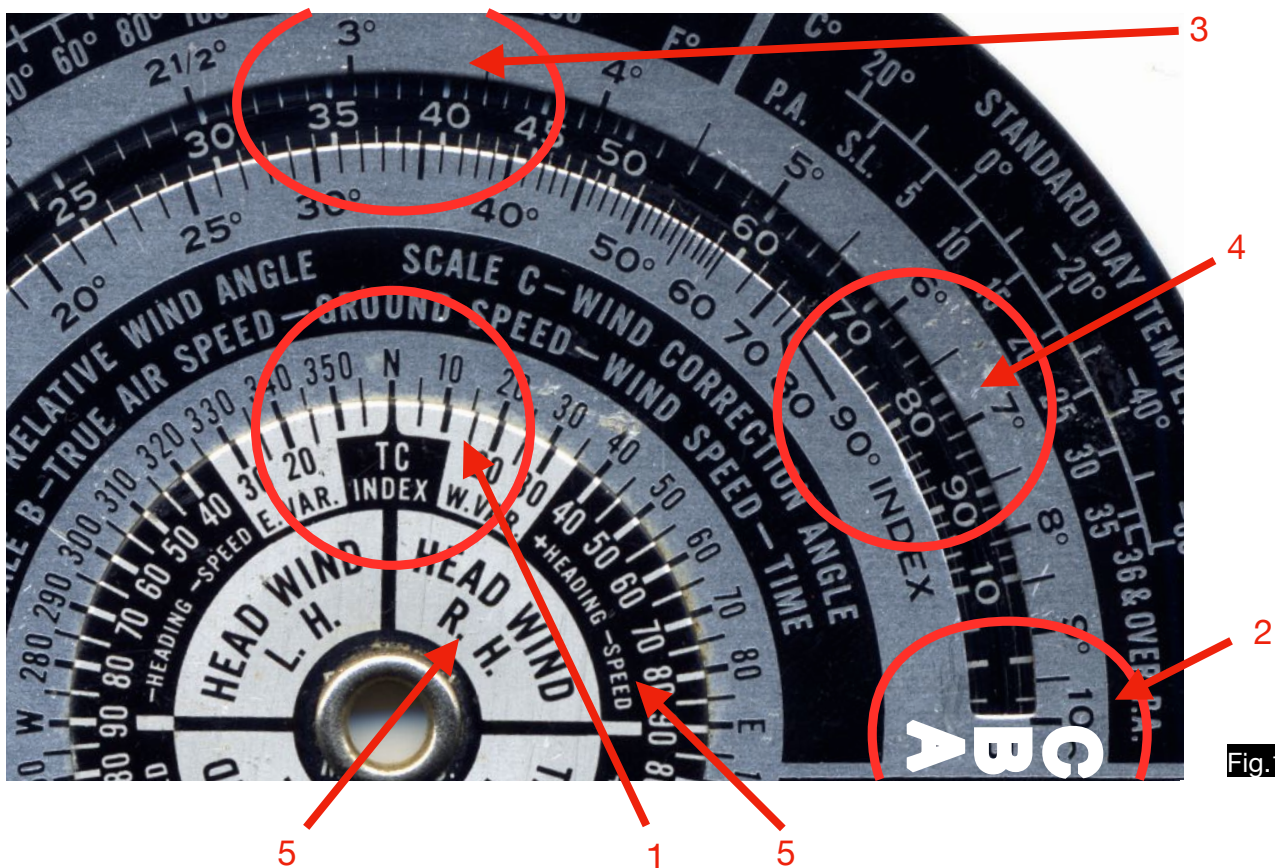
The following example explains the application (see Fig.1.27/2):

true course	360°	wind correction angle	3.31°
true airspeed	120 kt	ground speed	80.5 kt
true wind	010°/40 kt		

Solution:

The TC index is set to 360° (N) on the inner scale and the wind angle 10° is indicated opposite the wind direction 10° (1). Then the figure 12 (TAS) in scale B is positioned opposite the wind angle 10° in scale C (2). At the figure 40 (wind speed) in scale B one finds in scale C the WCA = 3.3° (3). The ground speed is calculated to 80 kt (4) opposite the figure 6.7° (wind angle - WCA = 10° - 3.3°). A help is the additional information (5):

HEAD WIND R.H.  
- SPEED (GS < TAS)  
+ HEADING (HDG = CRS + WCA).



### 1.27.3. Use of the computer for wind determination from drift and ground speed

The following example explains the application (see Fig.1.27/3):

true heading	090°
true airspeed	100 kt
drift angle	-16.5°
ground speed	124 kt
wind angle	61,8°
true wind	208.2°/40.0 kt

As already described under 1.3.2., it is efficient to use the method of "fitting" the drift angle between the values of the airspeed and ground speed for the wind calculation by means of log sine scales. In order to "fit" the drift angle of  $16.5^\circ$  between the digits 10 (corresponds to TAS = 100 kt) and 12.4 (corresponds to GS = 124 kt) the sine scale must be brought into the position shown in Fig.1.27/3. Then the wind speed of 40 kt can be read opposite the drift angle  $16.5^\circ$  and opposite the 12.4 (ground speed 124 kt) the wind angle  $62^\circ$ .

Since the drift angle is negative (drift to the left) and the ground speed is greater than the airspeed, there is a tailwind from the right. After setting true course (TC) to  $90^\circ$  the wind direction is found in the quadrant with this inscription:

> TAIL WIND R.H. <

Opposite the wind angle of  $62^\circ$  the wind direction  $210^\circ$  is found here.

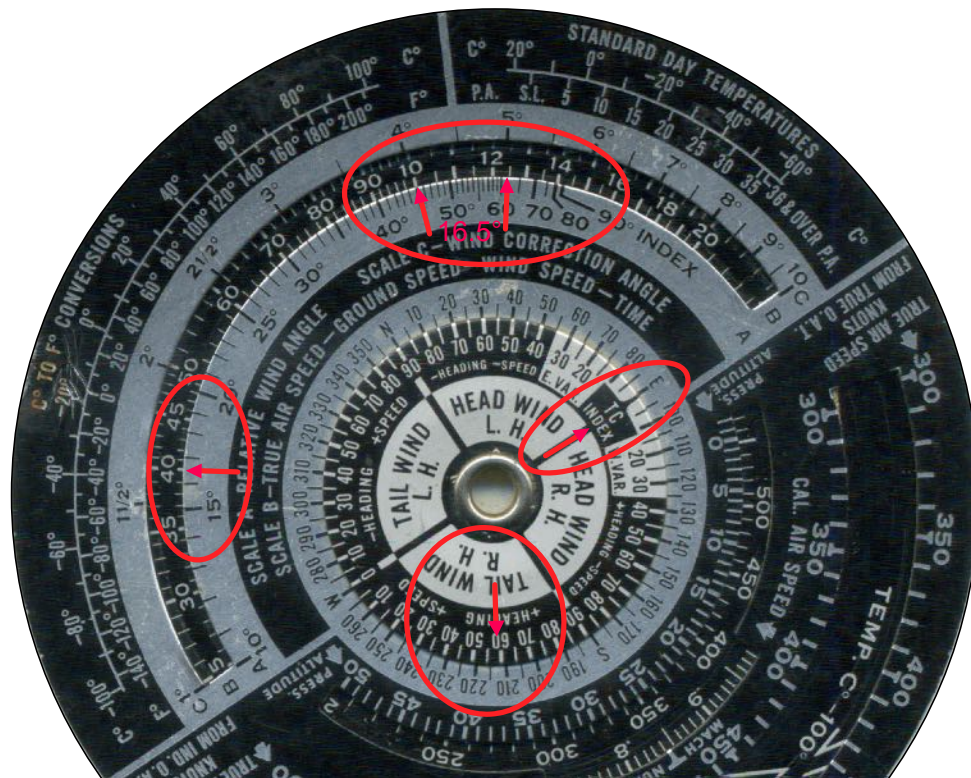


Fig.1.27/3

## 1.28. Calculation of the wind triangle according to Braun

The „BRAUN COURSE FINDER“ allows the direct determination of the true heading and the ground speed depending on the wind and airspeed vectors. The device consists of a board made of transparent material which is placed on the map. The handling is facilitated by a belonging ruler of the same speed scale. Using this example of a flight from point A to point B the application is described:

true course	097°
true airspeed	130 km/h
true wind	045°/ 10 m/s (36 km/h)
true heading	084.4°
ground speed	104.7 km/h



- $$v_G = 130 \frac{\overline{AW}}{\overline{AM}}$$

## 1.29. Calculation of the wind triangle using the computer DME plus

### 1.29.1. General

The computer is originally designed to determine the distance to a VOR by cross bearing to a second station. However, calculations of the wind triangle are also possible. The calculation procedure is described with the aid of Fig.1.29/1 using the following example:

magnetic course	120°	magnetic heading	131.7°
magnetic wind	190°/40 kt	ground speed	167.5 kt
true airspeed	185 kt		

### 1.29.2. Use of the computer for wind correction- and ground speed calculations

Solution:

1. Set mark "B" to magnetic course 120°.
2. Set wind arm to 190°.
3. Turn both disks and the wind arm together so that one of the red lines runs parallel to the connecting line of points W (wind speed) and C (true airspeed). The hatched triangle WCM represents the wind triangle.
4. At the mark "A" the magnetic heading of 132° can be read.
5. The ground speed is determined between the two black arrows (at points W and C) to 168 kt (180 - 12).

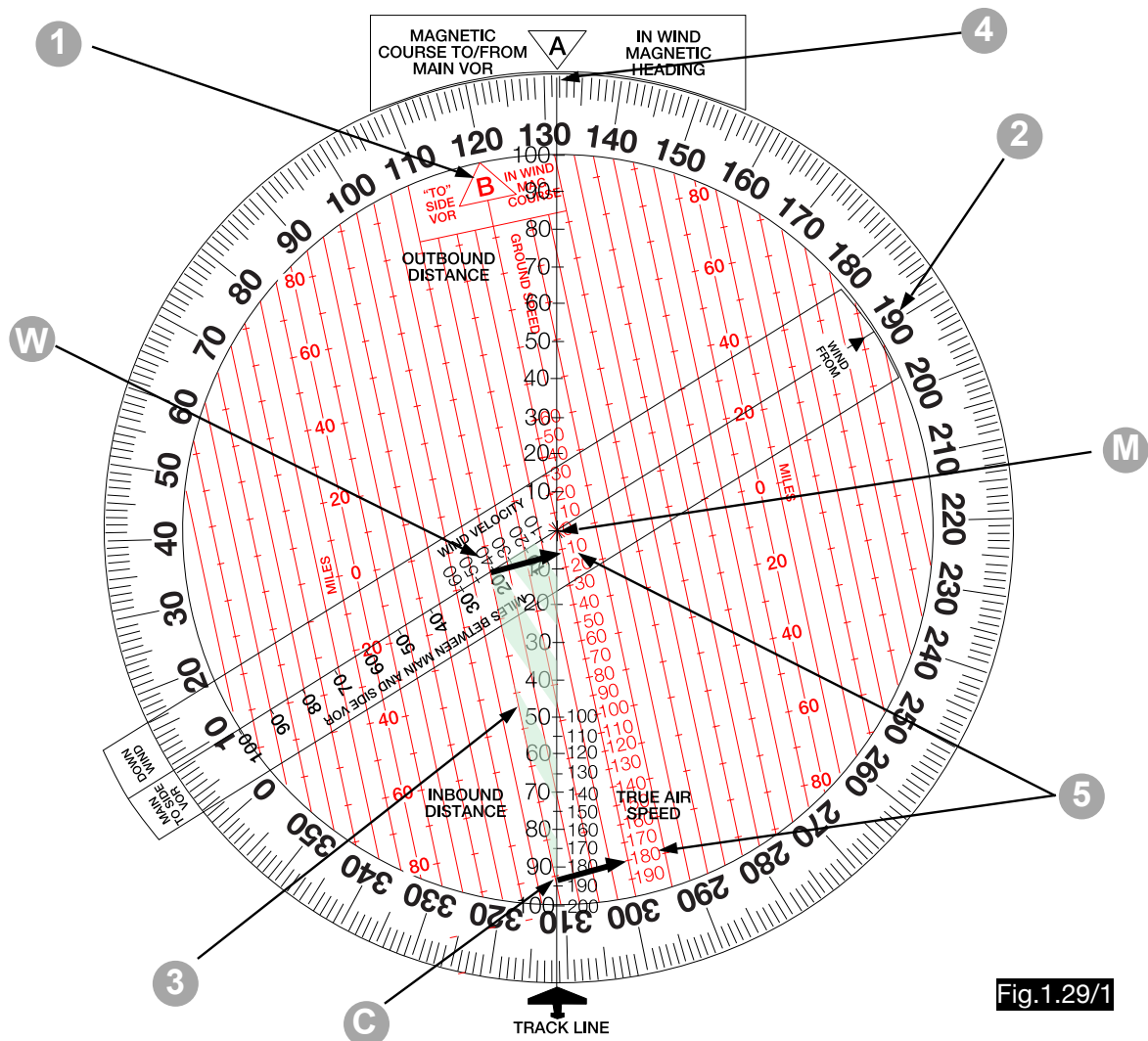


Fig.1.29/1



## 1.30. Calculation of the wind triangle using the computer IPC-1

### 1.30.1. General

IPC-1 computers are intended primarily for the calculation of flight manoeuvres, such as holding patterns, etc. For the calculation of drift angle and ground speed, the components of the wind ( $v_{WC}$  and  $v_{WL}$ ) are determined and then the drift angle and ground speed are calculated approximately.

### 1.30.2. Use of the computer for drift angle- and ground speed calculations

The following example explains the application:

true course	010°	drift angle	6.8°
true wind	220°/30 kt	ground speed in the drift triangle	126.8 kt
true airspeed	100 kt		

Solution:

1. After setting the true course 010° and the wind direction 220° the wind components  $v_{WL} = 26$  kt and  $v_{WC} = 15$  kt can be determined (see Fig.1.30/1). The ground speed is approximately:

$$V_G \approx V_{TAS} + V_{WL} = 100 \text{ kt} + 26 \text{ kt} = 126 \text{ kt}$$

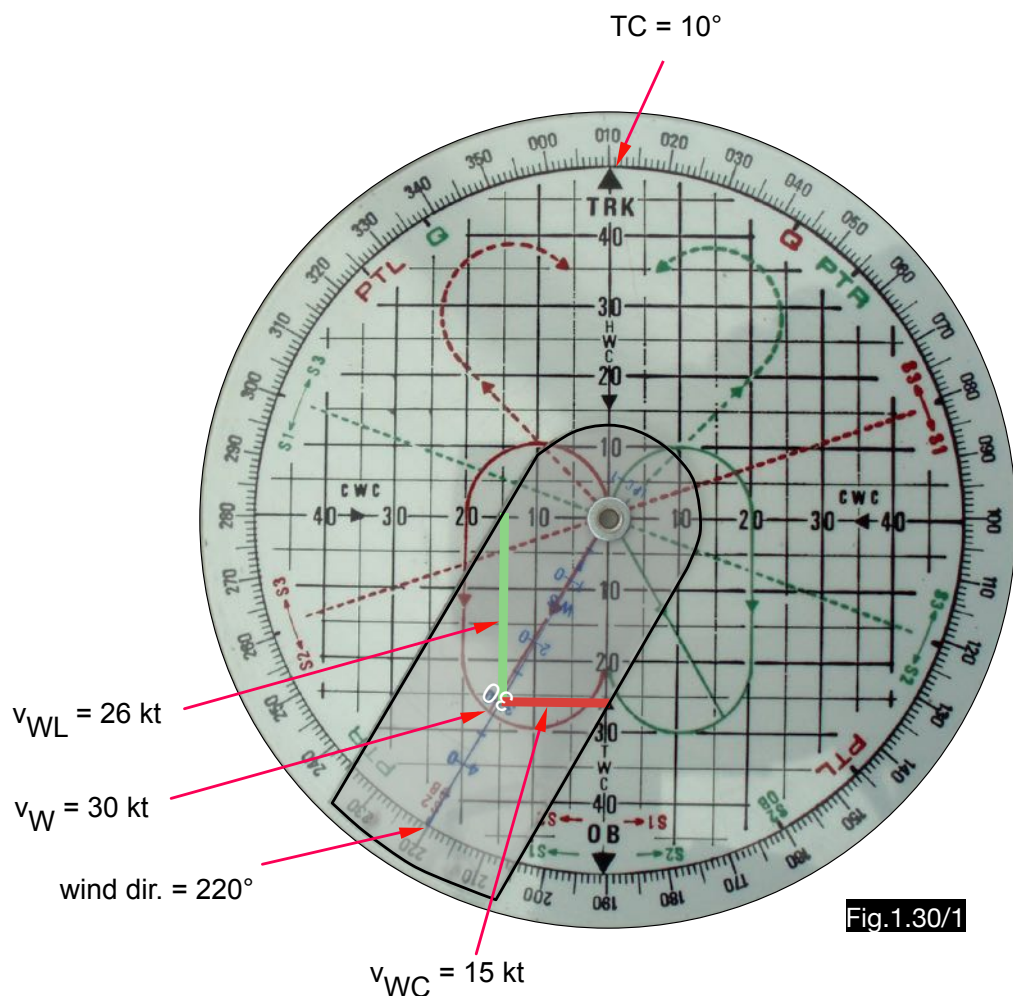
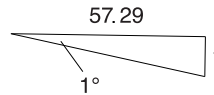


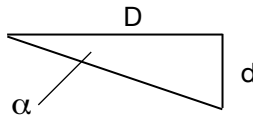
Fig.1.30/1

2. A simple approximation formula for a rectangular triangle can be derived:

$$\operatorname{tg} 1^{\circ} = \frac{1}{57.29}$$



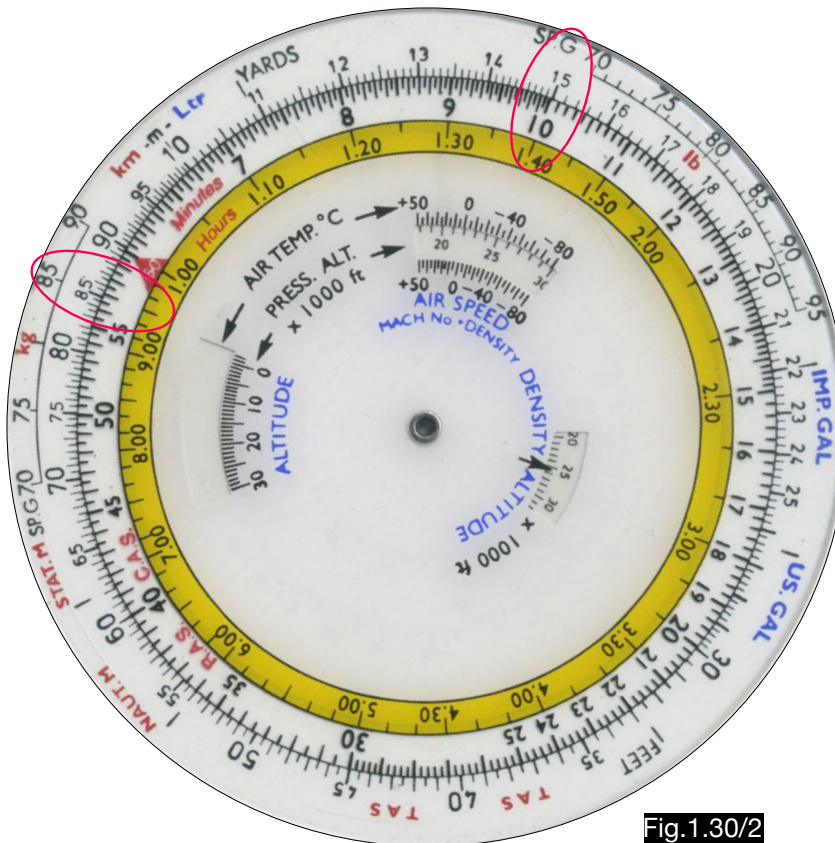
From this follows for small angles  $\alpha$ :



$$\alpha [^{\circ}] \approx \frac{d}{\frac{D}{57.3}} \rightarrow \alpha [^{\circ}] \approx \frac{d \times 57.3}{D} \quad [1.30/1]$$

Therefore, in the drift triangle applies:  $\alpha_{DA} [^{\circ}] \approx V_{WC} \frac{57.3}{V_{TAS}} \rightarrow \alpha_{DA} [^{\circ}] \approx 15 \frac{60}{100} = 9$

One does not find always the solution so easily by mental arithmetic. The calculation is simpler in the logarithmic scale on the other side of the calculator (see Fig. 1.30/2), in which the example used here was set. This calculation leads to the result  $\alpha_{DA} = 8.6^{\circ}$ .



$V_{WC} = 15$	$\alpha_{DA} = 8.6$
$V_{TAS} = 100$	57.3

**Note:**

A mark which facilitates the finding of the coefficient 57.3 was not provided at this calculator. However, on the logarithmic scales of other calculators these are sometimes attached (two examples see Fig.1.30/3 and 4).

Fig.1.30/2



Fig.1.30/3



Fig.1.30/4

## 1.31. Calculation of the wind triangle according to Schmidt

With the method specified by F. Schmidt, the WCA can be calculated approximately and without aids only using mental arithmetic.

From the sine theorem (see equ. 1.1/1) one obtains:

$$\sin \alpha = \frac{V_W}{V_{TAS}} \sin \epsilon$$

Since  $\sin 1^\circ = \frac{1}{57.3}$  the following applies to small angles:  $\sin 2^\circ \approx \frac{2}{57.3}$  or  $\sin \alpha \approx \frac{\alpha^\circ}{57.3}$  and consequently:

$$\alpha^\circ \approx 57.3 \sin \epsilon \frac{V_W}{V_{TAS}} \quad [1.31/1]$$

For simplification, the product  $57.3 \sin \epsilon$  is multiplied by 0.1 and grouped together to form the factor F. This requires that  $V_{TAS}$  in the denominator must also be multiplied by 0.1, so that the formula now takes this form:

$$\alpha_{WCA}^\circ \approx \frac{F V_W}{0.1 V_{TAS}} \quad [1.31/2]$$

Fig.1.31/1 shows the products  $57.3 \sin \epsilon$  and the selected factors F for the wind angles  $0^\circ - 90^\circ$ . Table 1.31/1 gives information about the occurring errors for this example:  $V_{TAS} = 100$  kt,  $V_W = 40$  kt.

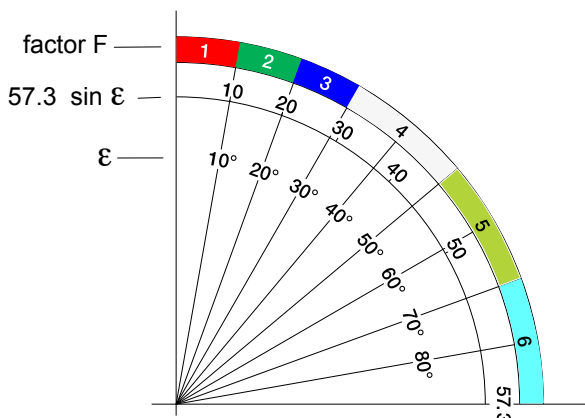


Fig.1.31/1

$\epsilon [^\circ]$	WCA $[^\circ]$ true value	WCA $[^\circ]$ = $F V_W / 0.1 V_{TAS}$	error $[^\circ]$
0 - 10	0 - 4	$40 \times 1:10 = 4$	0 - 4
10 - 20	4 - 7.9	$40 \times 2:10 = 8$	0.1 - 4
20 - 30	7.9 - 11.5	$40 \times 3:10 = 12$	0.5 - 4.1
30 - 40	11.5 - 14.9	$40 \times 4:10 = 16$	1.1 - 4.5
40 - 50	14.9 - 17.9	$40 \times 4:10 = 16$	1.1 - 1.9
50 - 60	17.9 - 20.2	$40 \times 5:10 = 20$	0.2 - 2.1
60 - 70	20.2 - 22	$40 \times 5:10 = 20$	0.2 - 2
70 - 80	22 - 23	$40 \times 6:10 = 24$	1 - 2
80 - 90	23 - 23.5	$40 \times 6:10 = 24$	0.5 - 1

Table 1.31/1

The computer WCA-2 (Fig.1.31/2) is used to determine the factors F as a function of course and wind direction. It does not help to calculate the wind correction angle by means of equ.1.31/1.



Fig.1.31/2



## 1.32. Calculation of the wind triangle according to Maleysson

### 1.32.1. General

With the computer Le Dériveur, invented by Charles Maleysson, the wind triangle is calculated using approximation formulas.

### 1.32.2. Calculation of wind correction angle, ground speed and flight time

The calculation method is described with the aid of Fig.1.32/1 and 2 in the following example (the French designations are added in brackets):

true wind direction $\delta$ ( $v_W$ )	340°
wind speed $v_W$ ( $F_W$ )	40 kt
true airspeed $v_{TAS}$ ( $v_p$ )	110 kt
true course TC ( $R_v$ )	005°
distance D	85 NM
wind angle $\epsilon$ (a)	25°
wind cross component $v_{WC}$ ( $X^\circ$ )	16.9 kt
wind longitudinal component $v_{WL}$ ( $V_e$ )	34.6 kt
wind corr. angle $\alpha_{WCA}$ ( $x^\circ$ )	8.8°
ground speed $v_G$ ( $v_s$ )	74.4 kt
flight time t ( $T_e$ )	68.5 min

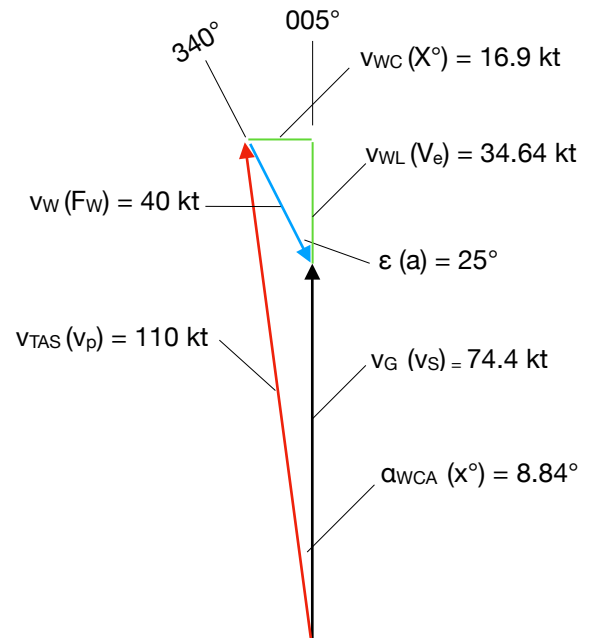


Fig.1.32/1

Solution (see Fig.1.32/2):

1. The true course 005° is set on the computer.
2. Along the line from the wind direction 340° to the center the  $\sin \epsilon \approx 0.42$  is found. The construction of the determination circle for the sine values is based on laws which are illustrated in Fig.1.32/3.
3. Since  $|\cos \alpha| = |\sin (\alpha \pm 90)|$  is, one finds along the line from the course 250° (340° - 90°) to the center the  $\cos \epsilon \approx 0.9$  (the marker ■ is used for finding the courses  $\epsilon - 90^\circ$  more easily).

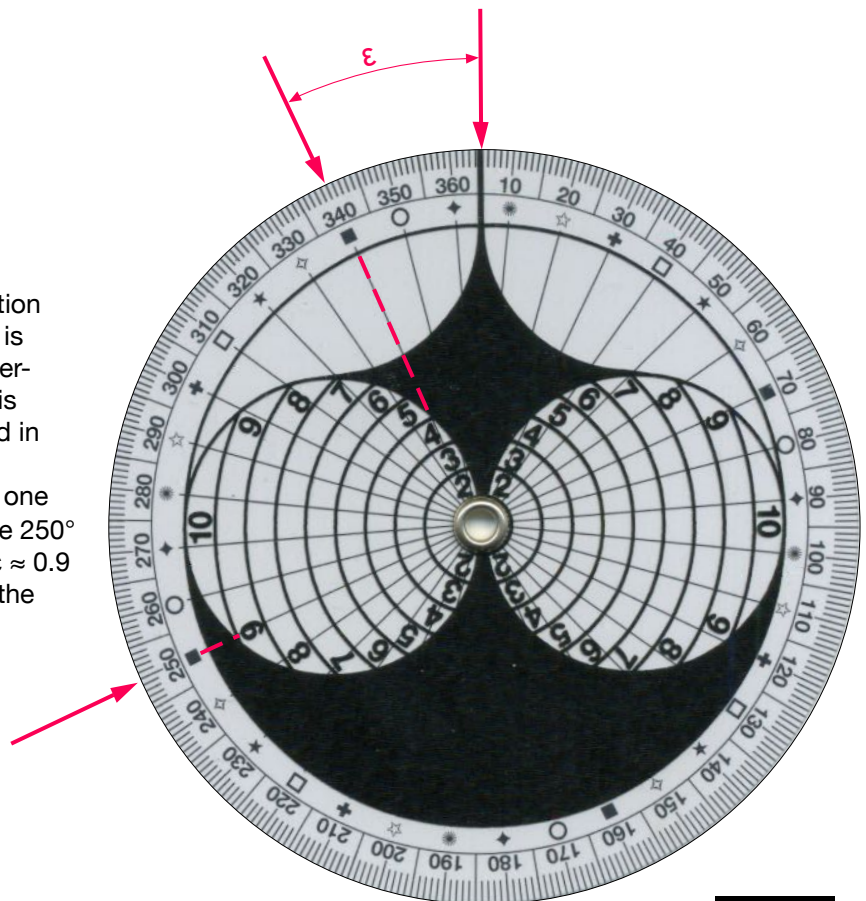


Fig.1.32/2



4. The wind correction angle  $\alpha_{WCA}$  ( $x^\circ$ ) is calculated according to the following formula (see equ. 1.31/1):

$$\alpha^\circ \approx 57.3 \sin \epsilon \frac{V_W}{V_{TAS}} \approx \frac{60}{V_{TAS}} V_W \sin \epsilon$$

In order to facilitate the calculation, the factor  $Fb = 60/V_{TAS}$  is introduced and the formula takes this form with the French designations:

$$x^\circ \approx Fb Fw \sin a$$

All formulas required here are listed next to the table of the factors  $Fb$  on the back of the calculator (see Fig. 1.32/4). For  $V_{TAS} = 110$  kt the factor  $Fb = 0.55$  is found here, so that the wind correction angle is obtained:

$$x^\circ \approx 0.55 \times 40 \times 0.42 = 9.2^\circ$$

5. The ground speed is calculated approximately according to the following formula:

$$V_G \approx V_{TAS} \pm V_W \cos \epsilon$$

With the abbreviations used here one gets:

$$V_S \approx V_P \pm F_W \cos a = 110 - 40 \times 0.9 = 74 \text{ kt}$$

6. The flight time  $t$  in minutes for the distance  $D$  in NM and  $V_G$  in kt results to:

$$t = \frac{60}{V_G} D = Fb D$$

On the back of the calculator this relation is shown like this:

Temps estimé avec vent = distance (NM) x NFb

With the factor  $Fb = 0.8$  (see Fig. 1.32/4) and  $D = 85$  NM one obtains:

$$Te = Fb D = 0.8 \times 85 = 68 \text{ min}$$

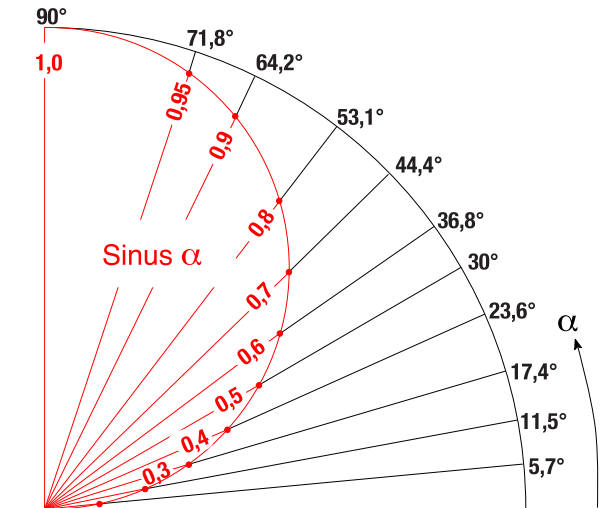


Fig.1.32/3

V	Fb	V	Fb	V	Fb	V	Fb	V	Fb	Ve = Fw x cosa	Vs = Vp +/- Ve
30	2.00	80	0.75	100	0.60	120	0.50	140	0.43		
35	1.71	82	0.73	102	0.59	122	0.49	145	0.41		
40	1.50	84	0.71	104	0.58	124	0.48	150	0.40		
45	1.33	86	0.70	106	0.57	126	0.48	155	0.39		
50	1.20	88	0.68	108	0.56	128	0.47	160	0.38		
55	1.09	90	0.67	110	0.55	130	0.46	165	0.36		
60	1.00	92	0.65	112	0.54	132	0.45	170	0.35		
65	0.92	94	0.64	114	0.53	134	0.45	175	0.34		
70	0.86	96	0.63	116	0.52	136	0.44	180	0.33		
75	0.80	98	0.61	118	0.51	138	0.43	185	0.32		

Temps estimé avec vent = distance (NM) x NFb

Fig.1.32/4

## 1.33. Calculation of the wind triangle using the GATCO 1 calculator

### 1.33.1. General

The GATCO-1 calculator is provided for the approximate determination of the wind correction angle and ground speed for air speeds from 90 to 130 (kt, m.p.h., km/h).

### 1.33.2. Use of the computer for wind correction- and ground speed calculations

The following example explains the application (see Fig.1.33/1):

true course	010°
true wind	220°/30 kt
true airspeed	95 kt
wind correction angle $\alpha_{WCA}$	9.1°
$V_{TASeff}$	93.8 kt
$V_{WL}$	25.0 kt
ground speed $V_G$	119.8 kt

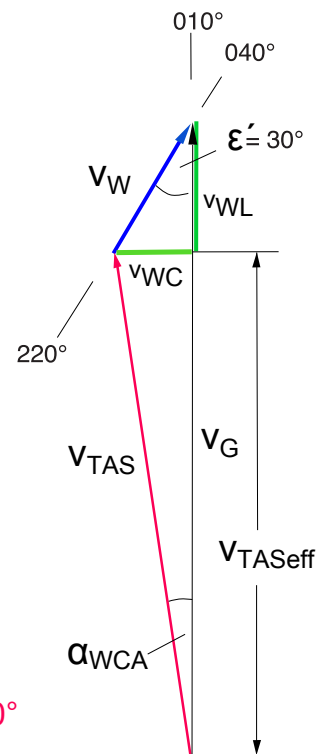


Fig.1.33/1

Solution (see Fig.1.33/2):

After setting the true course 010°, the pointer is set to the wind direction 220°. In the red DRIFT CORRECTION scale (wind correction to the left) the factor 0.3 is read, which has to be multiplied by the wind speed to find the wind correction angle:

$$\alpha_{WCA} = 30 \text{ kt} \times 0.3 \text{ deg/kt} = 9^\circ$$

In the light green GROUND SPEED CORRECTION scale ( $V_G > V_{TAS}$ ) the factor 0.75 is read, which must be multiplied by the wind speed to calculate the speed increase and thus the ground speed:

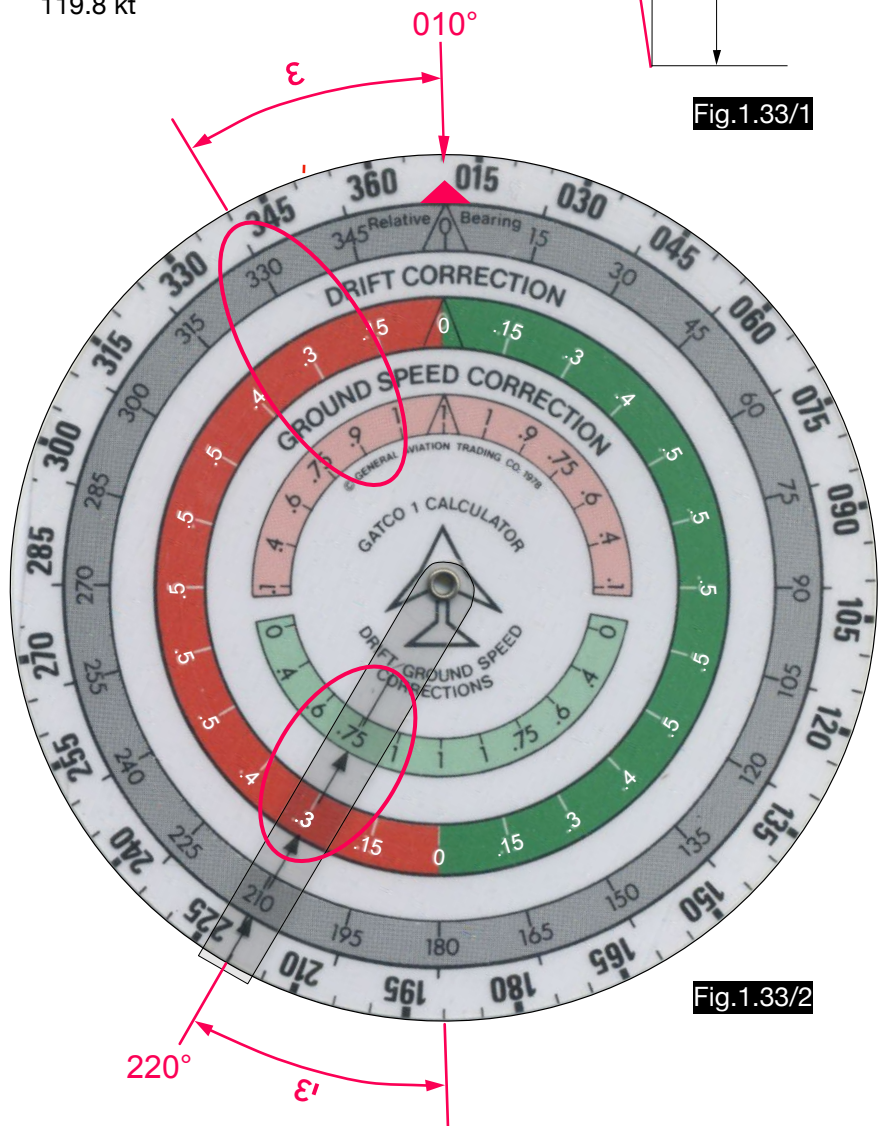


Fig.1.33/2

$$V_G = V_{TAS} + V_W \times 0.75 = 95 \text{ kt} + 30 \text{ kt} \times 0.75 = 95 \text{ kt} + 22.5 \text{ kt} = 117.5 \text{ kt}$$

### Approximate solution for the wind correction angle:

According to equ. 1.31/1, the wind correction angle for the assumed  $v_{TAS} = 95$  kt applies:

$$\alpha_{WCA} [^\circ] \approx v_w \frac{57.3}{95} \sin \epsilon = k v_w$$

$\frac{57.3}{95} \sin \epsilon$	0.156	0.302	0.522	0.582
$\cos \epsilon$	0.96	0.87	0.5	0.26
$\epsilon$	15°	30°	60°	75°

**Tab.1.33/1**

The factors  $k = \frac{57.3}{95} \sin \epsilon$  were entered in the computer (see Fig.1.33/2) in the circular ring DRIFT CORRECTION. Table 1.33/1 shows four recalculated factors.

### Approximate solution for the ground speed:

According to Fig.1.33/1 applies:  $v_G \approx v_{TAS} \pm v_w \cos \epsilon$

In the computer, the values for the  $\cos \epsilon$  can be read in the inner circle ring as a function of the wind angle. The values in the upper pink ring are negative i.e. after multiplication with the wind speed one gets a negative correction value, so that the ground speed is lower than the airspeed. The correction values in the lower ring highlighted in light green apply to tailwind and are positive (i.e.  $v_G > v_{TAS}$ ).

As Tab. 1.33/1 shows, the factors of the calculator are not exactly equal to the  $\cos \epsilon$ . Since the  $v_{TAS_{eff}}$  is always lower than the  $v_{TAS}$ , too less is subtracted in headwinds and too much is added in tailwinds. To reduce these errors, the factors have been adjusted. In the set example (Fig.1.33/2) at  $\epsilon = 30^\circ$  instead of 0.87 (=  $\cos 30^\circ$ ) the value 0.75 is read, which is obviously too small. The correct coefficient would be 0.82:

$$v_G = 95 \text{ kt} + 30 \text{ kt} \times 0.82 = 119.6 \text{ kt}$$

On the headwind side, the higher value 0.906 must therefore be entered instead of  $\cos 30^\circ = 0.87$  at a wind angle of  $30^\circ$ , which is actually the case. Table 1.33/2 summarizes the results of the example used here when the specified span of  $90 \text{ kt} < v_{TAS} < 130 \text{ kt}$  is used. The results shown in red in the table were obtained if the value 0.82 is used instead of the factor 0.75.

$v_{TAS} [\text{kt}]$		90	100	105	110	130
$\alpha_{WCA}$	GATCO	9				
	exact	9.6	8.62	8.2	7.8	6.6
$v_G [\text{kt}]$	GATCO	112.5 (114.6)	122.5 (124.6)	127.5 (129.6)	132.5 (134.6)	152.5 (154.6)
	exact	114.7	124.8	129.8	134.8	155.0

**Tab.1.33/2**



# 1.34. Tabular calculation of the wind triangle using a mechanical computer

## 1.34.1.General

Tab.1.34.1. shows the section of a table with the help of which wind triangles can be calculated. However, the precondition for their application is the prior calculation of the wind angle, which is more convenient with mechanical computers, which also often allow a simple reading of wind correction angle and ground speed.

TABLE I.—GROUND-SPEED AND DRIFT TABLE												
Wind angle	10-mile wind		15-mile wind		20-mile wind		25-mile wind		30-mile wind		40-mile wind	
	Wind corr.	Track speed, m.p.h.	Wind corr.	Track speed, m.p.h.	Wind corr.	Track speed, m.p.h.	Wind corr.	Track speed, m.p.h.	Wind corr.	Track speed, m.p.h.	Wind corr.	Track speed, m.p.h.
Air Speed 180 Miles per Hour												
0°	0°	170	0°	165	0°	160	0°	155	0°	150	0°	145
10°	1°	170	1°	165	1°	160	1°	155	1°	150	1°	145
20°	1°	170	2°	166	2°	161	3°	156	3°	151	3°	146
30°	2°	171	2°	167	3°	162	4°	157	4°	152	5°	147
40°	2°	172	3°	168	4°	163	5°	158	5°	153	6°	148

Tab.1.34/1

## 1.34.2.Calculation of the wind triangle using the Schiffmann-Dodd Navigator

The following example explains the application:

true course                    360°  
wind direction 1            050°  
wind direction 2            230°  
wind speed                  40 kt  
true airspeed                100 kt

wind angle                    50°  
wind corr. angle α WCA    ±17.8°  
ground speed 1              69.5 kt  
ground speed 2              120.9 kt

Solution (see Fig.1.34/1):  
After setting the course, the wind angle of 50° can be read opposite the wind direction. The same angle is determined opposite the wind direction 230° (not shown in the figure). In the center of the computer for the wind from 050° is indicated >HEAD WIND right< and for the wind from 230° >TAILWIND left<.The upper disk is then adjusted so that these values can be read in the window for TAS =100 kt and the wind angle 50° below the wind speed 40 kt (see Fig. 1.34/2):

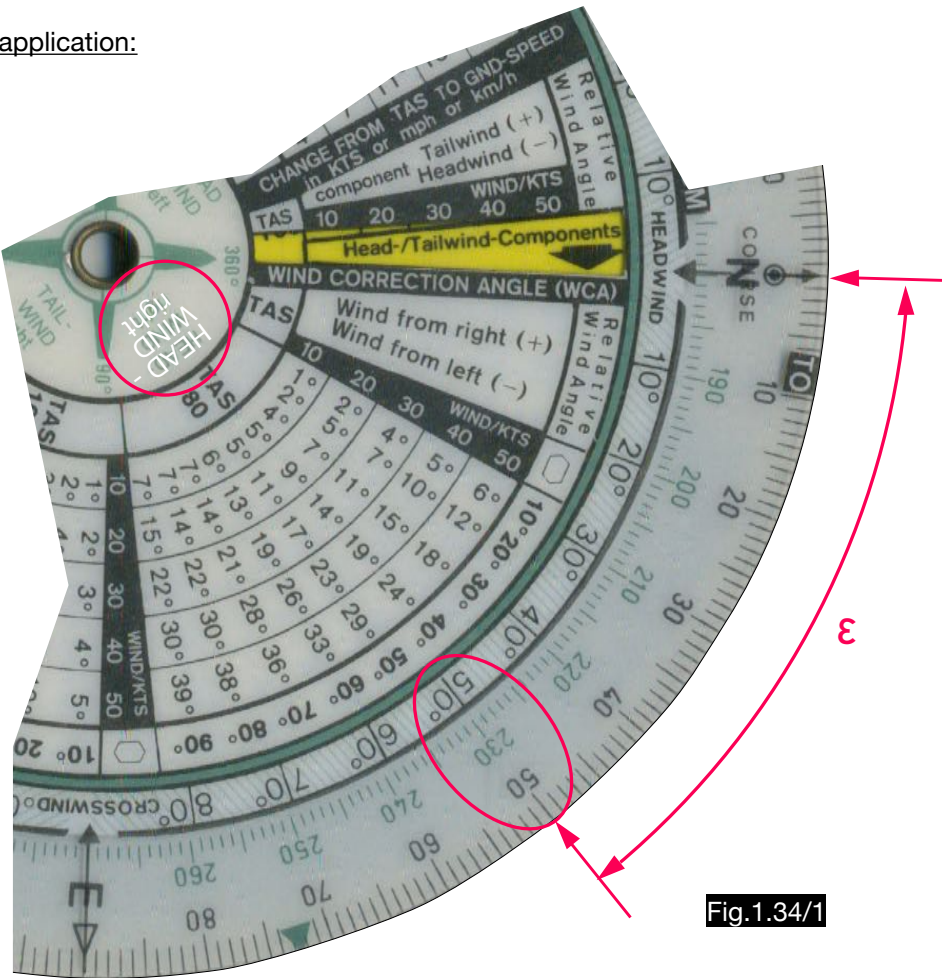


Fig.1.34/1

wind correction angle                    18°  
change from TAS to GND -SPEED (tailwind)            +21  
change from TAS to GND -SPEED (headwind)           - 31



With these values one gets for the wind direction 050° a wind correction angle of +18° and a ground speed of 69 kt (100 - 31) and for the wind direction 230° a wind correction angle of -18° and a ground speed of 121 kt (100 + 21).

Note 1:

The ground speed calculation using the amount "change from true air speed to ground speed" (see note 1 in Fig.1.34/2) is not done with the longitudinal component of the wind (which here is 25.7 kt), but with a signed true airspeed correction. This avoids the need to calculate a  $V_{TAS_{eff}}$ .

Note 2:

On this computer, the angle between course and wind direction is called the "relative wind angle", but this usually applies to the angle between heading and wind direction (see Fig.1.1/2). It can be an indication that the ambiguity of the results must be taken into account, since the wind angles used here are never greater than 90° (see note 2 in Fig.1.34/2).

note 2

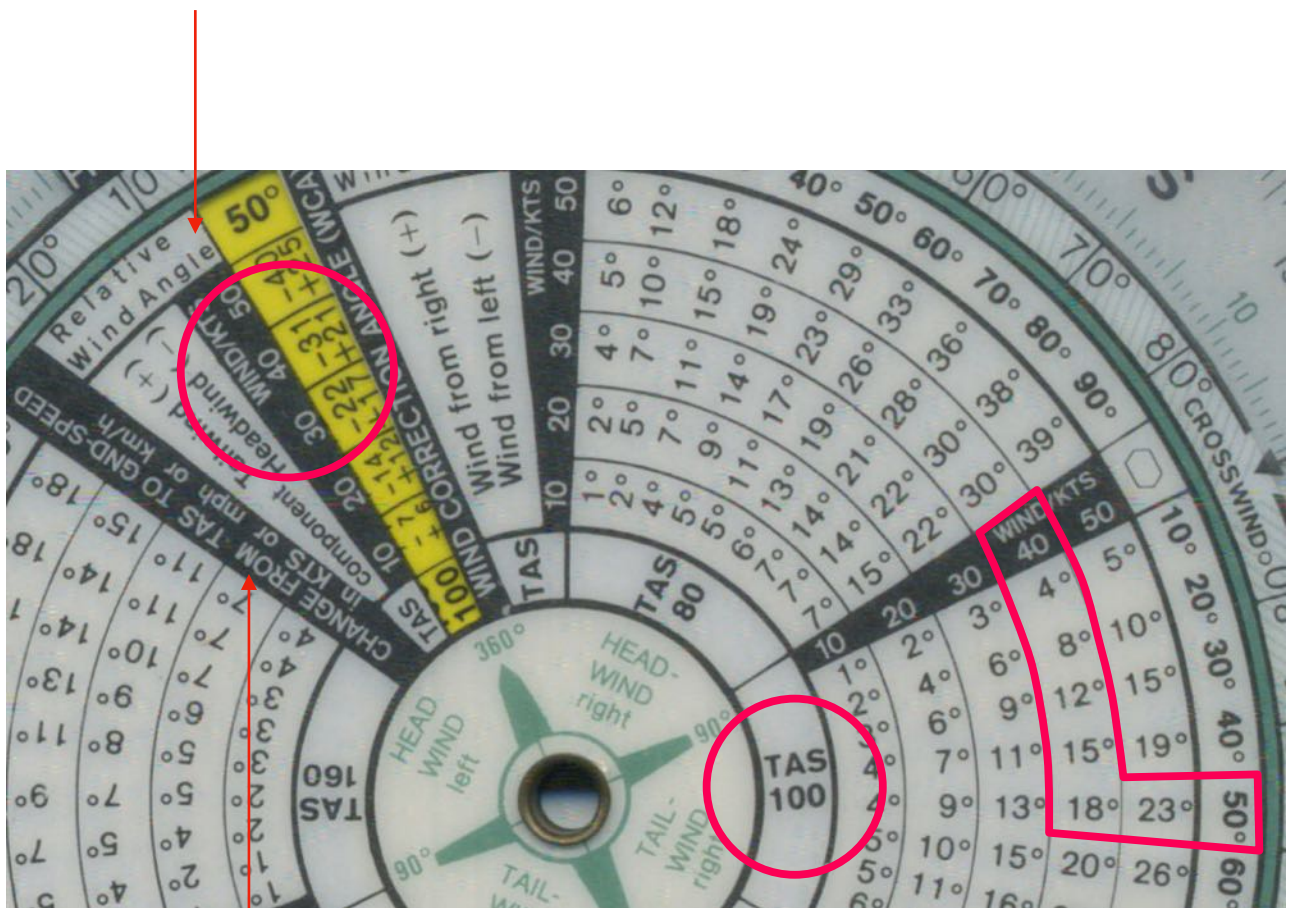


Fig.1.34/2

note 1

### 1.34.3. Calculation of the wind triangle using the Phelps Speedy Flight Computer

#### 1.34.3.1. General

Fig.1.34/3 shows the computer invented by A. T. Phelps where the wind triangle is calculated using tables. The wind angle is calculated using the compass rose at the bottom. To limit the size of the tables, the wind angles are below  $90^\circ$ . In order to simplify the determination of the signs of the speed and the wind correction, the four quadrants of the compass rose are marked with >HEAD WINDS L or R < and >TAIL WINDS L or R<.

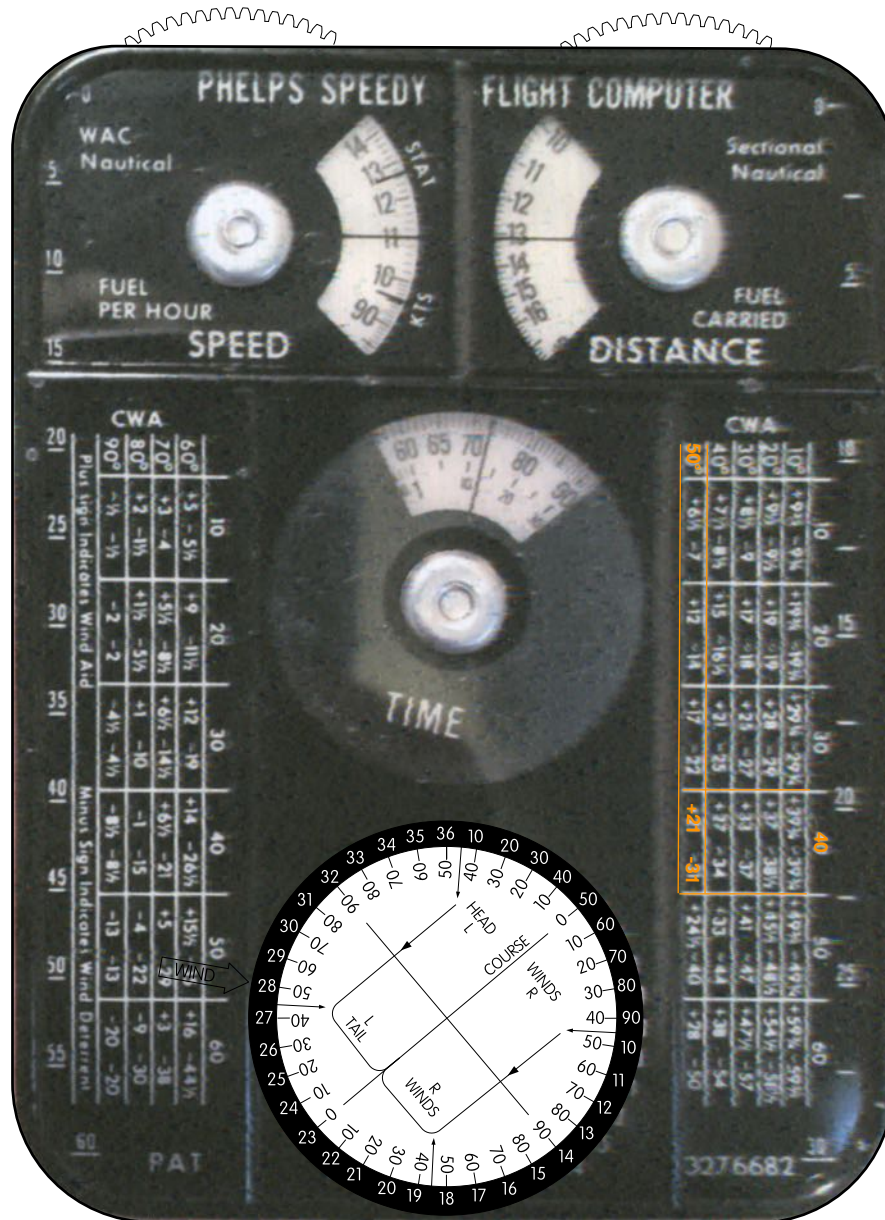


Fig.1.34/3

### 1.34.3.2. Use of the computer for wind correction and ground speed calculations

The following example explains the application:

true wind                      280°/40 kt  
true airspeed                  100 kt  
true course                    050°

$\alpha_{WCA}$                         17.8°  
ground speed  $v_G$             120.9 kt

Calculation of the wind correction angle:

With the help of the compass rose a wind angle of  $\varepsilon = 50^\circ$  and at the same time the information >TAIL WIND LEFT< is found (shown in Fig.1.34/3).

A table of the wind correction angle for different values of wind angle, wind speed and airspeed is very voluminous and cannot be displayed on a handheld calculator in a well readable font size. To eliminate the variable wind speed, instead of the wind correction angle the factor  $f$  was inserted into the table, which is defined as follows:

$$f = \frac{\alpha_{WCA} [\text{deg}]}{v_W [\text{kt}]}$$

Tab.1.34/2 gives a part of the table of the back of the computer in which one finds the factor  $f = 0.45$  for the airspeed 100 kt and the wind angle  $50^\circ$ . Therefore, the following applies:

$$\alpha_{WCA} = f \cdot v_W = 0.45 \text{ deg/kt} \times 40 \text{ kt} = -18^\circ$$

The negative sign of the wind correction angle is taken from the information >TAIL WIND LEFT<. The true heading is therefore  $18^\circ$  lesser than the true course.

This is of course an approximate solution. Tab.1.34/3 contains the exact (ext.) factors  $f$  and the factors contained in the table (tab.) for two airspeeds and three wind angles.

	TAS	100	120	140	160	180	200
WIND ANGLE	10°	.10	.09	.08	.07	.06	.05
	20°	.20	.18	.16	.14	.12	.10
	30°	.30	.25	.22	.19	.17	.15
	*						
	50°	.45	.38	.33	.29	.26	.24
	*						
	80°	.58	.47	.42	.37	.32	.29
90°	.60	.49	.43	.38	.34	.30	
		WIND CORRECTION ANGLE					

WIND CORRECTION ANGLE

Tab.1.34/2

wind angle [°]	$v_W$ [kt]	$v_{TAS}$ [kt]					
		100			200		
		$\alpha_{WCA}$	$\frac{\alpha_{WCA}}{v_W}$		$\alpha_{WCA}$	$\frac{\alpha_{WCA}}{v_W}$	
			ext.	tab.		ext.	tab.
50°	20	8.81	.44		4.39	.21	
	40	17.84	.45	.45	8.81	.22	.24
	60	27.36	.46		13.29	.22	
70°	20	10.83	.54		5.39	.27	
	40	22.10	.55	.54	10.83	.27	.27
	60	34.32	.57		16.37	.27	
90°	20	11.53	.58		5.74	.29	
	40	23.58	.59	.60	11.54	.29	.30
	60	36.87	.61		17.46	.29	

Tab.1.34/3

Calculation of the ground speed:

On the front of the calculator there are beside the compass rose two tables for the ground speed (partly shown in Tab.1.34/5). These contain the correction amounts  $\Delta v = v_{TAS} - v_G$ , which are a function of true airspeed, wind angle and wind speed. These were calculated for an airspeed of 100 (kt, m.p.h., km/h), but can also be used in good approximation for other speeds. Three examples are shown in Fig.1.34/4 and in Tab.1.34/4.

	example 1	example 2	example 3	
true airspeed $v_{TAS}$	$v_1 = 100$ kt	$v_2 = 100$ kt	$v_3 = 200$ kt	chosen
true course	$050^\circ$	$050^\circ$	$050^\circ$	
true wind direction	$100^\circ$	$180^\circ$	$180^\circ$	
wind speed $v_w$	$u_1 = 40$ kt	$u_2 = 40$ kt	$u_3 = 40$ kt	precise
wind angle $\epsilon$	$50^\circ$	$50^\circ$	$50^\circ$	
ground speed $v_G$	$w_1 = 69.47$ kt	$w_2 = 120.86$ kt	$w_3 = 223.3$ kt.	
wind corr. angle $\alpha_{WCA}$	$\alpha_1 = 17.8^\circ$	$\alpha_2 = 17.8^\circ$	$\alpha_3 = 8.81^\circ$	Phelps
ground speed $v_G$	$w_1 = 69$ kt	$w_2 = 121$ kt	$w_3 = 221$ kt	
wind corr. angle $\alpha_{WCA}$	$\alpha_1 = 18^\circ$	$\alpha_2 = 18^\circ$	$\alpha_3 = 9.6^\circ$	

Tab.1.34/4

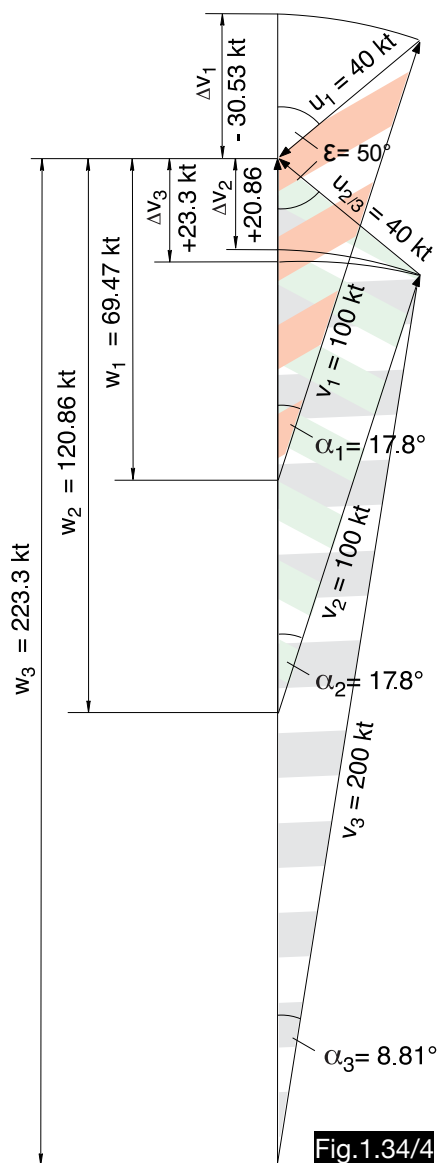


Fig.1.34/4

wind angle	$\Delta v = v_{TAS} - v_G$					
	wind speed					
	10	20	30	40	50	60
$10^\circ$	$+9\frac{3}{4}$ $-9\frac{3}{4}$	*	*	$+39\frac{3}{4}$ $-39\frac{3}{4}$	*	$+59\frac{1}{4}$ $-59\frac{1}{4}$
$20^\circ$	*	*	*	*	*	*
$30^\circ$	$+8\frac{1}{2}$ $-9$	*	*	$+33$ $-37$	*	$+47\frac{1}{2}$ $-57$
$40^\circ$	*	*	*	*	*	*
$50^\circ$	$+6\frac{1}{2}$ $-7$	*	*	$+21$ $-31$	*	$+28$ $-50$
$60^\circ$	*	*	*	*	*	*
$70^\circ$	$-4$ $+3$	*	*	$+6\frac{1}{2}$ $-21$	*	$+3$ $-38$
$80^\circ$	*	*	*	*	*	*
$90^\circ$	$-1\frac{1}{2}$ $-1\frac{1}{2}$	*	*	$-8\frac{1}{2}$ $-8\frac{1}{2}$	*	$-20$ $-20$

Tab.1.34/5



### 1.34.4. Calculation of the wind triangle using the „QuDieM“-Navigator

#### 1.34.4.1. General

The "QuDieM-Navigator" (see Fig.1.34/5) is intended for aircraft flying at an air speed of approx. 100 kt and is used to calculate course, heading and flight time using a 1:500,000 scale map.

#### 1.34.4.2. Use of the computer for wind correction- and ground speed calculations

The application is explained using the following example (see Fig.1.34/5):

true airspeed	100 kt	magnetic course	050°
true course	060°	wind corr. angle	+ 3.9°
variation	10° E	flight time	17.7 min
true wind	220°/20 kt		

##### Step 1:

The device is placed on the map in such a way that the center aperture is above the departure point, the aperture of the bearing arm is above the destination point, and the north-south axis is parallel to a meridian.

##### Step 2:

The arrow TO TARGET shows the true course 60° on the outer compass rose and the magnetic course 50° on the inner yellow one. For this it is necessary that the variation 10° E has been set on the back of the device.

##### Step 3:

In the window of the bearing arm one can read the distance (35 NM) and the still-air time (21 min).

##### Step 4:

The wind arrow is set to the true wind direction (outer wind rose) 220°.

##### Step 5:

The wind speed of 20 kt is set.

##### Step 6:

In the wind correction window, the figure 4 appears on the left and a white triangle (+) below it, as well as the figure 18 on the right and a black triangle (-) below it. Thus the wind correction angle is determined to +4° and the true heading to 64°. The flight time is 18% less than the still-air time, which means:

$21\text{ min} - (21\text{ min} \times 0.18) = 17.2\text{ min}.$

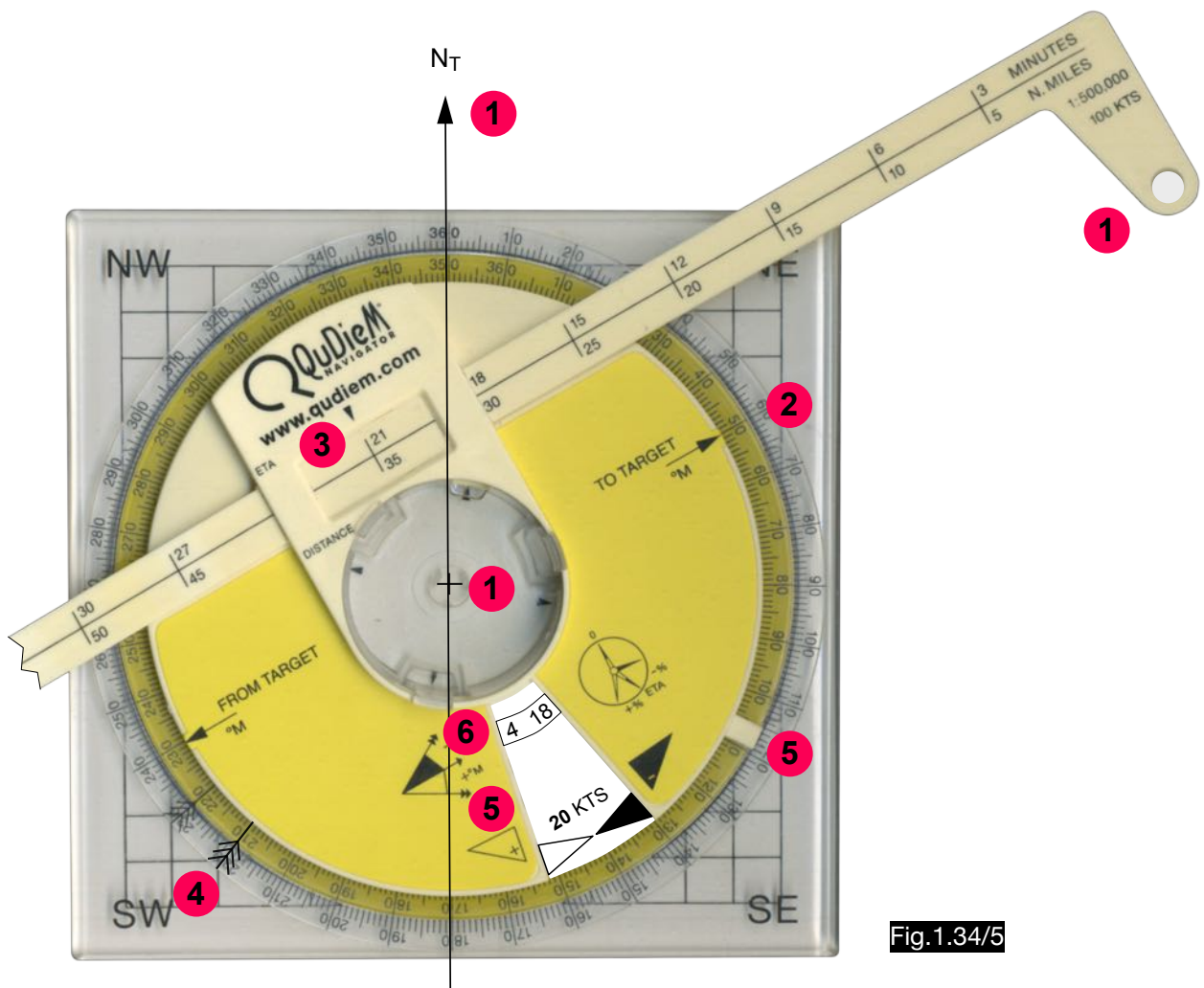


Fig.1.34/5

Fig.1.34/6 shows the table of correction figures, which is located inside the calculator, so that after setting the course, the wind direction and the wind speed the wind angle  $\epsilon$  is set and the applicable wind correction window opens. Tab.1.34/6 lists the results for three other examples to illustrate accuracy.

Example	true course = 360°		v <sub>TAS</sub> = 100 kt		distance = 50 NM		still-air time = 30 min	
	1	true wind            225°/40 kt wind angle ε <sub>1</sub> 45°	α <sub>WCA</sub> +16.4° v <sub>G</sub> 67.6 kt time      44.4 min		Indication of wind corr. window: +16 +25 α <sub>WCA</sub> +16° time:     t = 30 + (30 x 0.25) = 37.5 min			
	2	true wind            250°/25 kt wind angle ε <sub>2</sub> 70°	α <sub>WCA</sub> -13.6° v <sub>G</sub> 105.8 kt time      28.4 min		Indication of wind corr. window: -13 -7 α <sub>WCA</sub> -13° time:     t = 30 - (30 x 0.07) = 27.9 min			
	3	true wind            340°/15 kt wind angle ε <sub>3</sub> 20°	α <sub>WCA</sub> -2.9° v <sub>G</sub> 85.8 kt time      35.0 min		Indication of wind corr. window: -3 +12 α <sub>WCA</sub> -3° time:     t = 30 + (30 x 0.12) = 33.6 min			
		presumed		accurate results		results of the QuDieM - Navigator		

Tab.1.34/6

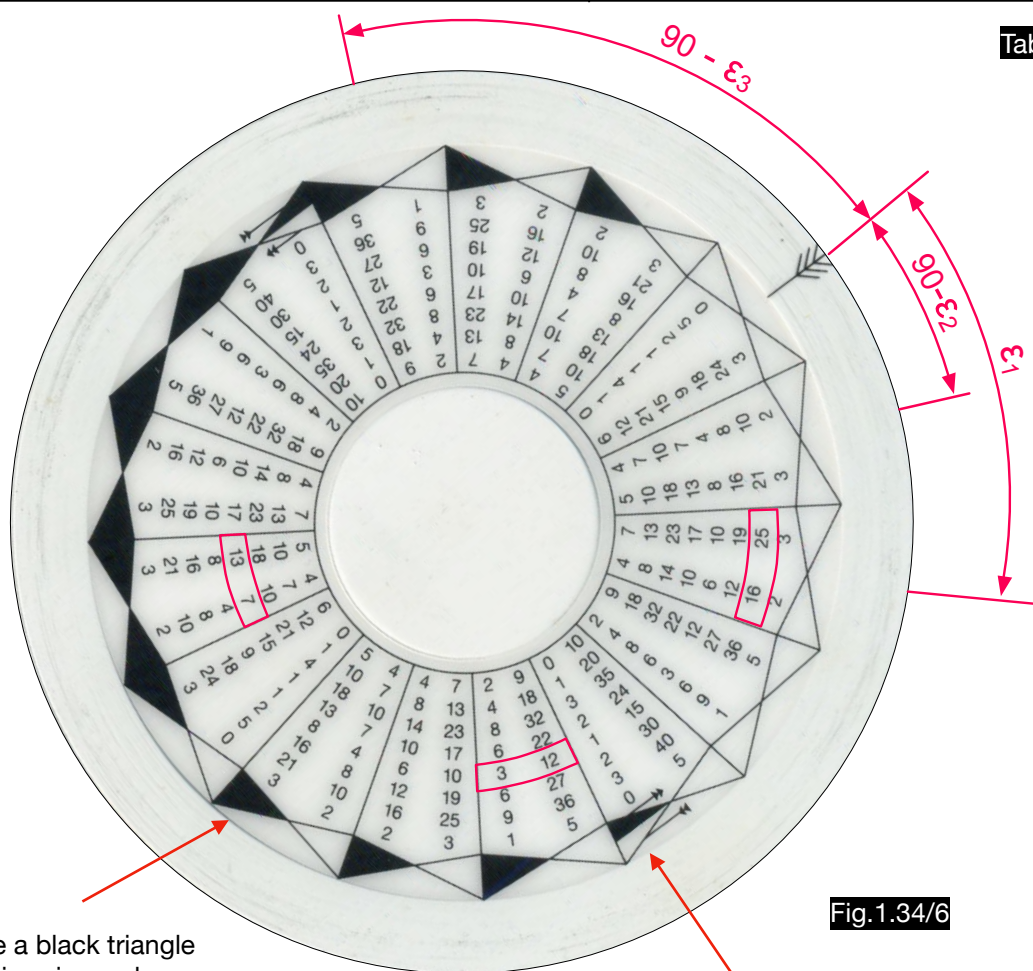


Fig.1.34/6

Values above a black triangle have a negative sign, values above a white triangle a positive sign.

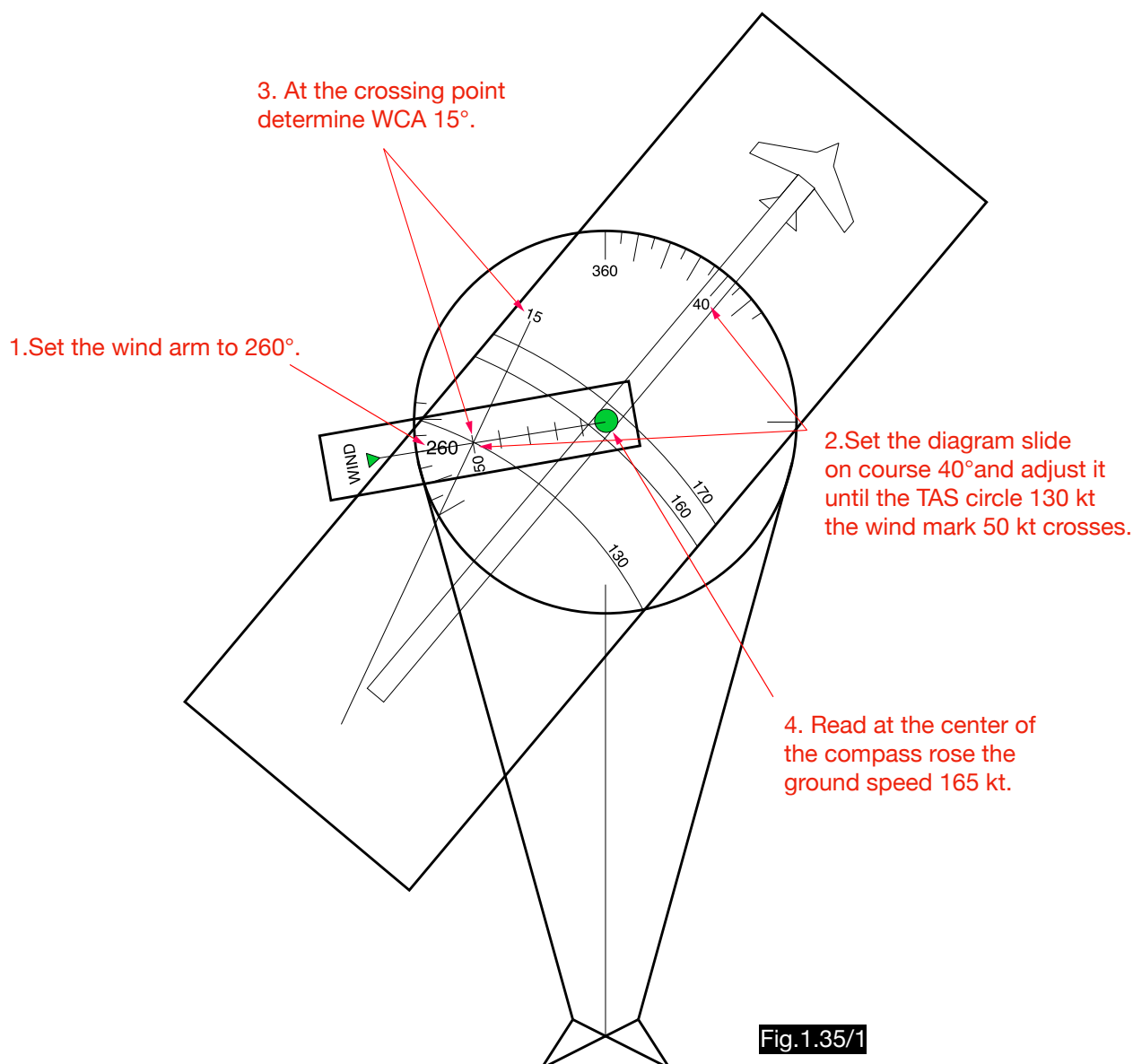
With tail wind or head wind, i.e. with wind angles  $\epsilon < 10^\circ$ , only the sign of the time correction can be clearly defined. The sign of the wind correction angle can be recognized by the direction of the wind arrow in relation to the course. This is described in the manual: „If the wind is within 10° of track, the application of heading compensation is determined by the direction from which the Compensation Reference Mark was centred. The heading compensation would be added if centred clockwise and subtracted if counter-clockwise“.

## 1.35. Calculation of the wind triangle using mechanical replicas

### 1.35.1. STEPHAN NAVIMAT 501

The application is explained using the following example (see Fig.1.35/1):

true course	040°
true airspeed	130 kt
true wind	260°/50 kt
wind correction angle	-14.3°
ground speed	164.3 kt



### 1.35.2. The „Simplified Flight Calculator“ according to Nelson

The application is explained using the following example (see Fig.1.35/2):

true course	090°
true airspeed	120 m.p.h.
true wind	145°/30 m.p.h.
wind correction angle	+11.8°
ground speed	100.3 m.p.h.
true heading	101.8°

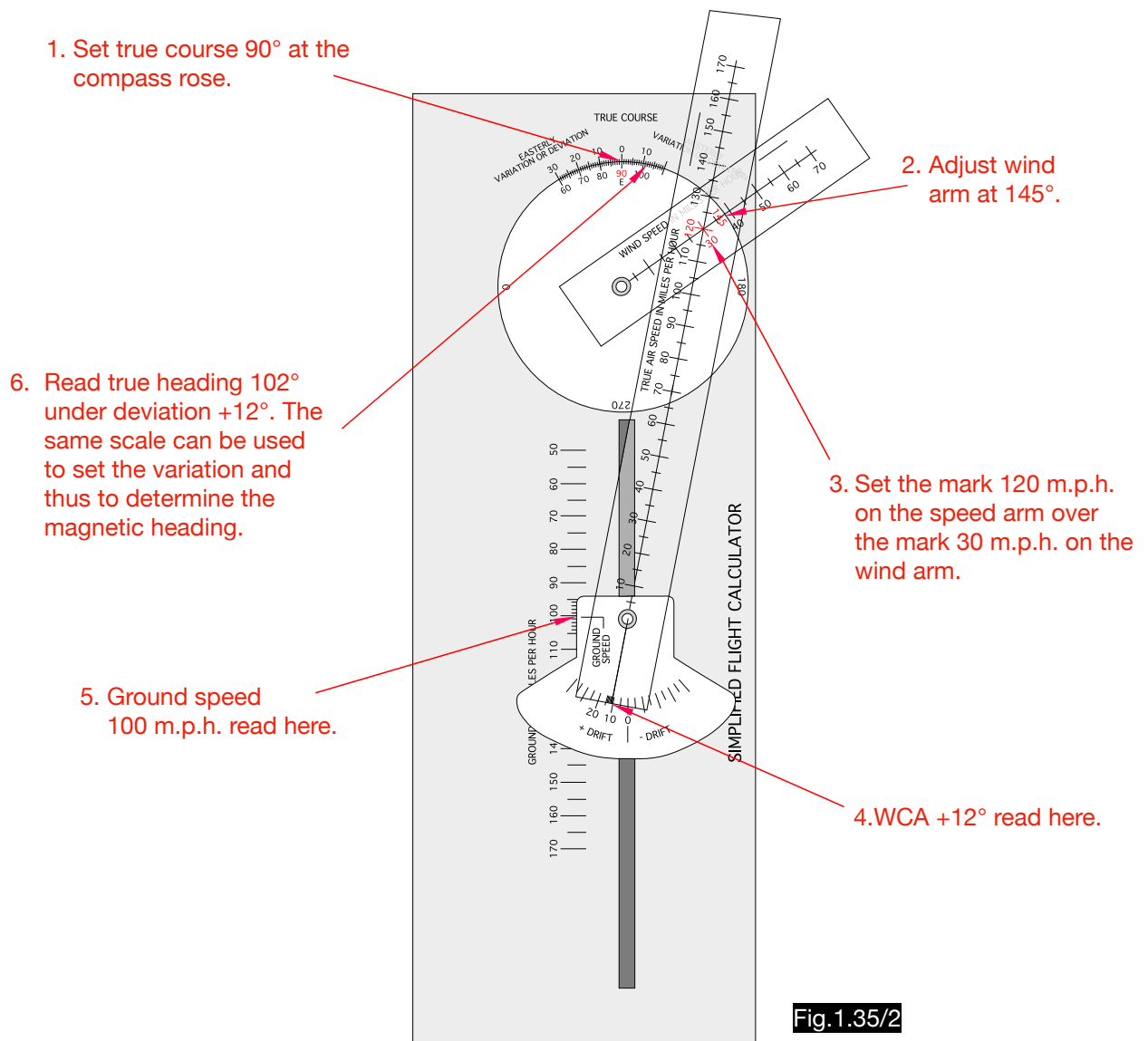


Fig.1.35/2

#### Note:

The handling of the calculator for wind determinations from drift and ground speed is analogous to the methodology described in the subsequent paragraph, which is why a more detailed explanation is not necessary.



### 1.35.3. Air line flight calculator according to Hokanson

The application of the calculator invented by E. C. Hokanson is explained using the following example (see Fig.1.35/3):

true course	120°
true airspeed	185 m.p.h.
true wind	190°/40 m.p.h.
variation	11°E

wind correction angle	+11.7°
ground speed	167.5 m.p.h.
true heading	131.7°
magnetic heading	120.7°

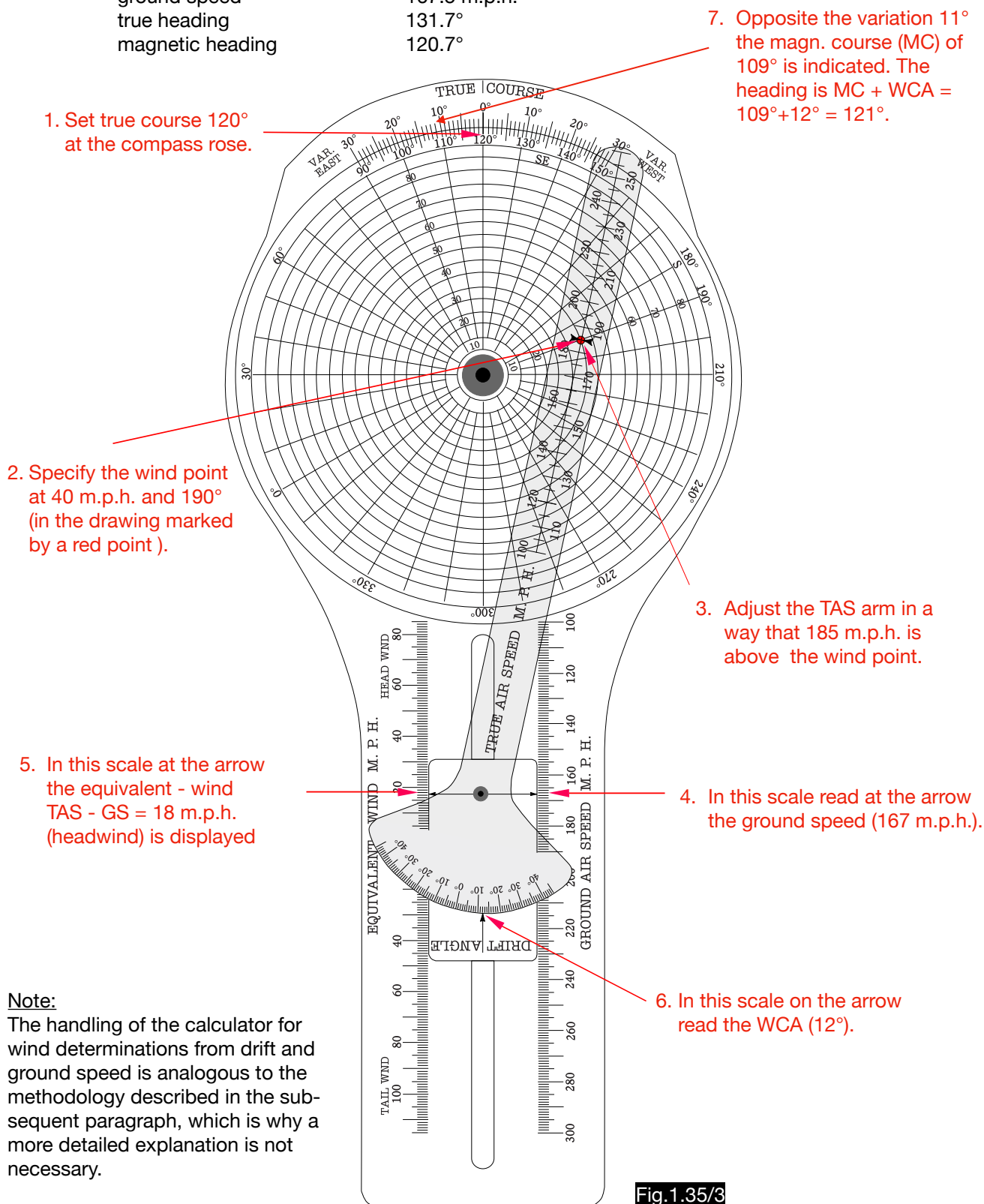


Fig.1.35/3

### 1.35.4. Calculation of the wind triangle according to Pilehn

The application of the Pilehn-Calculator is explained using the following example (see Fig.1.35/4):

true heading	090°
true airspeed	90 kt
true wind	045°/40 kt

drift angle	+24.6°
ground speed	67.9 kt

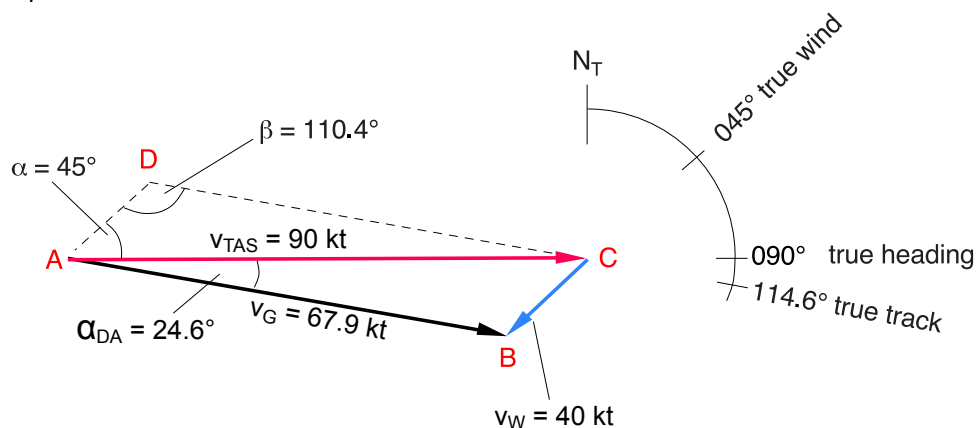


Fig.1.35/4

In Fig.1.35/4 the drift triangle ABC is supplemented by the congruent triangle ACD. In this triangle the angle A-C-D corresponds to the drift angle and the line CD to the ground speed. In Fig.1.35/5 the calculator which simulates the triangle ACD is shown in the setting of the example chosen here.

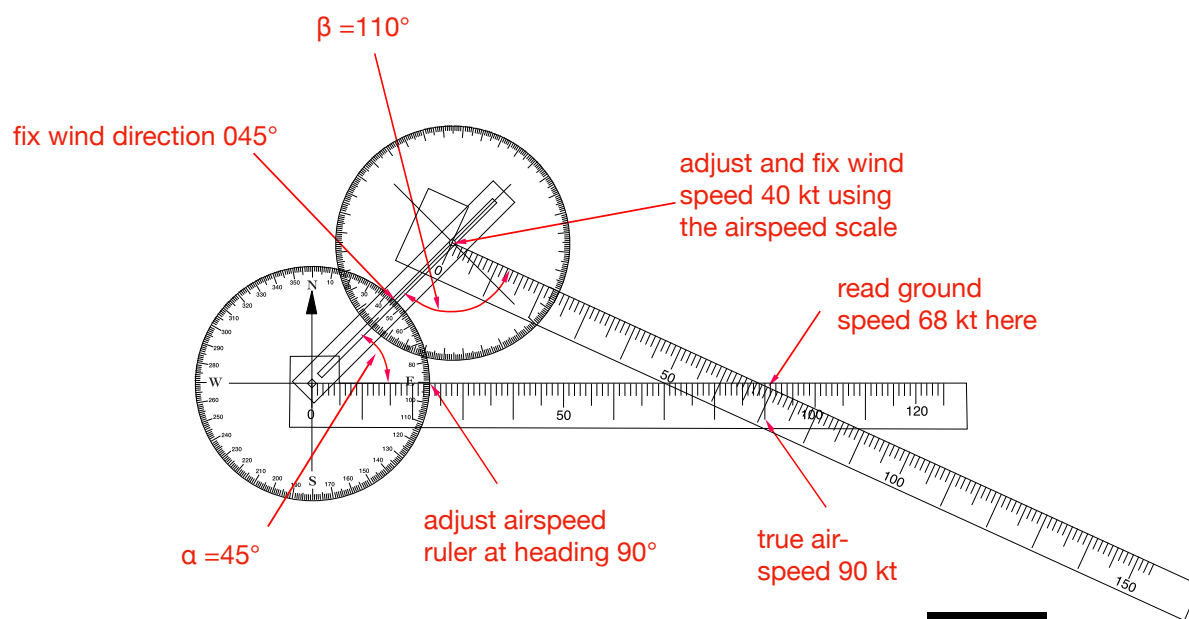


Fig.1.35/5

The ground speed can be read directly from the speed arm. However, the drift angle can only be calculated using the angles  $\alpha$  and  $\beta$  and results in  $\alpha_{DA} = 180^\circ - \alpha - \beta = 180^\circ - 45^\circ - 110^\circ = 25^\circ$ .

### 1.35.5. Calculation of the wind triangle with computers of typical Japanese design

#### 1.35.5.1. Calculation of wind correction angle and ground speed

The triangular calculator illustrated in Fig.1.35/6 is in design and use similar to the constructions described under 1.6. and 1.9. Like these, it is suitable for calculating the wind correction and drift triangle as well as for determining the wind from drift and ground speed or from two drift values. A different method is used here to display or set the drift or wind correction angle. Under the transparent and pencil writeable disk 1 is field 2 of parallel green and red lines with the corresponding scale 3. On the speed arm 4 the mark 5 is positioned and one can see that at the shown position an angle of about  $9^\circ$  is displayed (see red dashed line). This display remains even if a different speed is set on the scale 7 by means of screw 6. The wind arm 8 is used for setting and reading wind speed and wind direction.

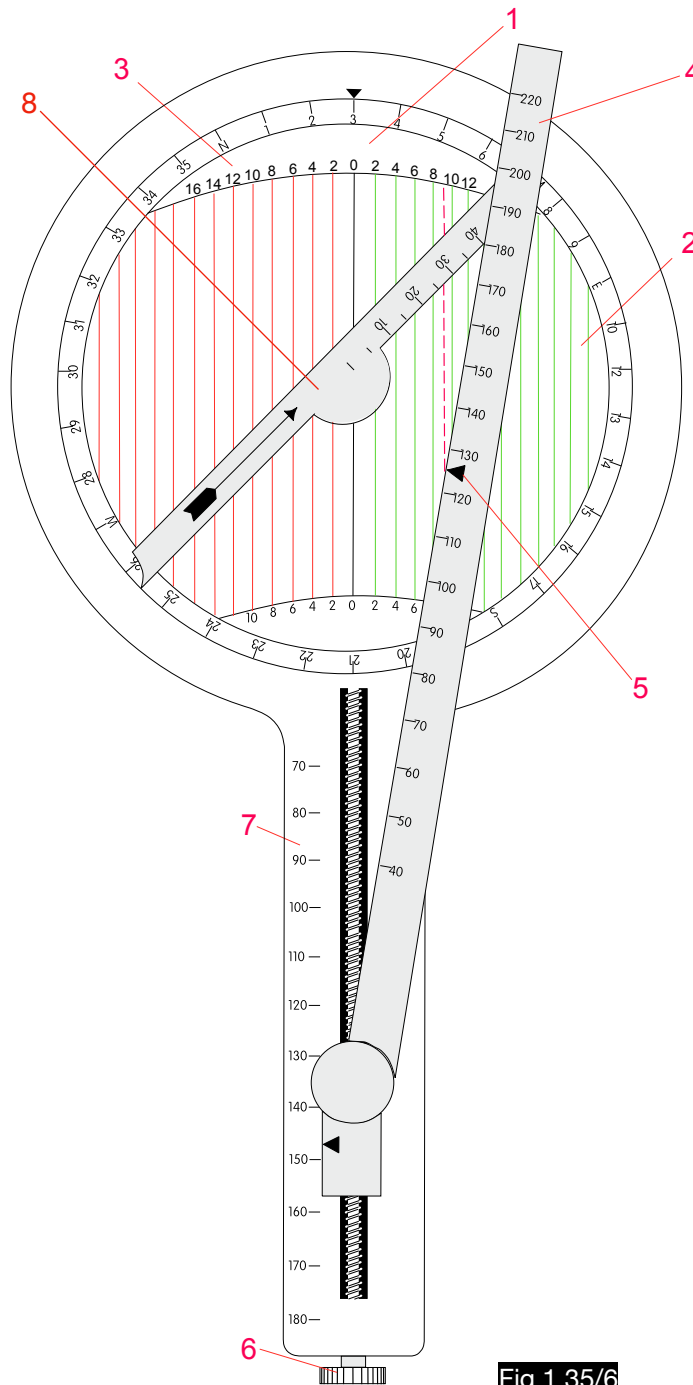


Fig.1.35/6

### 1.35.5.2. Use of the computer for wind determination from drift angle and ground speed

1. Set the course at the compass rose.
2. Set ground speed on scale 7.
3. Adjust the speed arm 4 in such a way that the mark 5 is on the line of the drift angle.
4. Set the wind arm 8 to the value of true airspeed on the speed arm 4 and thus find the wind point W.

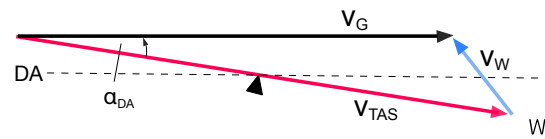


Fig.1.35/7

### 1.35.5.3. Use of the computer for wind determination from two drift angles

1. Set true airspeed on scale 7.
2. Set HDG 1 on the compass rose.
3. Adjust the speed arm 4 so that mark 5 is at the line of the first drift angle (DA 1).
4. Draw a pencil line along the speed arm 4 on the surface of the compass rose.
5. Set HDG 2 on the compass rose.
6. Adjust the speed arm 4 so that the mark 5 is at the line of the second drift angle (DA 2).
7. Draw a pencil line along the speed arm on the surface of the compass rose.
8. The wind point W is located at the intersection of the two lines plotted. After the wind arm 8 has been attached to this point, the wind direction and wind speed can be read from it.

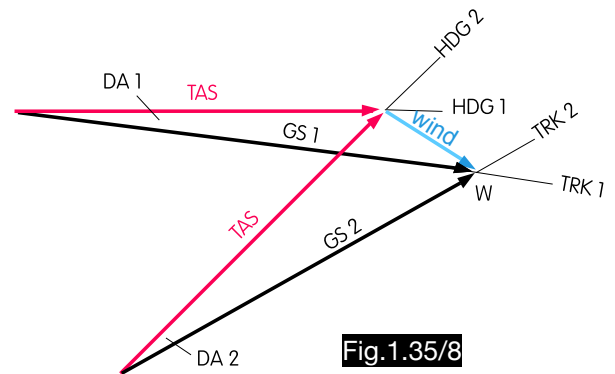


Fig.1.35/8

### 1.35.5.4. Use of the computer for wind determination from two ground speeds and tracks

1. Set ground speed 1 on scale 7.
2. Set track 1 at the compass rose.
3. With the help of the speed arm, make a circular arc of the radius true airspeed with pencil on the surface of the compass rose.
4. Set track 2 at the compass rose.
5. Set ground speed 2 on scale 7.
6. The wind point is found at the intersection of the airspeed mark 5 of the speed arm 4 with the drawn arc.
7. After the wind arm 8 has been attached to this point, the wind direction and wind speed can be read from it.

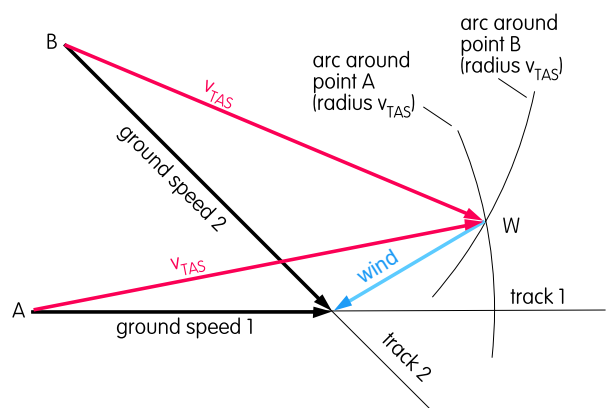


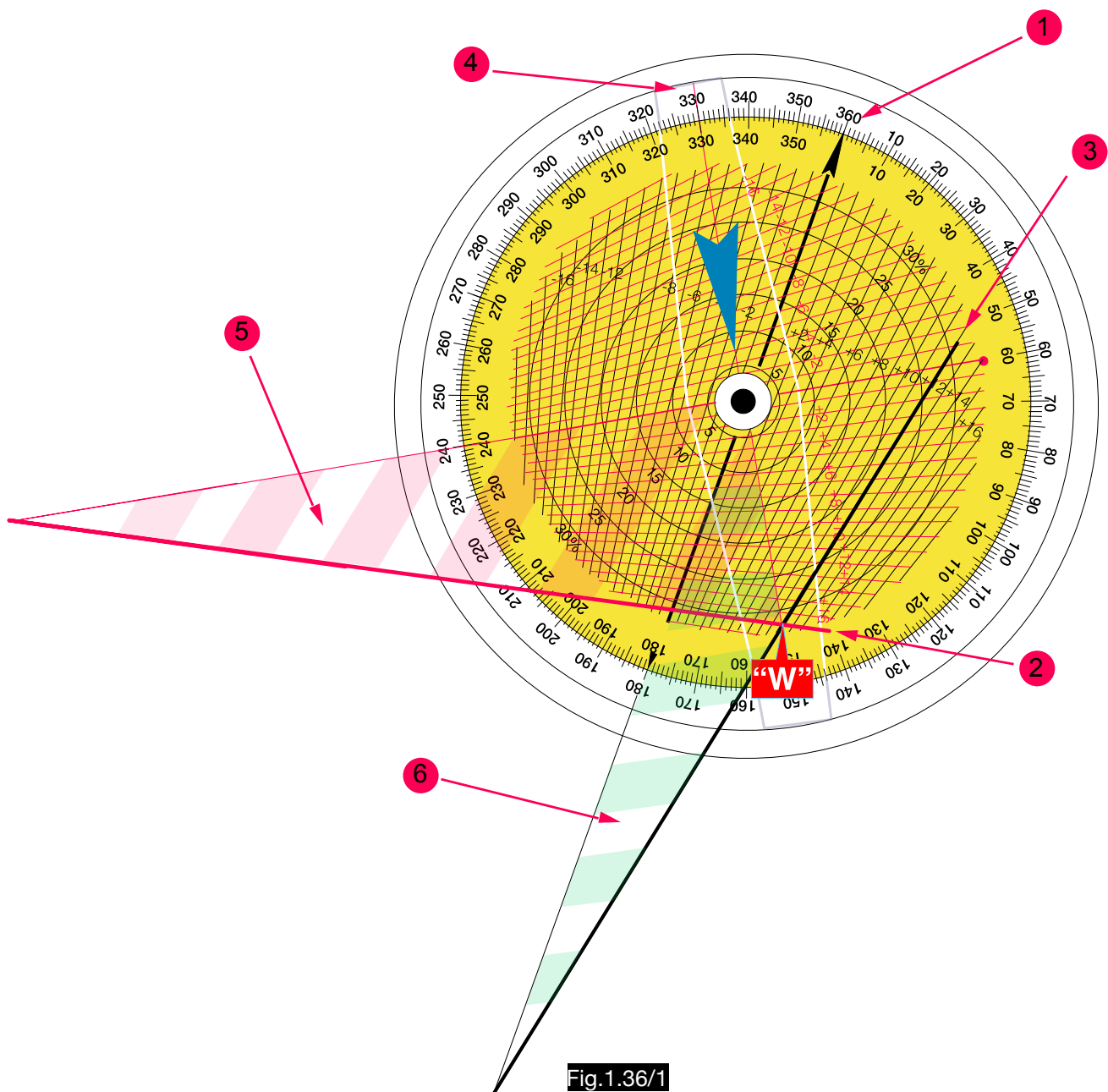
Fig.1.35/9



## 1.36. Computers specially designed for wind determination

### 1.36.1. Calculator DR 3 for wind determination from two drift angles

This type of calculator DR 3 is intended for calculation of wind from two drift angles found on two headings differing by 60°. The example described under 1.7.3. is also shown in Fig.1.36/1. After setting the compass rose to HEADING 1 = 360° (1), the two track lines are selected. The first (black) drift line +12° (3) crosses with the second (red) drift line +17° (2) at wind point W. The figure shows the corresponding wind triangles (5 and 6) for illustration purposes. The wind vector is the connecting line from the center to the wind point. After the centerline of the wind arm has been placed above the wind point, the wind direction 330° (4) can be read. The wind point lies approximately on the speed circle 31%, so that a wind speed of 31 kt results ( $V_{TAS} \times 0.31 = 100 \text{ kt} \times 0.31 = 31 \text{ kt}$ ).



## 1.36.2. Wind determination using the Four-Point-Bearing-Method with a Computer according to Burt and Wright

### 1.36.2.1. Description of the method

The red line in Fig.1.36/3 represents the track which occurs when an aircraft performs the flight figure according to Fig.1.36/2 under the influence of a north-easterly wind. The stopwatch is set in motion at the departure point  $P_0$  and the times  $t_1$ ,  $t_2$  and  $t_3$  are recorded when the positions  $P_1$ ,  $P_2$  and  $P_3$  are reached.

In the figure, the distance  $S$  ( $P_0 \leftrightarrow P_1$ ) represents the wind track  $u \times t$  (wind speed  $u$  multiplied by time  $t$ ), the components of which were drawn red for position 2 and green for position 3. Point  $P_1$  is reached when the described flight figure is completed (see note 1). The points  $P_2$  and  $P_3$  are defined by the bearings of  $90^\circ$  and  $135^\circ$  respectively. The triangle  $A P_1 P_2$  is similar to the drift triangle, its sides are the distances driven in the time  $t_2 - t_1$ . The distance  $A \leftrightarrow P_2$  is the air distance  $v(t_2 - t_1)$ ,  $A \leftrightarrow P_1$  is the wind way  $u(t_2 - t_1)$  and  $P_1 \leftrightarrow P_2$  is the distance flown over ground. This applies analogously to the triangle  $P_1 B P_3$  and the time  $t_3 - t_1$ .

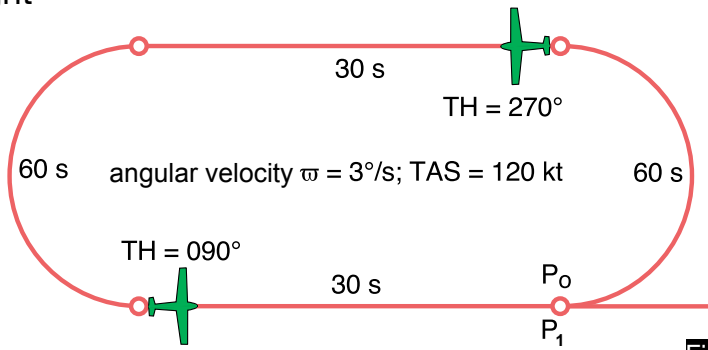


Fig.1.36/2

The following applies to the position 2:

$$S_2 \cos \gamma = v(t_2 - t_1) \text{ and with } S_2 = u \cdot t_2 \text{ one gets: } u \cos \gamma = v \left(1 - \frac{t_1}{t_2}\right) \quad [1.36/1]$$

The following applies to the position 3:

$$S_3 \sin \gamma + S_3 \cos \gamma = v(t_3 - t_1) \text{ and with } S_3 = u \cdot t_3 \text{ one gets: } u(\sin \gamma + \cos \gamma) = v \left(1 - \frac{t_1}{t_3}\right) \quad [1.36/2]$$

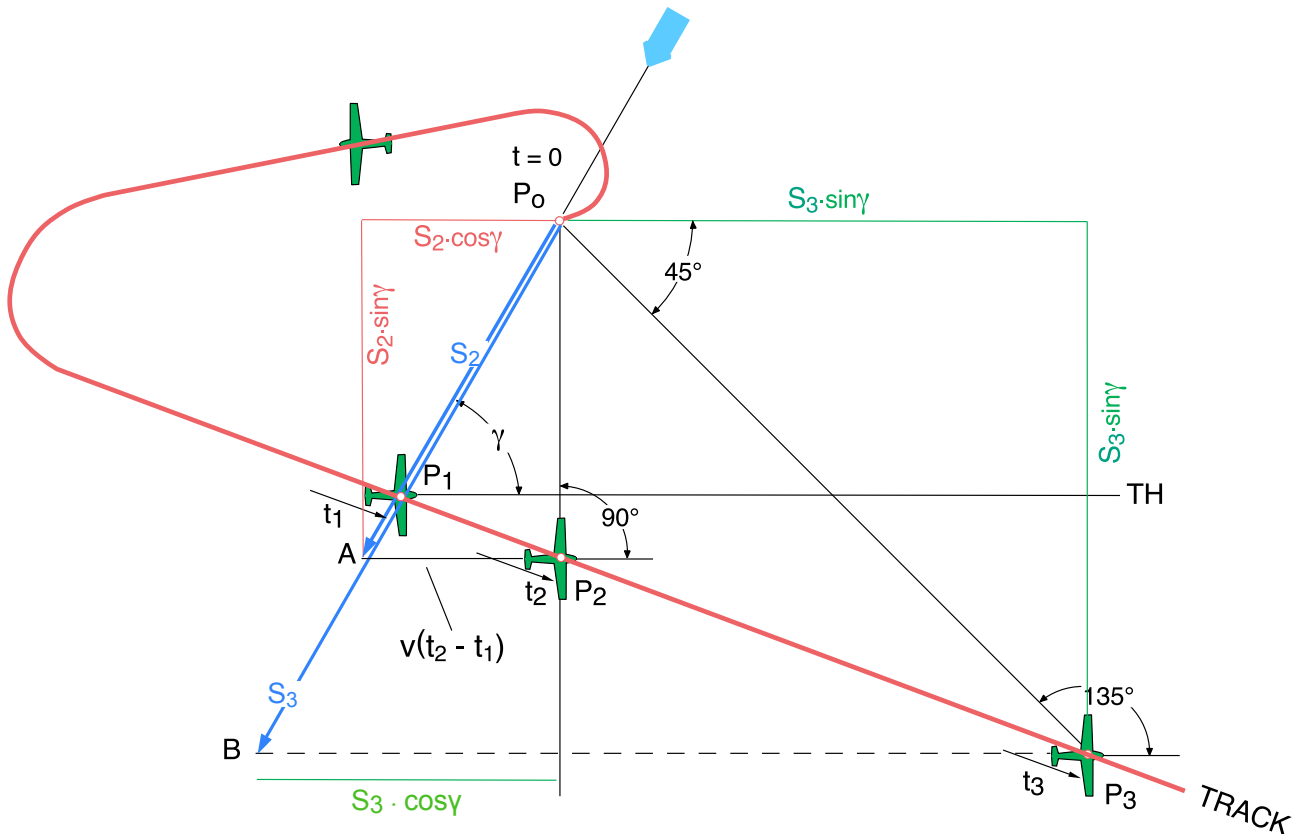


Fig.1.36/3

Example:

true heading	090°	wind speed u	30.3 kt
true airspeed v	120 kt	angle γ	60.1°
t <sub>1</sub>	180 s	true wind direction	030.0°
t <sub>2</sub>	206 s		
t <sub>3</sub>	275 s		

Solution:

In equ. 1.36/1 air speed and times entered one obtains:  $u = \frac{120}{\cos \gamma} (1 - \frac{180}{206}) = \frac{15.15}{\cos \gamma}$

In equ. 1.36/2 air speed and times entered one obtains:  $u = \frac{120}{\sin \gamma + \cos \gamma} (1 - \frac{180}{275}) = \frac{41.45}{\sin \gamma + \cos \gamma}$

After equation results:  $\frac{\sin \gamma + \cos \gamma}{\cos \gamma} = \frac{41.45}{15.15} = \tan \gamma + 1 = 2.63 \rightarrow \gamma = 60.05^\circ$

According to equ. 1.36/1, this results for the wind speed:  $u = \frac{15.15}{\cos 60} = 30.3 \text{ kt}$

The formulas for wind speed u and wind angle γ are not too complicated, but their calculation without today's electronic calculator is quite time-consuming. To simplify these calculations, F.S. Burt and C.S. Wright invented a special calculator, which is described here.

### 1.36.2.2. Computer for the „Four point bearing procedure“ according to Burt and Wright

The formulas 1.36/1 and 2 can be simplified if the airspeed v = 100 is used. One will get the false wind speed u', but it is easy to correct with this relation:

$$u = u' \frac{v}{100}$$

The formulas now get this shape:

$$u' \cos \gamma = 100 (1 - \frac{t_1}{t_2}) \quad [1.36/3]$$

$\underbrace{\hspace{10em}}_A$

$$u' (\sin \gamma + \cos \gamma) = 100 (1 - \frac{t_1}{t_3}) \quad [1.36/4]$$

$\underbrace{\hspace{10em}}_B$

One side of the calculator has the logarithmic scales 1 and 2, which are used for multiplication by the factor v/100, and the special scale 3, for the calculation of the variables A and B. For the example used here one gets (see Fig. 1.36/4):

$$A = 100 (1 - \frac{180}{206}) = 12.6$$

$$B = 100 (1 - \frac{180}{275}) = 34.5$$





4. The pointer scale shows  $u' = 25$  kt. The wind speed can be calculated at the logarithmic scales 1 and 2 on the back (see Fig.1.36/4). For  $u' = 25$  kt and  $v = 120$  kt one gets  $u = 30$  kt:

## 2. General navigational problems and mechanical computers

### 2.1. Calculation of courses and bearings

These equations can be derivated from Fig.1.1/1-3 and Fig.2.1/1:

$$\text{compass} \begin{cases} \text{heading (CH)} \\ \text{course (CC)} \\ \text{track (CT)} \end{cases} + \text{deviation} = \text{magnetic} \begin{cases} \text{heading (MH)} \\ \text{course (MC)} \\ \text{track (MT)} \end{cases}$$

$$\text{magnetic} \begin{cases} \text{heading (MH)} \\ \text{course (MC)} \\ \text{track (MT)} \end{cases} + \text{variation} = \text{true} \begin{cases} \text{heading (TH)} \\ \text{course (TC)} \\ \text{track (TT)} \end{cases}$$

$$\text{true} \begin{cases} \text{heading (TH)} \\ \text{course (TC)} \\ \text{track (TT)} \end{cases} - \text{variation} = \text{magnetic} \begin{cases} \text{heading (MH)} \\ \text{course (MC)} \\ \text{track (MT)} \end{cases}$$

$$\text{magnetic} \begin{cases} \text{heading (MH)} \\ \text{course (MC)} \\ \text{track (MT)} \end{cases} - \text{deviation} = \text{compass} \begin{cases} \text{heading (CH)} \\ \text{course (CC)} \\ \text{track (CT)} \end{cases}$$

$$\begin{matrix} \text{true} \\ \text{magnetic} \\ \text{compass} \end{matrix} \begin{matrix} \nearrow \\ \rightarrow \\ \rightarrow \end{matrix} \text{course} + \text{wind correction angle (WCA)} = \begin{matrix} \text{true} \\ \text{magnetic} \\ \text{compass} \end{matrix} \begin{matrix} \nearrow \\ \rightarrow \\ \rightarrow \end{matrix} \text{heading}$$

$$\begin{matrix} \text{true} \\ \text{magnetic} \\ \text{compass} \end{matrix} \begin{matrix} \nearrow \\ \rightarrow \\ \rightarrow \end{matrix} \text{heading} + \text{drift angle (DA)} = \begin{matrix} \text{true} \\ \text{magnetic} \\ \text{compass} \end{matrix} \begin{matrix} \nearrow \\ \rightarrow \\ \rightarrow \end{matrix} \text{track}$$

$$\begin{matrix} \text{true} \\ \text{magnetic} \\ \text{compass} \end{matrix} \begin{matrix} \nearrow \\ \rightarrow \\ \rightarrow \end{matrix} \text{heading} + \text{relative bearing (RB)} = \begin{matrix} \text{true} \\ \text{magnetic} \\ \text{compass} \end{matrix} \begin{matrix} \nearrow \\ \rightarrow \\ \rightarrow \end{matrix} \text{bearing to (MB}_{\text{to}}\text{)}$$

$$\begin{matrix} \text{magn. bearing to (QDM or MB}_{\text{to}}\text{)} \\ \text{magn. bearing from (QDR or MB}_{\text{from}}\text{)} \end{matrix} \begin{matrix} \rangle \\ \rangle \end{matrix} \pm \text{Variation (VAR.)} = \begin{matrix} \langle \\ \langle \end{matrix} \begin{matrix} \text{true bearing to (QUJ or TB}_{\text{to}}\text{)} \\ \text{true bearing from (QTE or TB}_{\text{from}}\text{)} \end{matrix}$$

$$\begin{matrix} \text{magn. bearing to (QDM or MB}_{\text{to}}\text{)} \\ \text{true bearing to (QUJ or TB}_{\text{to}}\text{)} \end{matrix} \begin{matrix} \rangle \\ \rangle \end{matrix} \pm 180 = \begin{matrix} \langle \\ \langle \end{matrix} \begin{matrix} \text{magn. bearing from (QDR or MB}_{\text{from}}\text{)} \\ \text{true bearing from (QTE or TB}_{\text{from}}\text{)} \end{matrix}$$

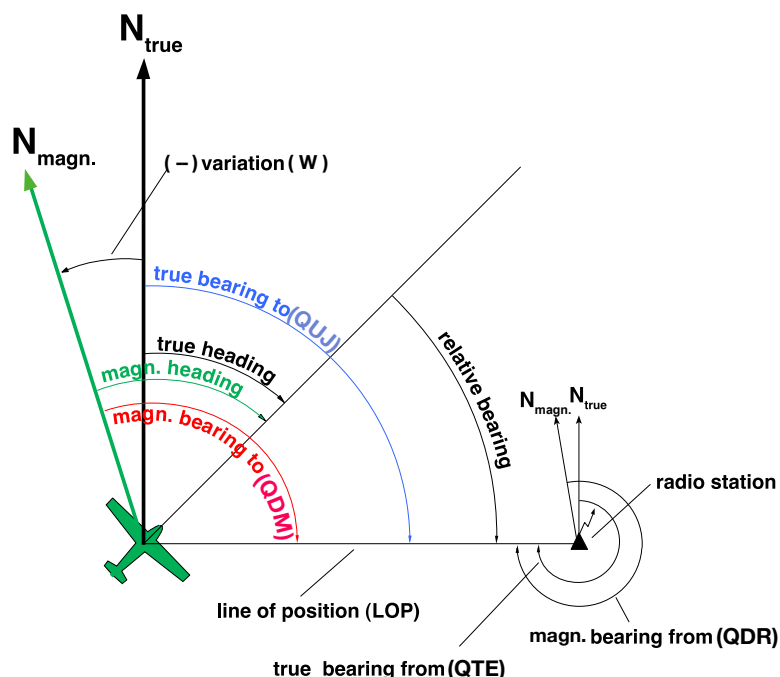


Fig.2.1/1

Different calculators make the conversion of courses and bearings easier by inserting formulas or calculation schemes. Here are some examples:



Fig.2.1/2

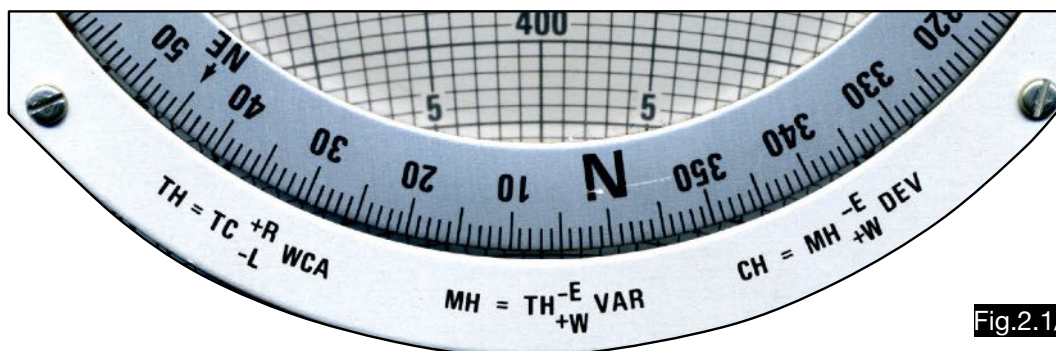


Fig.2.1/3

Fig.2.1/4 illustrates the following calculations:

$$TH = 027^\circ, VAR = 15^\circ W \Rightarrow MH = 042^\circ$$

$$MH = 027^\circ, MT = 042^\circ \Rightarrow DA = +015^\circ$$

$$TC = 027^\circ, WCA = +15^\circ \Rightarrow TH = 042^\circ$$

$$MH = 027^\circ, RB = 040^\circ \Rightarrow MB_{to} (QDM) = 067^\circ, MB_{from} (QDR) = 247^\circ$$

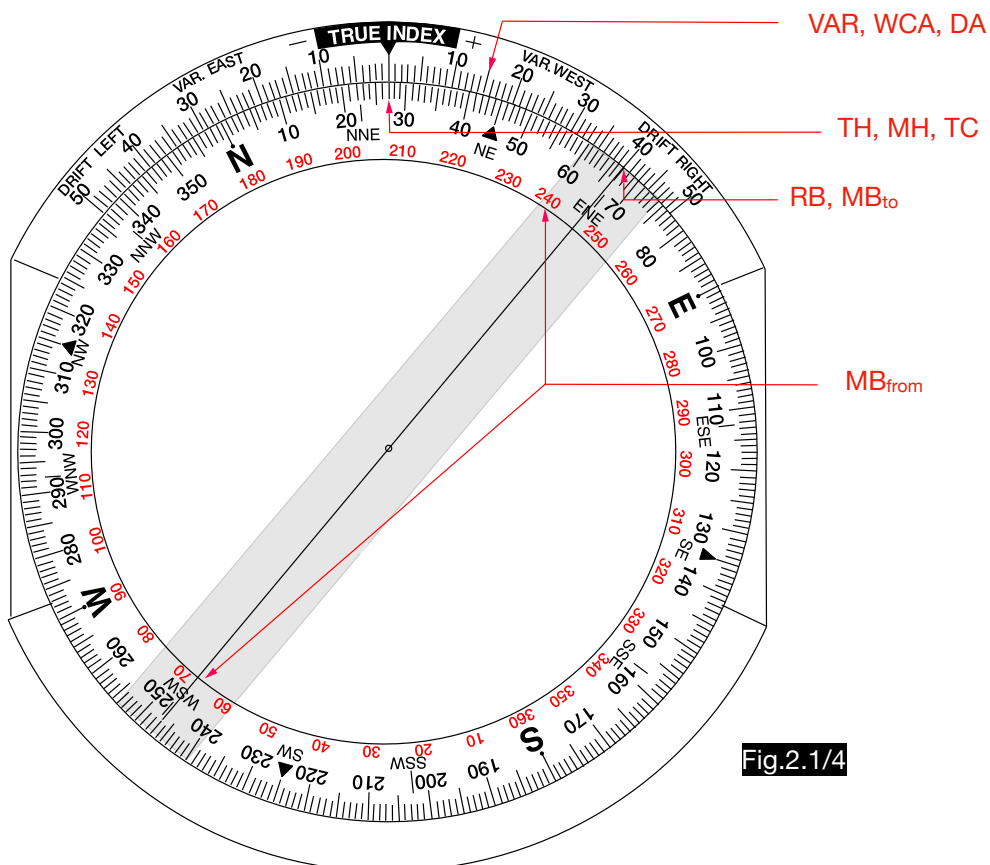


Fig.2.1/4

## 2.2.Conversion of units of measurement

Table 2.2/1 contains the conversion factors for units of measurement that are often required in aviation. The usual solutions are illustrated by examples.

1 nautical mile (NM) = 1.852 km 1 km = 0.54 NM	1 kt = 1.852 km/ h 1 km/ h = 0.54 kt	1 stat.mile = 1.609344 km 1 km = 0.621371 stat.miles
1 ft per minute = 0.00508 m/ s 1 m/ s = 196.8504 ft/ min	1 ft = 0.3048 m 1 m = 3.28084 ft	1 m/ s = 3.6 km/ h 1 km/ h = 0.277 m/ s
1 US gallon = 3.785412 Liter 1 Liter = 0.264172 US Gallons	1 Imperial Gallon = 4.54609 Ltr. 1 Liter = 0.219969 Imp.Gal.	1 pound (lb) = 0.453592 kg 1 kg = 2.204622 lb
1 m.p.h. = 1.609344 km/ h 1 km/ h = 0.621371 m.p.h.	1 yard = 0.9144 m 1 m = 1.093613 yards	1 inch = 25.4 mm 1 mm = 0.039370 inch
$t_{°C} = (t_{°F} - 32) \times 0.55$	$t_{°F} = (t_{°C} \times 1.8) + 32$	$t_{°C} = T_K - 273.15$ $T_K = t_{°C} + 273.15$

Tab.2.2/1

### Example 1 (see Fig.2.2/1)

The conversion factors 3.6 (m/s  $\rightleftharpoons$  km/h) and 1.852 (NM  $\rightleftharpoons$  km) represent a distance on the logarithmic inner scale which was graphically entered as an arc. In the example shown here, the following readings are possible:

200 km = 108 Sm (sea mile)  
200 km/h = 108 kt = 55.6 m/s

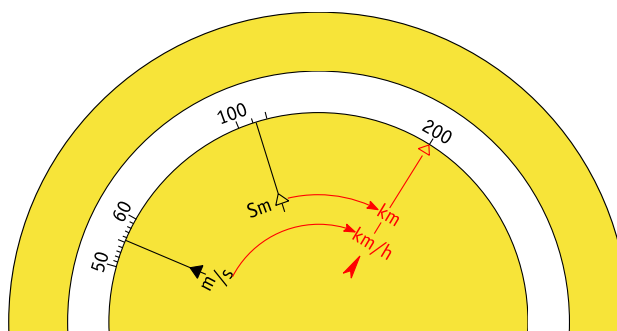


Fig.2.2/1

### Example 2 (see Fig.2.2/2)

The following results are achieved with the setting shown here:

50 km(km/h) = 27 NM(kt) = 31.07 STAT.MILES (m.p.h.)  
50 m = 54.68 yards = 164.04 feet  
50 Liter = 10.998 Imp.Gal. = 13.21 US Gallons  
33.4 m/s = 6575 ft/min

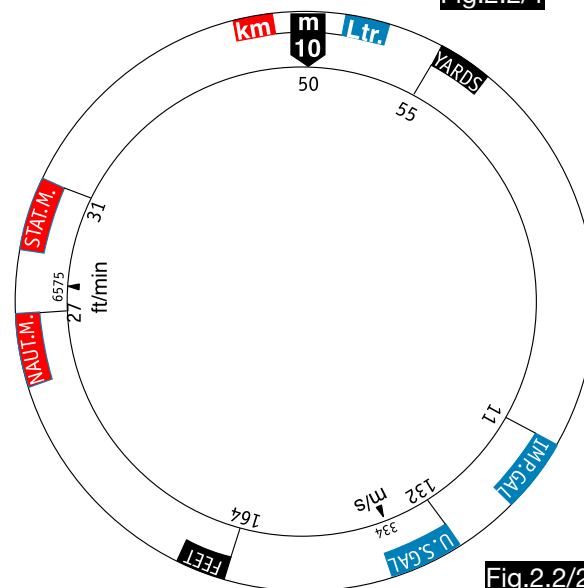


Fig.2.2/2

### Example 3 (see Fig.2.2/3)

With the help of the logarithmic scales of the calculators of the NL series, multiplications with all factors are possible. Factor 3.6 (red circle) was highlighted for conversions from m/s to km/h and vice versa (example: 51 km/ h = 14.16 m/s). For conversions from meters or kilometers to feet or miles, markings are provided at 1-6-1 (mile), 1-8-5 (n mile) and 3-0-4 (Feet).

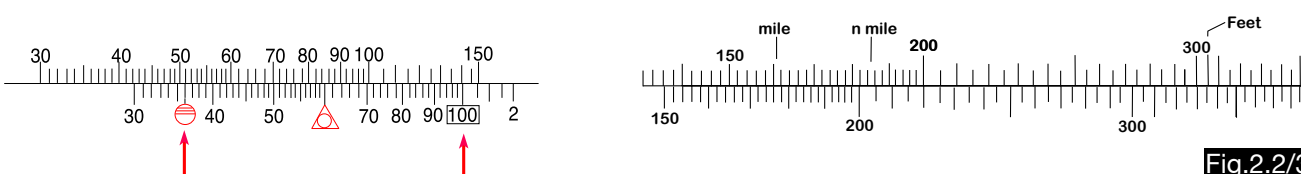


Fig.2.2/3



#### Example 4 (see Fig.2.2/4)

In this original solution, all conversions based on multiplication by a factor are made on the logarithmic outer scales. The correct factor is set by means of several small holes in the upper disc and a single hole in the lower disc. The two holes must be aligned before reading. To enable this, the manufacturer recommends inserting a sharp object into the hole next to the desired conversion and turning the upper disc until it sits in the only hole of the lower disc (set example: 60 lb = 27.21 kg). In this way, the following conversions are possible quickly and easily:

KNOTS $\rightleftharpoons$ FT. PER SEC.	OIL-QT $\rightleftharpoons$ LB	FT $\rightleftharpoons$ M
LB. $\rightleftharpoons$ KG	GAS GAL $\rightleftharpoons$ LB	KM $\rightleftharpoons$ STAT.
STAT. $\rightleftharpoons$ NAUT.	JP-4 FUEL GAL. $\rightleftharpoons$ LB.	U.S.GAL. $\rightleftharpoons$ IMP.GAL.;

The conversion NM  $\rightleftharpoons$  stat.Mile can also be done at the two arrows "NAUT" and "STAT".

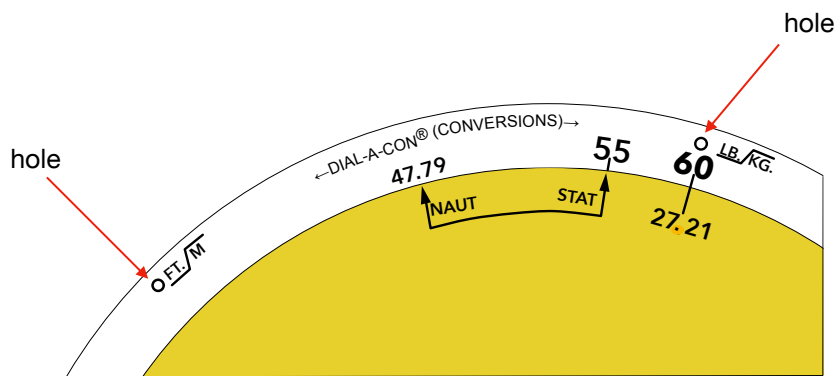


Fig.2.2/4

#### Example 5 (see Fig.2.2/5)

The conversion degrees Fahrenheit  $\rightleftharpoons$  degrees Celsius allow many calculators over additional scales of this type:

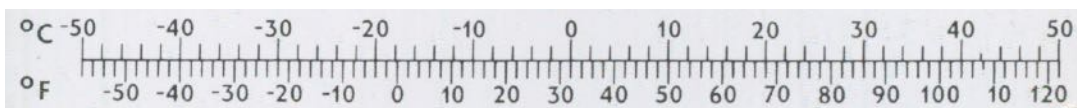


Fig.2.2/5

## 2.3. Time - speed - distance problems

### 2.3.1. Triangle calculator with log sin scale

On computers, which calculate the wind triangle with the aid of the sine theorem (see Fig.2.3/1), the determination of the flight time can be done thereafter on the same scales (see Fig.2.3./2).

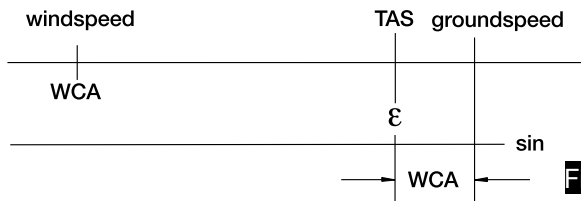


Fig.2.3/1



Fig.2.3/2

### 2.3.2. Triangle calculators with diagram slide

On computers with diagram slides, the second setting is made on the logarithmic scales on the back, which is inconvenient and can lead to errors. A. Heitor therefore proposed an additional device for these computers. Fig. 2.3/3 shows the figure from the patent specification.

The tongue (1) of an additional logarithmic slide rule is moved over the pin (2) and the groove (3) of the diagram slide in such a way that the groundspeed (after the calculation of the wind triangle in the center of the wind rose) is opposite the mark "1 hour". In this way, the flight time can be read immediately below the distance, so that no turning of the computer is necessary. The same effect could be achieved by simpler means if the diagram slide gets an additional line network (shown red in Fig.2.3/4). After the wind triangle calculation the ground speed (here 150 kt) is below the center of the compass rose. At the top or bottom of the computer, the flight time can now be read from the red lines opposite the distance. For a distance of 50 NM, for example, both scales show 20 minutes. If the distance (e.g. 80 km) is set over the time (e.g. 32 min.), the ground speed (150 km/h) can be read in the centre.

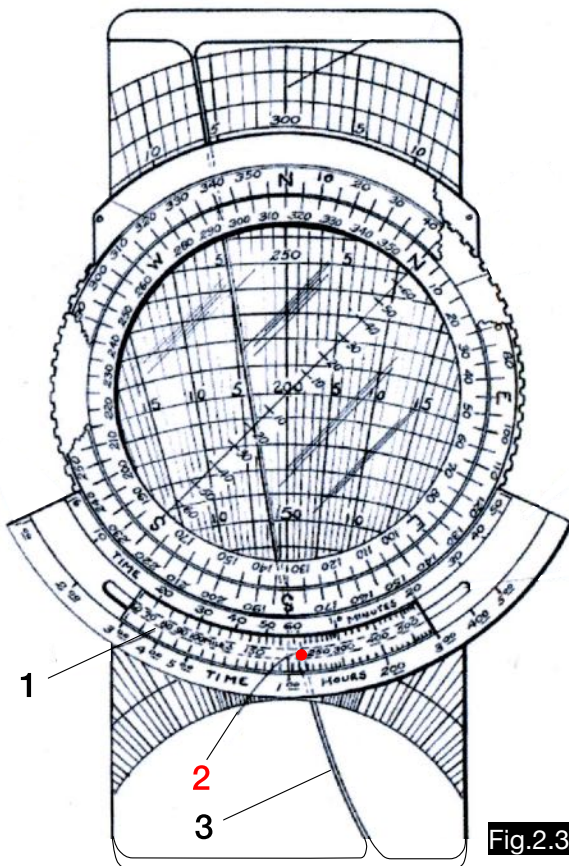


Fig.2.3/3

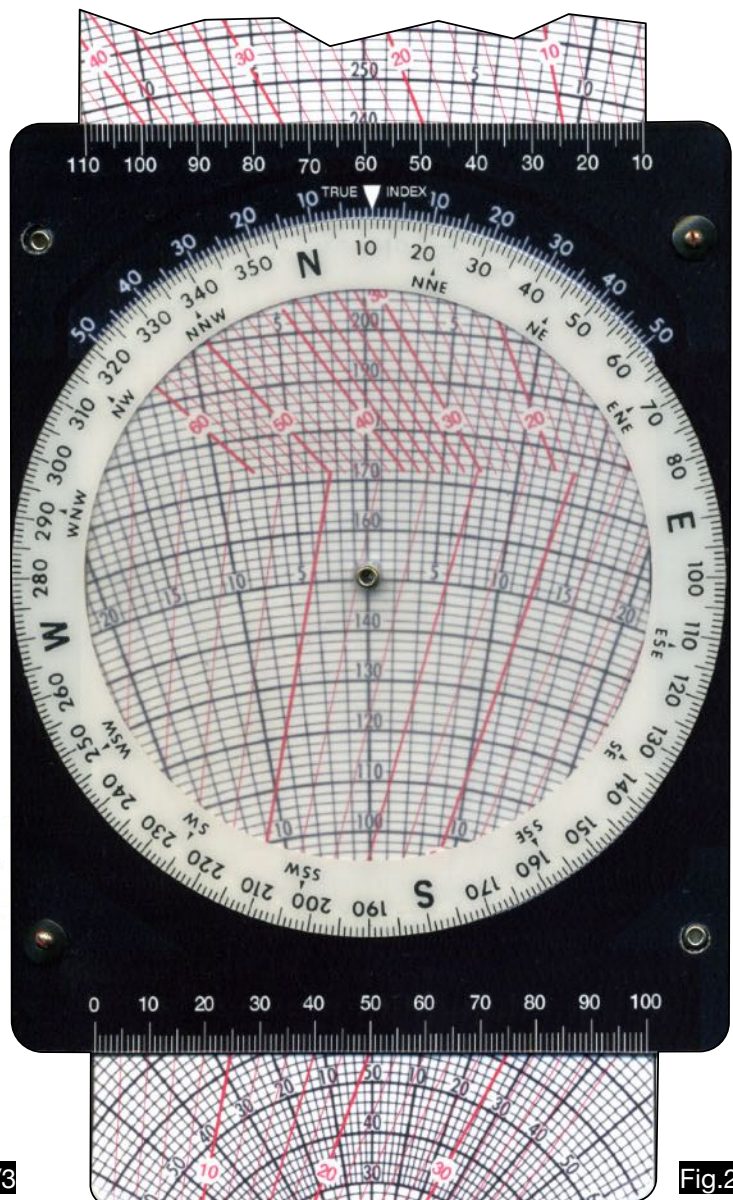


Fig.2.3/4

### 2.3.3. Time-speed-distance computer according to Keator

Fig. 2.3/5 shows the Time-Speed-Distance-Calculator of F.W. Keator which simulates the straight line by means of a thread from the coordinate origin. In the line grid the quotients  $\text{speed} = \text{distance} / \text{time}$  or  $\text{time} = \text{distance} / \text{speed}$  can be read. The figure shows this example (red lines):

Speed: 110 kt , distance: 140 NM, time: 1 hour 16 minutes.

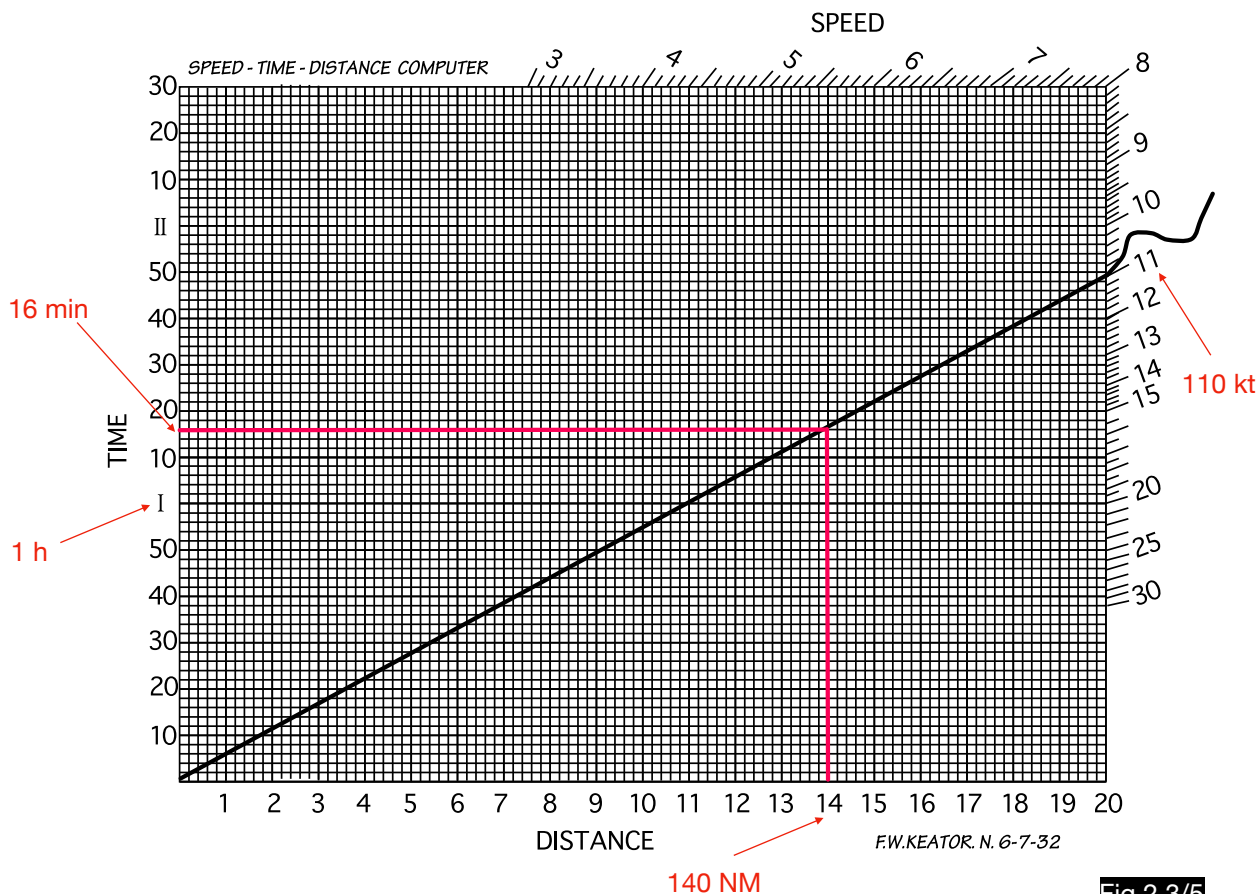


Fig.2.3/5

### 2.3.4. Time-speed-distance-Computer according to Phelps

Fig.2.3/6 represents the upper part of the Phelps Speedy Flight Computer, which is provided for time-speed-distance calculations. This quotient is shown in the figure:

$$t = \frac{s}{v} = \frac{131 \text{ NM}}{110 \text{ kt}} = 1.19 \text{ h} = 71.5 \text{ min}$$

The same setting allows the calculation of the flight time from the hourly consumption (11 l/h) and the fuel quantity (131 l).

In addition to the middle pointer, there are two other marks on the speed window, the upper one is labeled with STAT (stands for stat.mile) and the lower one with KTS (see note).



Fig.2.3/6



Fig.2.3/7 illustrates the basic design of the machine. All scales are logarithmic, so that the product relation distance = speed x time is reduced to additions (subtractions), which can be realized by a simple gear mechanism. The gear wheels 1 - 4 have the same number of teeth. The gear wheels 1 and 2 with the distance protrude beyond the housing in order to be able to set speed and distance values. The gear wheels 3 and 4 are stacked on one axle. Wheel 3 with the time scale is in constant engagement with wheel 1. Wheel 4 (shown in red) with the window and the reading mark for the time, is constantly in engagement with wheel 2. Three examples are used to explain how it works.

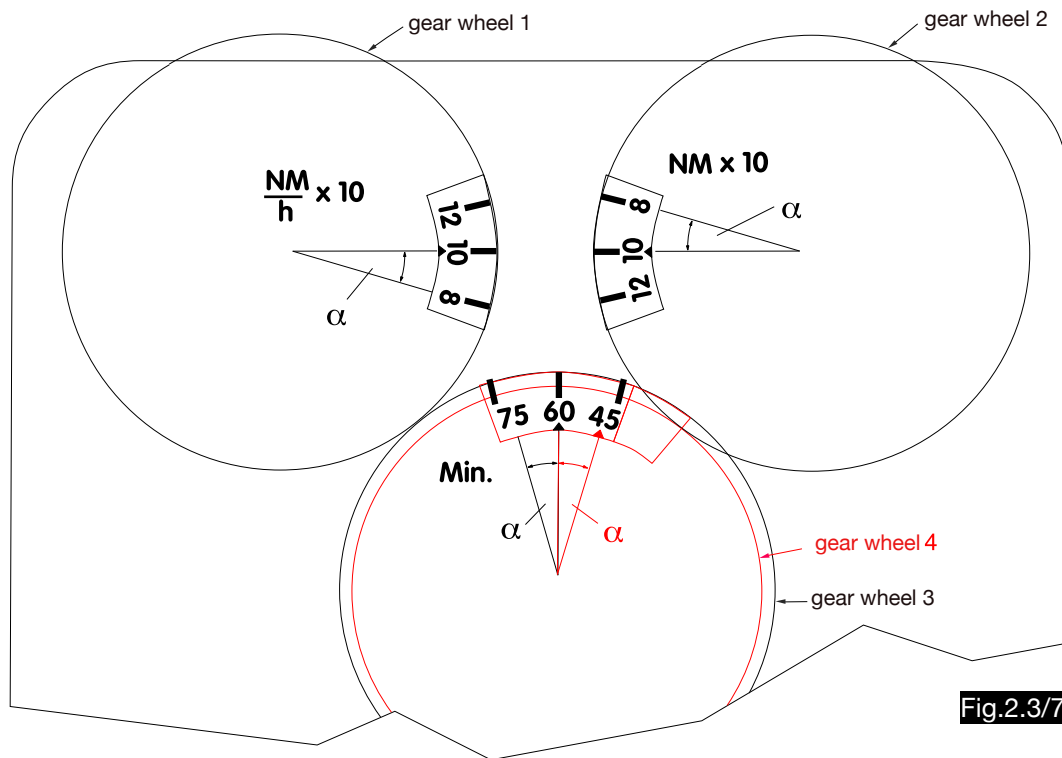


Fig.2.3/7

The 1st example is shown in Fig.2.3/7: speed: 100 kt; distance: 100 NM; time: 60 min.

For the 2nd example these values were assumed: speed: 80 kt; distance: 100 NM; time: 75 min. In order to adjust the speed 80 at wheel 1, it must be turned by the angle  $\alpha$  to the left. Wheel 3 rotates to the right by the same angle and 75 minutes of flight time are displayed ( $100:80 = 1.25$  hours = 75 minutes). For the 3rd example these values were assumed: speed: 80 kt; distance: 80 NM; time: 60 min. For this, wheel 1 must be turned to the left by the angle  $\alpha$ . Wheel 3 rotates to the right by the same angle so that the number 75 is at the top. Since wheel 2 must be set to 80 NM, wheel 4 moves with the time window by the angle  $\alpha$  to the right, so that the time mark is opposite the number 60.

#### Note:

The three markers on the speed window are advantageous if one wants to use land and nautical miles simultaneously. The amounts of the speeds of both dimensions differ by a factor of 1.15 ( $\text{kt} \times 1.15 = \text{m.p.h.}$ ). The multiplication occurs at the logarithmic speed scale (see Fig.2.3/8) by the distance between the speed marks, which corresponds to the amount 1-1-5 on the logarithmic scale. Thus it is possible to set the speed in m.p.h. at the STAT mark, while nautical miles are used at the distance scale. Conversely, the speed can be set in knots at the KTS marker and stat. miles can be used in the distance scale.

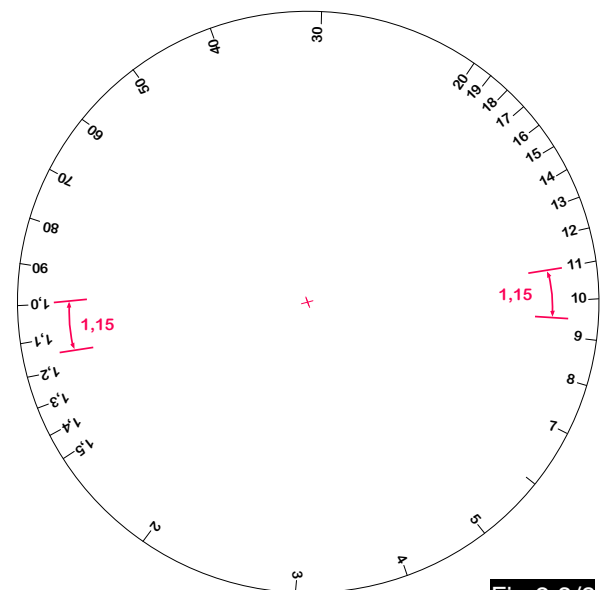


Fig.2.3/8



### 2.3.5. Time-speed-distance computer according to Brazel

The calculator Naviquick (see Fig.2.3/9), invented by J. Brazel, has the shape of a circle and is suitable for working with the map as well as for calculating distance, time and speed. In the distance field 3, the yellow point P indicates the distance S in NM, which was picked up in a map of the scale 1:500,000. The time scale 4 shows the flight time for this distance in minutes when flying with the ground speed set with the screw 2 in the speed scale 1. Fig.2.3/10 schematically shows the kinematics inside the device. When adjusting the ground speed using nut 2, the lever 6 and the speed mark 7 move on spindle 8, which is rotatably mounted in the movable circular leg 5. The pin 9 is attached to the lever 6 and slides in the groove 10 when the circle is opened. The leg 5 is also connected via the pin 11 to the rack 12, which slides on the rail 13 and thus rotates the gear wheel 14 and the minute pointer attached to it. When setting a higher ground speed, lever 6 and pin 9 move downwards. This changes the transmission ratio between leg 5 and rack 12, so that shorter times can be read off the scale at higher speeds. If the circle is closed as shown in Fig.2.3/10, i.e. the distance S is zero, the speed adjustment has no effect on the time display because the groove 10 is parallel to spindle 8. The yellow point P is attached to the leg 5 for the display of the distance S. This point moves along the lines in the distance field 3 when different distances S are set. Schematically is shown the position of leg 5 at high speeds in red and at low speeds in green.

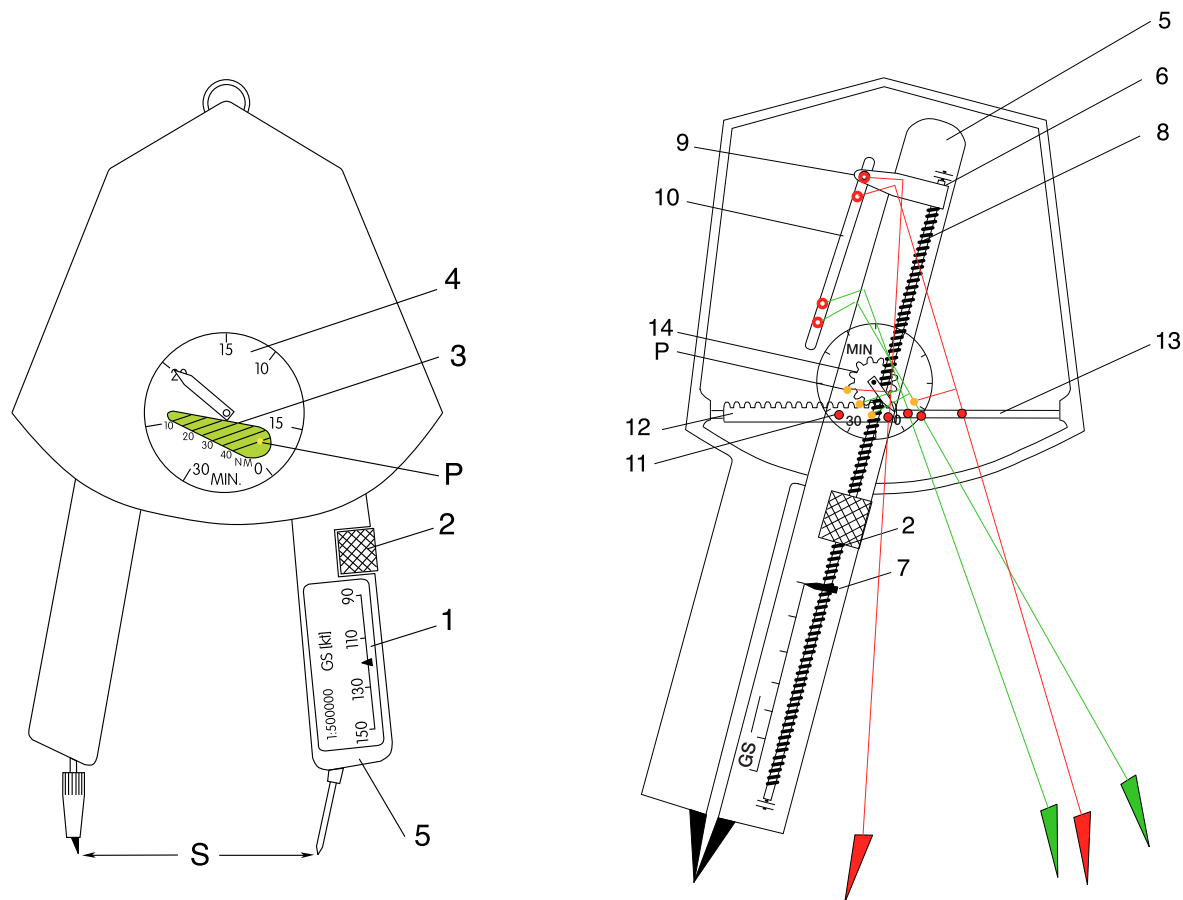
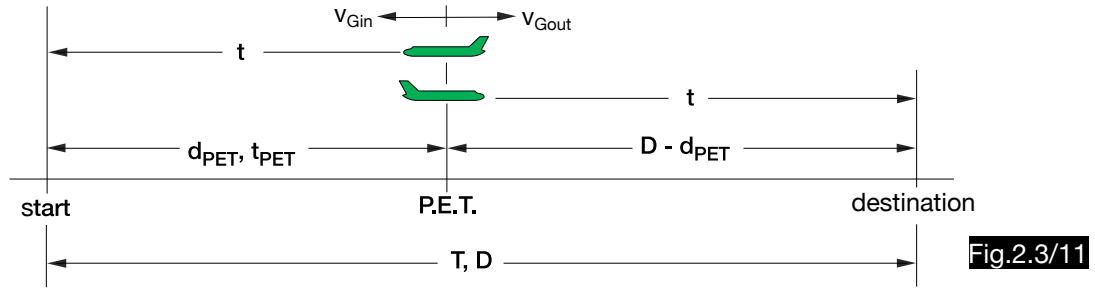


Fig.2.3/9

Fig.2.3/10

### 2.3.6. Point of equal times (P.E.T.)

At the point of equal times (P.E.T.) the times for the continuation of the flight to the destination and for the return to the starting point are the same. In Fig.2.3/11 the distances and times are described.



$V_{Gin}$  ground speed in  
 $V_{Gout}$  ground speed out  
 $d_{PET}$  distance start - P.E.T.  
 $t_{PET}$  time start - P.E.T.  
 $D$  distance start - destination  
 $T$  time start - destination \*)

$$t = \frac{d_{PET}}{V_{Gin}} = \frac{D - d_{PET}}{V_{Gout}} \rightarrow \frac{V_{Gout}}{V_{Gin}} = \frac{D}{d_{PET}} - 1 \rightarrow d_{PET} = \frac{D}{\frac{V_{Gout}}{V_{Gin}} + 1} \rightarrow d_{PET} = \frac{D \cdot V_{Gin}}{V_{Gout} + V_{Gin}} \quad [2.3/1]$$

$$d_{PET} = \frac{D \cdot V_{Gin}}{V_{Gout} + V_{Gin}} \quad \left| \begin{array}{l} D = T \cdot V_{Gout} \\ d_{PET} = t_{PET} \cdot V_{Gout} \end{array} \right| \quad t_{PET} \cdot V_{Gout} = \frac{T \cdot V_{Gout} \cdot V_{Gin}}{V_{Gout} + V_{Gin}} \rightarrow t_{PET} = \frac{T \cdot V_{Gin}}{V_{Gout} + V_{Gin}} \quad [2.3/2]$$

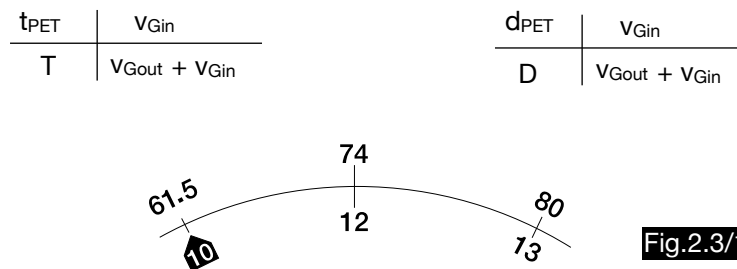
Example:

$D = 1000 \text{ km};$	$V_{Gout} = 500 \text{ km/h};$	$V_{Gin} = 800 \text{ km/h};$	$T = \frac{D}{V_{Gout}} = \frac{1000}{500} = 2 \text{ h} = 120 \text{ min}$
------------------------	--------------------------------	-------------------------------	---

$$d_{PET} = \frac{1000 \text{ km} \times 800 \frac{\text{km}}{\text{h}}}{1300 \frac{\text{km}}{\text{h}}} = 615 \text{ km}$$

$$t_{PET} = \frac{120 \text{ min.} \times 800 \frac{\text{km}}{\text{h}}}{1300 \frac{\text{km}}{\text{h}}} = 74 \text{ min}$$

The formulas for  $d_{PET}$  and  $t_{PET}$  require these settings in logarithmic scales (see Fig.2.3/12):



\*) The time  $T$  results from the available fuel, which in practice is always reduced by a reserve quantity.

### 2.3.7. Point of no return (P.N.R.) or radius of action

Depending on the total fuel quantity, one can just return from the point of no return (P.N.R.) to the starting point. If the fuel supply is reduced by a certain reserve, this point is shifted towards the starting point and is then called the point of safe return. The distance to the point of no return is also called the radius of action. The distances and times are defined in Fig.2.3/13.

$V_{Gin}$  ground speed out  
 $V_{Gout}$  ground speed in  
 $d_{PNR} (R)$  distance start - P.N.R.  
 (radius of action)  
 $t_{PNR}$  time start - P.N.R.  
 $t_{in}$  time P.N.R. - start  
 $T$   $t_{in} + t_{PNR}$

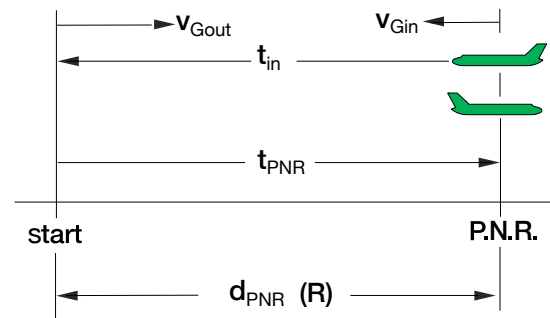


Fig.2.3/13

$$t_{PNR} V_{Gout} = (T - t_{PNR}) V_{Gin} \rightarrow \frac{T - t_{PNR}}{t_{PNR}} = \frac{V_{Gout}}{V_{Gin}} \rightarrow \frac{T}{t_{PNR}} = \frac{V_{Gout}}{V_{Gin}} + 1 \rightarrow t_{PNR} = \frac{T V_{Gin}}{V_{Gout} + V_{Gin}} \quad [2.3/3]$$

For the radius of action  $R = d_{PNR} = V_{Gout} \cdot t_{PNR}$  this formula results:

$$R = \frac{T V_{Gin} V_{Gout}}{V_{Gout} + V_{Gin}} \quad [2.3/4]$$

Example:

$V_{Gout} = 500 \text{ km/h}$	$V_{Gin} = 800 \text{ km/h}$	$T = 4 \text{ h}$
$t_{PNR} = \frac{4 \text{ h} \times 800 \frac{\text{km}}{\text{h}}}{1300 \frac{\text{km}}{\text{h}}} = 2.46 \text{ h} = 148 \text{ min}$		$R = t_{PNR} V_{Gout} = 2.46 \times 500 \text{ km} = 1230 \text{ km}$

The formulas for  $R$  and  $t_{PNR}$  require these settings in logarithmic scales (see Fig.2.3/14):

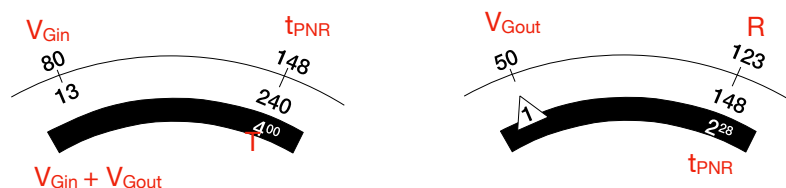


Fig.2.3/14

Fig.2.3/15 shows how the formulas for P.N.R. and P.E.T. are displayed on the MERCATOR computer.

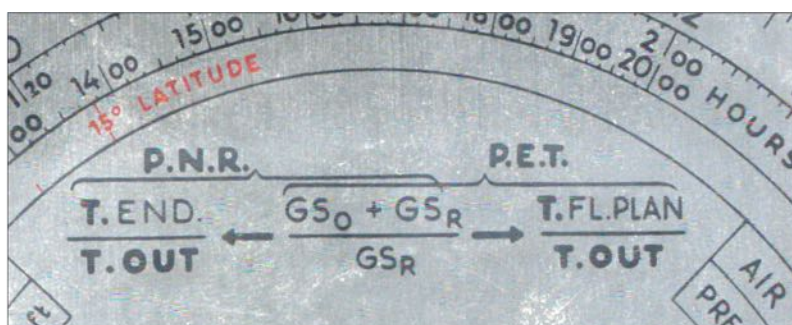


Fig.2.3/15

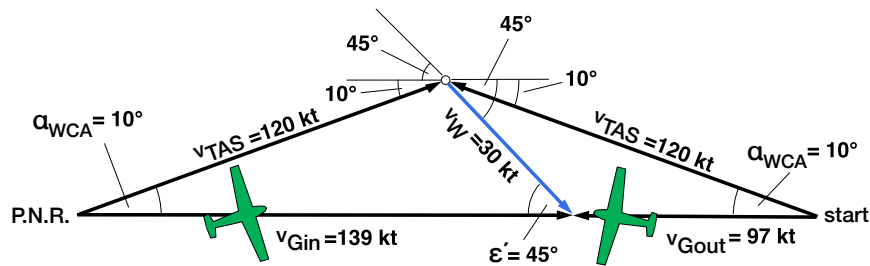


Fig.2.3/16

Flight times and ground speeds are inversely proportional:  $\frac{V_{Gout}}{V_{Gin}} = \frac{t_{in}}{t_{out}}$

In the two wind triangles the following applies according to the sine theorem:

$$V_{Gout} = V_W \frac{\sin(\epsilon' - \alpha_{WCA})}{\sin \alpha_{WCA}} \quad V_{Gin} = V_W \frac{\sin(\epsilon' + \alpha_{WCA})}{\sin \alpha_{WCA}} \quad \longrightarrow \quad \frac{t_{in}}{t_{out}} = \frac{\sin(\epsilon' - \alpha_{WCA})}{\sin(\epsilon' + \alpha_{WCA})} = x$$

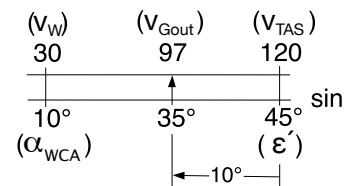
For the total flight time T apply:  $T = t_{in} + t_{out} = t_{out} + x t_{out} = t_{out} (1 + x)$

The following applies to the radius of action:  $R = t_{out} V_{Gout}$

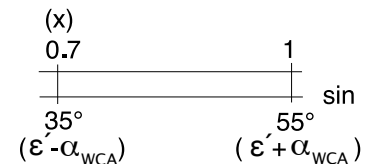
$$\left. \begin{array}{l} T = t_{in} + t_{out} = t_{out} (1 + x) \\ R = t_{out} V_{Gout} \end{array} \right\} \frac{R}{T} = \frac{V_{Gout}}{1 + x}$$

The following settings are required for this example on a computer with a sine scale:

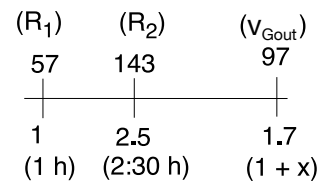
1. Calculation of  $\alpha_{WCA}$  ( $10^\circ$ ) and  $V_{Gout}$  (97 kt):



2. Calculation of the factor x (0.7):



3. Calculation of the radius of action  $R_1 = 57$  NM (for  $T = 1$  h) and  $R_2 = 143$  NM (for  $T = 2.5$  h):



#### Note:

In order to calculate the largest possible radius of action, the flight time T must be calculated as follows using the range diagram schematically shown in Fig.2.3/17:

1. Determination of the relative range at the curve maximum.
2. Determination of the corresponding true air speed ( $V_{max.range}$ ).
3. Determination of the hourly fuel consumption by division  $V_{max.range} : \text{relative range}$
4. The maximum possible flight time T is obtained from the fuel quantity (reduced by the necessary reserve) divided by the hourly consumption.

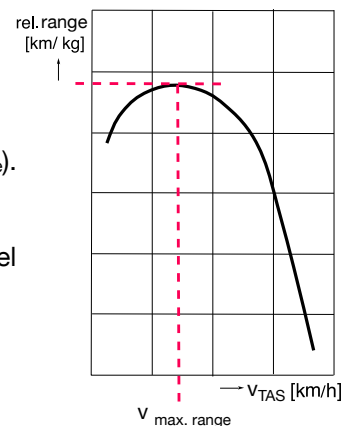


Fig.2.3/17



Example:

true course	280°/100°
true airspeed	60 kt
true wind	150°/30 kt
ground speed	$V_{Gout}$ 74.71 kt
	$V_{Gin}$ 36.14 kt

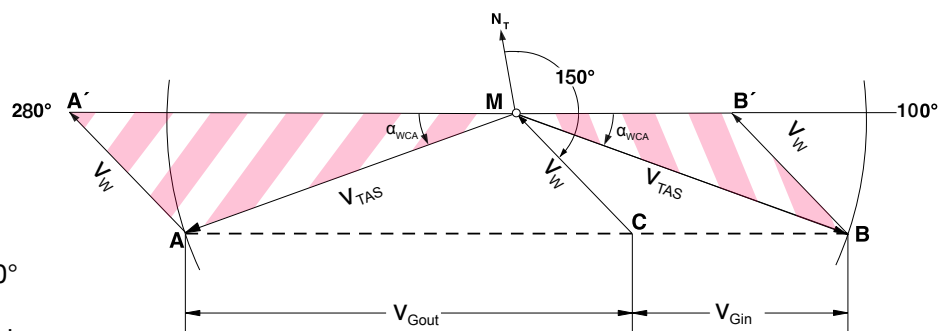


Fig.2.3/18

A drawing according to Fig.2.3/18 is used to graphically determine the ground speeds  $V_{Gout}$  and  $V_{Gin}$ . From point M on the route A' B' (280°/100°), the wind vector is drawn in the direction of 150° (30 length units). Then a parallel to the route is drawn through the endpoint C of the wind vector (dashed line). Two arcs around the point M of radius 60 units of length ( $V_{TAS}$ ) intersect the dashed line at points A and B. The actual wind triangles (M A' A for the outbound flight and M B B' for the return flight) are hatched in red. The ground speeds of the outbound and return flights are determined to  $V_{Gout} = 75$  and  $V_{Gin} = 36$  length units. This solution is also possible on certain computer types (see Fig.2.3/19). After setting the compass rose to course 280°/100° the wind point C is marked on the 30 knots circle in direction 150° and then on the dashed line through the wind point between points A (on the 60 kt circle) and point C the  $V_{Gout} = 75$  kt is counted. In the same way the ground speed  $V_{Gin} = 36$  kt is found between the points C and B.

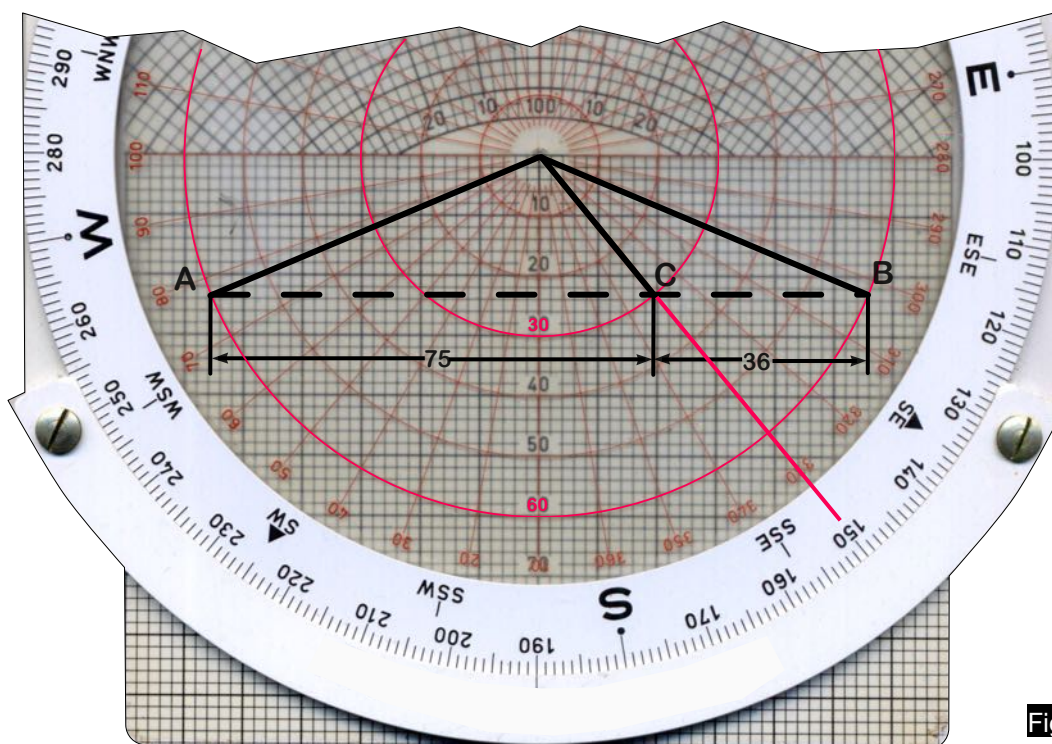


Fig.2.3/19

## 2.3.8. Meeting point calculation

### 2.3.8.1. General

The rendezvous calculations discussed here are applicable for aircraft flying on the same course or on courses that differ by 180°. Meetings between moving objects on different courses are described under intercepting.

When solving rendezvous problems, it does not matter whether the airspeeds or ground speeds of the aircraft are taken into account. Since different wind conditions can occur in different flight levels, more accurate results can be expected when ground speeds are used.

Flight computers are particularly suitable for these problems because of their logarithmic time scales.

### 2.3.8.2. Airplanes on opposite courses

For the data of an example given in Fig.2.3/20 the position and time of the meeting shall be calculated.

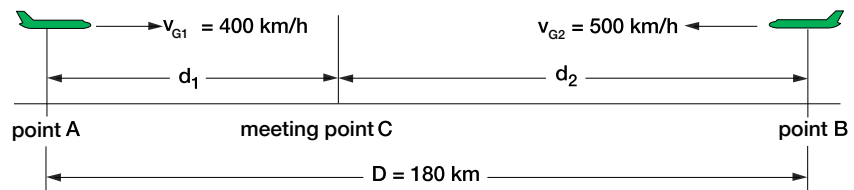


Fig.2.3/20

Solution:

$$t_{mp} = \frac{D}{v_{G1} + v_{G2}} = \frac{180 \text{ km}}{900 \text{ km/h}} = 0.2 \text{ h} = 12 \text{ min}$$

$$d_1 = t_{mp} \cdot v_{G1} = 400 \text{ km/h} \times 0.2 \text{ h} = 80 \text{ km}$$

$$d_2 = t_{mp} \cdot v_{G2} = 500 \text{ km/h} \times 0.2 \text{ h} = 100 \text{ km}$$

The settings of Fig.2.3/21 are required for this example on a flight computer.

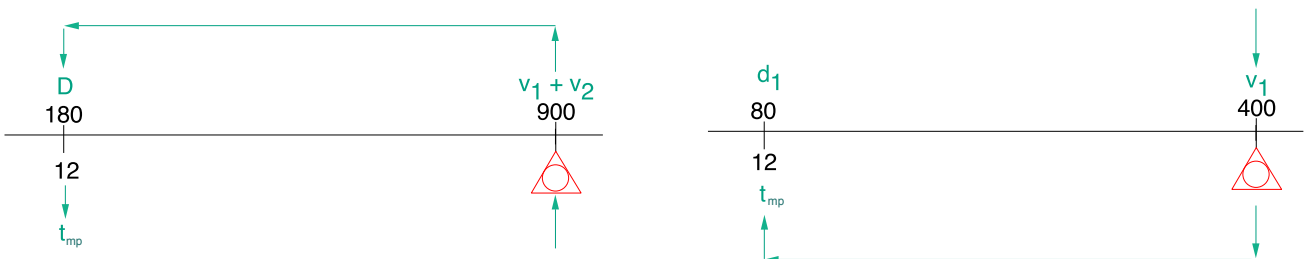


Fig.2.3/21

### 2.3.8.3. Airplanes on the same course

Fig.2.3/22 shows an example in which an aircraft with ground speed  $v_{G1} = 400 \text{ km/h}$  flies from point A to point B. A second aircraft follows on the same route with  $v_{G2} = 440 \text{ km/h}$ , but overflies the point A  $t_D = 12 \text{ min}$  later. It shall be calculated after which time  $T_{mp}$  and distance  $D$  the first aircraft is reached.

Fig.2.3/23 shows the settings on a flight computer.

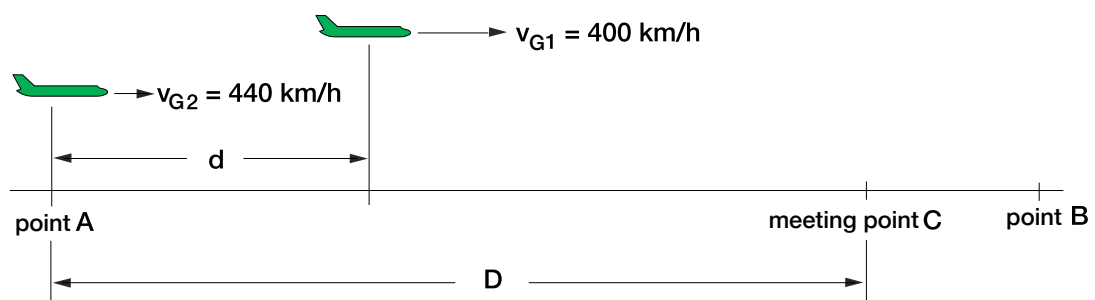


Fig.2.3/22

Solution:

1. Distance  $d = v_{G1} t_D = 400 \frac{\text{km}}{\text{h}} \frac{12}{60} \text{h} = 80 \text{ km}$

2. Time  $T_{\text{mp}} = \frac{d}{v_{G2} - v_{G1}} = \frac{80 \text{ km}}{40 \frac{\text{km}}{\text{h}}} = 2 \text{ h}$

3. Distance  $D = v_{G2} t_{\text{mp}} = 440 \frac{\text{km}}{\text{h}} 2 \text{ h} = 880 \text{ km}$

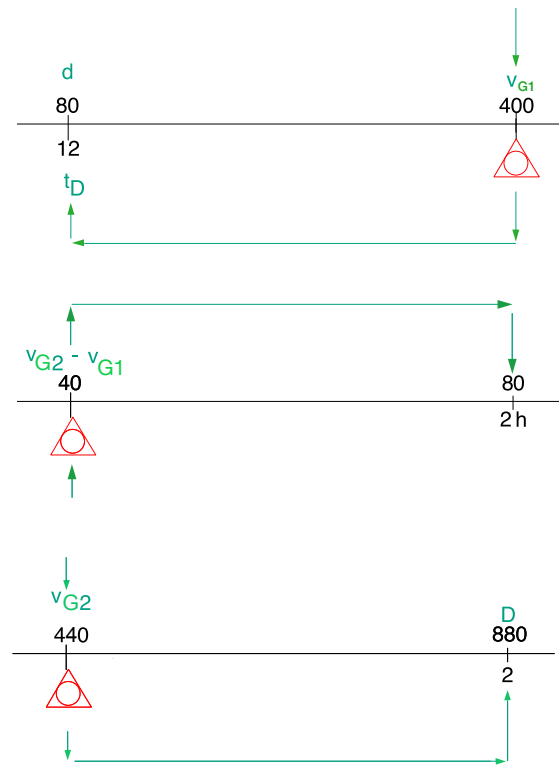


Fig.2.3/23

## 2.4. Angle calculations

### 2.4.1. Triangular calculations

#### 2.4.1.1. Calculators with log sine and log tangent scales

Example:

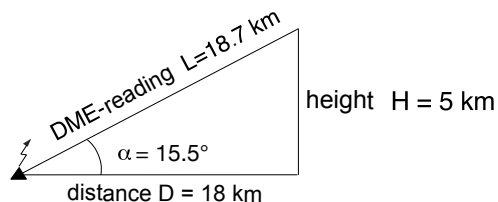


Fig.2.4/1

$\sin \alpha = \frac{H}{L}$	
$\tan \alpha = \frac{H}{D}$	
$\frac{\tan \alpha}{H} = \frac{1}{D}$	

#### 2.4.1.2. Calculators with log sine scales

Example 1:

Because at small angles  $\sin \alpha = \tan \alpha$ , with these two arithmetic operations a sufficiently accurate result is achieved:

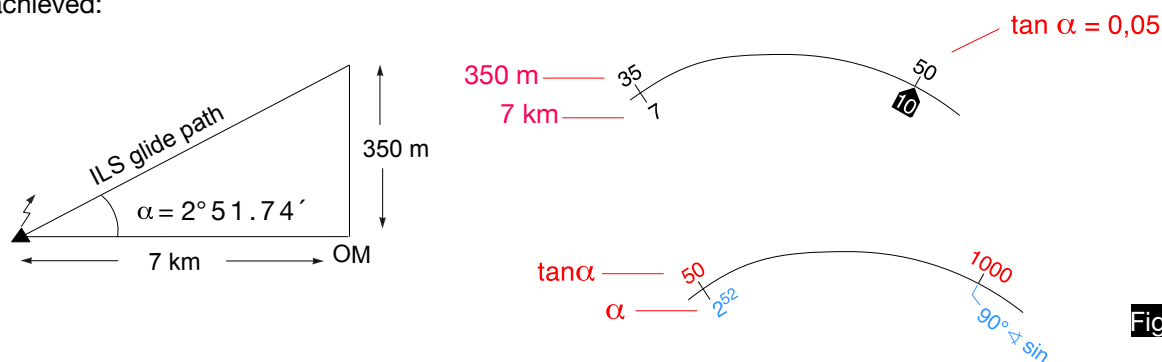


Fig.2.4/2

Example 2:

The lateral displacement  $V$  can be exactly determined using the sinus theorem:



Fig .2.4/3

#### 2.4.1.3. Calculators without log sine and log tangent scales

Example:

The Bellamy formula is used to determine a drift of  $d = 36$  NM over a distance of  $D = 400$  NM. The required wind correction angle is calculated:

$$\tan \alpha_{WCA} = \frac{36}{400} = 0.09 \quad \rightarrow \quad \alpha_{WCA} = 5.14^\circ$$

After equ.1.30/1 applies:  $\alpha [^\circ] \approx \frac{d \times 57.3}{D} \quad \rightarrow \quad \alpha_{WCA} [^\circ] \approx \frac{36 \times 57.3}{400} \approx 5.16^\circ$



On computers with logarithmic scales the setting according to Fig.2.4/4 is required:



Fig.2.4/4

Computers without sinus or tangent scales usually have a diagram slide with which this task can be solved graphically. After setting the compass rose to any course (360° was selected in Fig.2.4/5), a 36 km long distance is marked from the centre (red dot). After setting a 90° larger course (90° in Fig.2.4/6) and positioning the distance 400 below the centre point, the angle 5.5° is found under the drawn point.

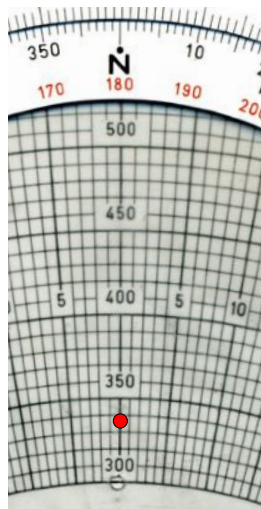


Fig.2.4/5

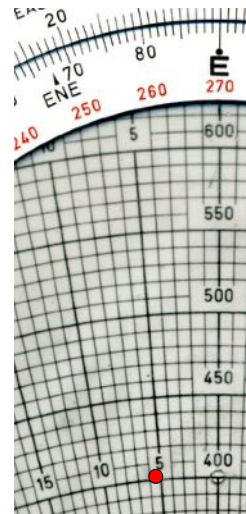


Fig.2.4/6

## 2.4.2. Wind component determination

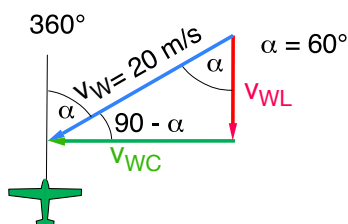


Fig.2.4/7

For the example shown in Fig.2.4/7 the components  $v_{WL} = 10$  m/s and  $v_{WC} = 17.3$  m/s are calculated according to the sine theorem.

$$\frac{v_{WC}}{\sin \alpha} = \frac{v_{WL}}{\sin (90 - \alpha)} = \frac{v_W}{\sin 90}$$

For calculators with logarithmic sine scales the setting according to Fig.2.4/7 is required.

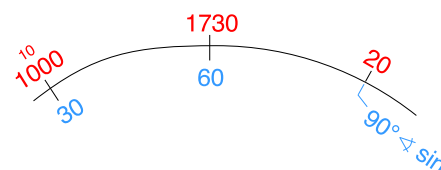


Fig.2.4/7

Computers with diagram slides often have a special wind component grid for determining the wind components (Fig.2.4/8). The computers described under 1.12. are used in the same way (see Fig. 2.4/9 and 10). For more accurate results it is recommended to double the wind speed so that the results have to be halved.

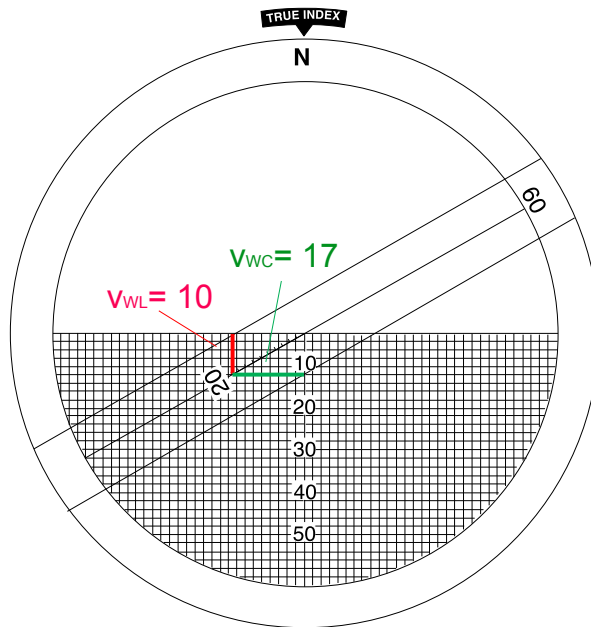


Fig.2.4/8

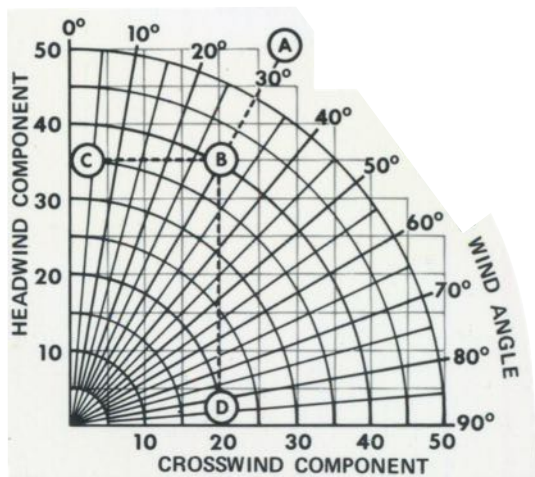


Fig.2.4/9

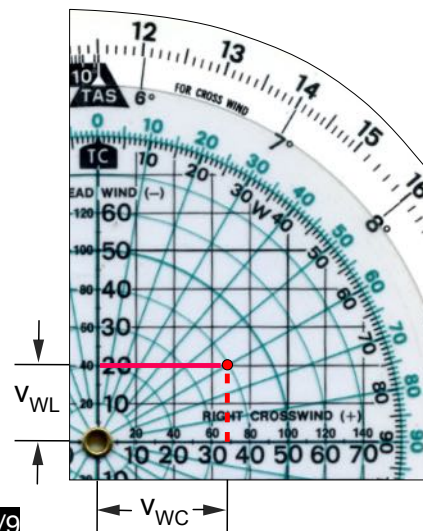


Fig.2.4/10

### 2.4.3. Drift correction

When flying without wind correction, the airplane drifts away from the route. If it is not possible to calculate the wind influence in advance, the distance off course and the distance flown can be calculated by finding the position.

On computers that have a tangent scale, the calculation of the corrected heading is simple. For this purpose, during the flight from A to B, the distance off course  $a$  and the distance flown  $b$  must be calculated by defining the position at point C. The angles  $\alpha$  and  $\beta$  are determined by the tangent. A heading correction by the angle  $\alpha$  leads to the continuation of the flight along a parallel to the route, the correction of the heading by the angle  $\alpha + \beta$  leads directly to target B. This approximation solution is based on the incorrect assumption that the drift angle is equal to the wind correction angle.

The following example explains the calculation (see Fig.2.4/11):

distance off course $a$	7 km	angle to parallel course $\alpha$	8° right
distance flown $b$	50 km	angle to intercept $\beta$	4° right
distance to be flown $c$	100 km	total correction $\alpha + \beta$	12° right

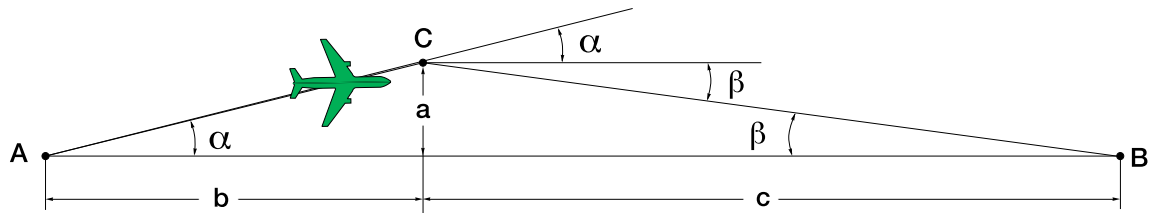


Fig.2.4/11

On computers with tangent scales these settings of Fig.2.4/12 are required:



Fig.2.4/12

Many navigation computers have a DRIFT CORRECTION window whose tangent scale works together with the logarithmic time-speed-distance scale. For example, to calculate the quotient  $7/100$  in the logarithmic scale, the 70 on the outer scale must be set opposite the 10 on the inner scale. The result is  $\tan \beta = 0.07$  or  $\tan \beta = 0.7$ , which corresponds to angles of 4° or 34.9°. Both values appear in the DRIFT-CORR. window. In order to determine the correct result, an approximate advance calculation is necessary (at a distance of about 60 km, a lateral deviation of 1 km results in an angle of 1°). Fig.2.4/13 shows the required settings:

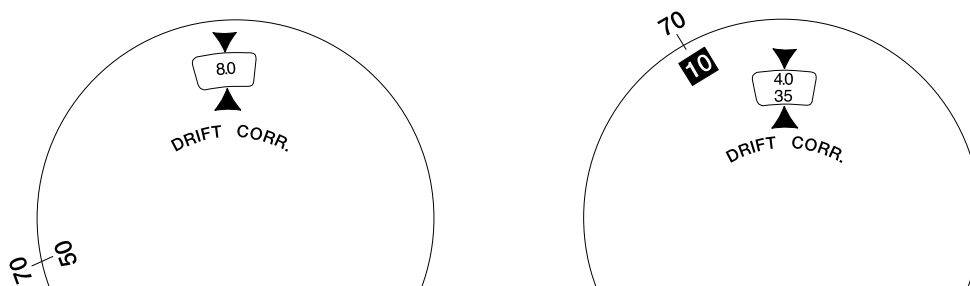


Fig.2.4/13

With the logarithmic scales of any navigation computer this calculation is also possible in good approximation. According to equ. 1.30/1 applies to small angles (see Fig.2.4/14):

$$\alpha [^\circ] \approx 57.3 \frac{a}{b} = 57.3 \frac{7}{50} = 8.02^\circ$$

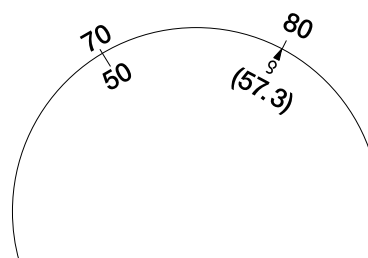


Fig.2.4/14

## 2.5. Level turn

### 2.5.1. Determination of the turn radius

Fig.2.5/1 shows the balance of forces in the stationary level turn ( $F_L$  = lift,  $F_W$  = weight).

For the radial acceleration applies:  $b = \frac{v^2}{R}$

So the centrifugal force is:  $F_c = m \frac{v^2}{R}$

From this can be derived for the radius R:

$$\tan \mu = \frac{F_c}{F_w} = \frac{m \frac{v^2}{R}}{m g} = \frac{v^2}{R g} \quad \boxed{R = \frac{v^2}{g \tan \mu}} \quad [2.5/1]$$

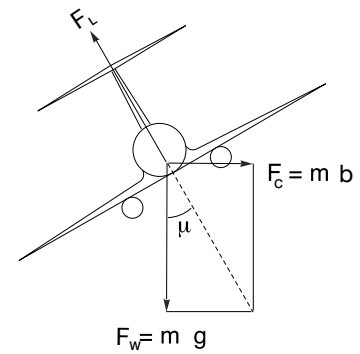


Fig.2.5/1

A computer of type NL has the mark  $\textcircled{R}$  in the tangent scale for this calculation, which is determined according to Fig.2.5/2.

$$\frac{\frac{1}{g}}{R} = \frac{\tan \mu}{v^2} \quad \left| \quad \frac{1}{g} = 7.87 \times 10^{-6} \left[ \frac{\text{h}^2}{\text{km}} \right] \quad \frac{\textcircled{R}}{7.87} \quad \frac{\tan \mu}{v^2} \right. \quad \left. \frac{\text{radius}[\text{km}]}{v:100 [\text{km/h}]} \right. \quad [2.5/2]$$

### 2.5.2. Determination of the three-sixty time $t_{360}$

$$v = \frac{2\pi R}{t_{360}} = \frac{2\pi v^2}{t_{360} g \tan \mu} \quad \boxed{t_{360}[\text{sec.}] = \frac{2\pi v_{[\text{km/h}]}}{3.6 g \tan \mu}} \quad [2.5/2]$$

With  $\frac{2\pi}{3.6 \times 9.81} = 0.178$  equ. 2.5/2 gets this form:  $t_{360}[\text{sec.}] = \frac{0.178 v_{[\text{km/h}]}}{\tan \mu}$

Therefore, in the tangent scale of the NL computer, the additional mark  $t_{360}$  is located at the position  $\tan 10^\circ = 0.176$ , so that this setting scheme results (see Fig.2.5/3):

$$\frac{0.178}{t_{360}} \quad \left| \quad \frac{\tan \mu}{v_{\text{km/h}}} \right. \quad [2.5/3]$$

If the curve radius is known, the curve time can be determined in the distance - time - scale. In the additional scale above, the time for curve angle  $\delta < 360^\circ$  can be determined.

$$v = \frac{2\pi R}{t_{360}} \quad \frac{v}{R} = \frac{2\pi}{t_{360}} \quad \boxed{\frac{v_{[\text{km/h}]}}{R_{[\text{km}]}} = \frac{2\pi \times 3600}{t_{360}[\text{sec}]}} \quad [2.5/3]$$

If the quotient  $v:R$  is set, the curve time in seconds for a three sixty circle results at the mark  $t_{360}$ , which is located at position 2-2-6-2 ( $2\pi \times 3600 = 22620$ ). An additional scale allows the determination of times for curve angles  $\delta < 360^\circ$ . Fig.2.5/4 shows the setting for the following example:

airspeed $v$	300 km/h
bank angle $\mu$	$15^\circ$
curve radius $R$	2642 m
time $t_{360}$ ( $\delta = 360^\circ$ )	199.1 s
time $t_{180}$ ( $\delta = 180^\circ$ )	100 s

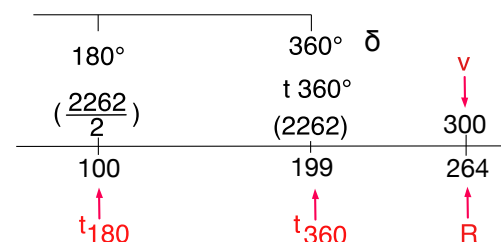


Fig.2.5/4



### 2.5.3. Determination of the angular speed $\omega_z$

With  $\omega_z = \frac{2\pi}{t_{360}}$  and equ. 2.5/2 one gets:

$$\omega_z \left[ \frac{\text{rad}}{\text{s}} \right] = \frac{g \tan \mu}{v} = \frac{3.6 \times 9.81 \times \tan \mu}{v_{[\text{km/h}]}} = \frac{35.3 \times \tan \mu}{v_{[\text{km/h}]}}$$

This formula supplies the angular speed  $\omega_z$  in radians. In practice, however, the result is required in degrees per second. Since for angles below  $15^\circ$  the tangent is nearly equal to the argument, the tangent scale of the NL calculator is suitable for this calculation.

$1^\circ = \frac{2\pi}{360}$  for  $\alpha = 15^\circ$  follows from this:  $\alpha \left[ \frac{\text{rad}}{\text{s}} \right] = \frac{2\pi \times 15}{360} = 0.2618$  (compare:  $\tan 15^\circ = 0.2679$ )

This results in the following setting scheme:

$\omega_z \left[ \frac{^\circ}{\text{s}} \right]$	$\tan \mu$
35.3	$v_{[\text{km/h}]}$

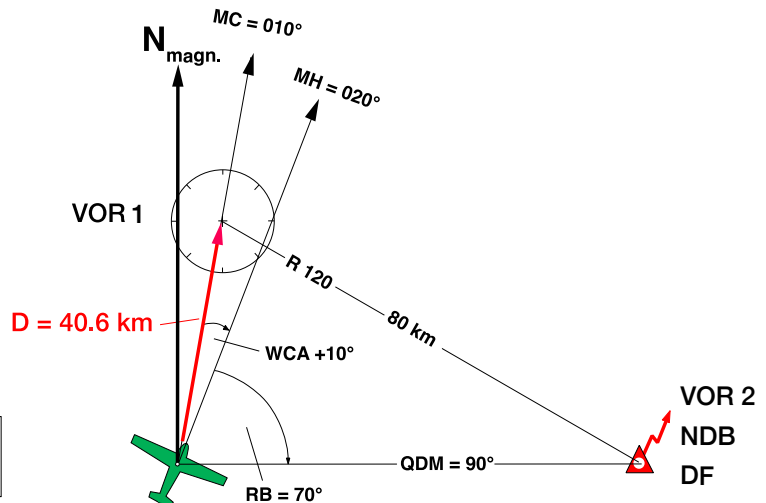
**Fig.2.5/5**

Note: The mark  $\omega_z$  calculated here does not exist in the computers of the NL series published until now.

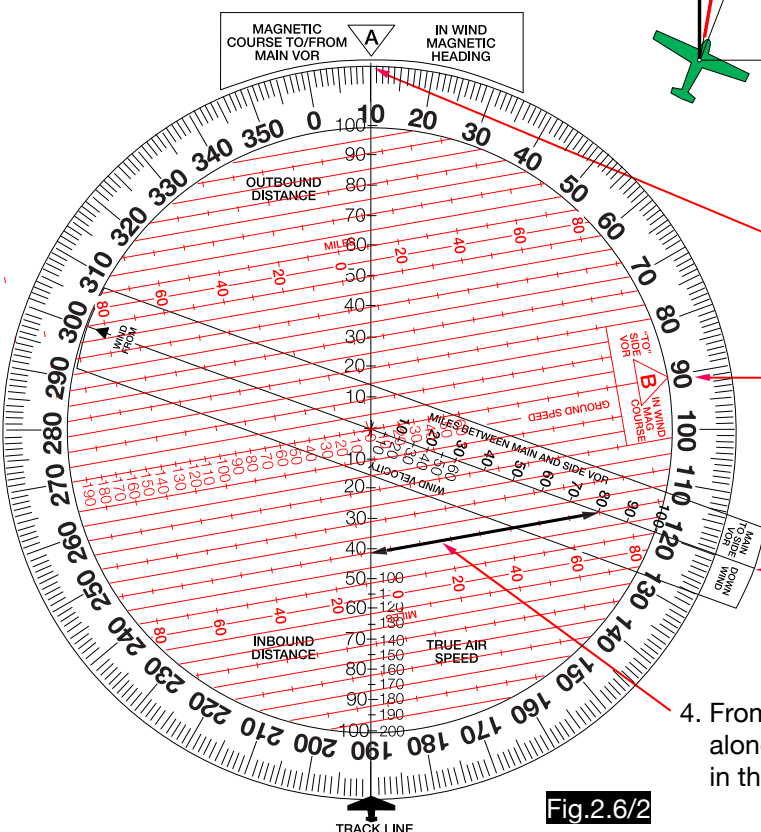
## 2.6. Radio navigation

### 2.6.1. Distance determination with the instrument "DME plus"

Fig.2.6/1 shows how distances are determined with the help of a bearing. The calculation of the distance D from the position to VOR 1 with the instrument "DME plus" is shown in Fig.2.6/2.



**Fig.2.6/1**



**Fig.2.6/2**

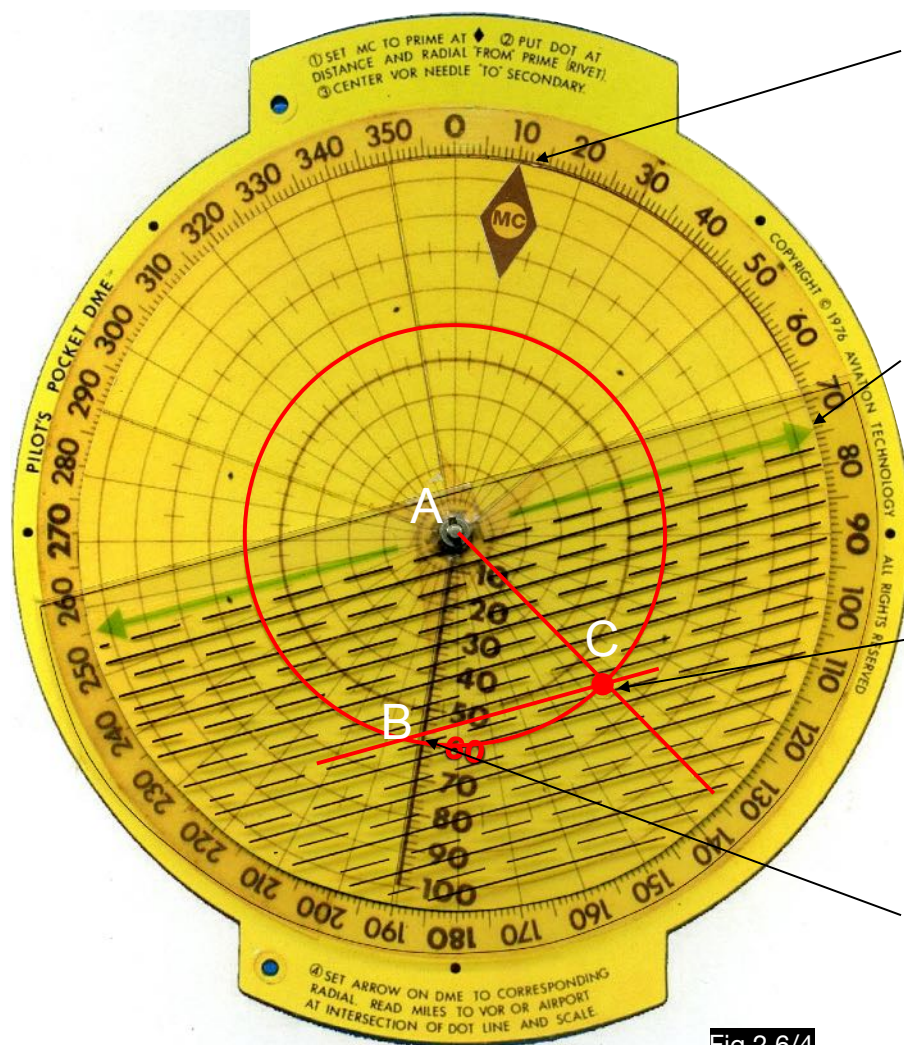
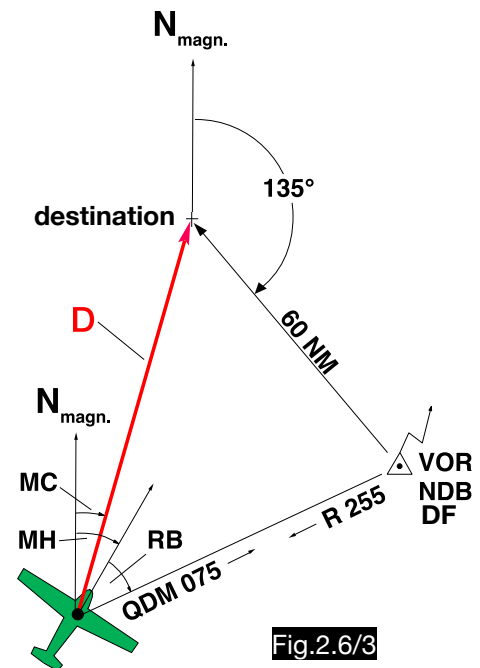
1. The magnetic course MC = 010° is set opposite the mark A.
2. The mark B is set to the magnetic bearing to (QDM):  $MB_{to} = RB + MH (70^\circ + 20^\circ = 90^\circ)$ .
3. The pointer is set to R 120°.
4. From the value 80 km (distance VOR 1 - VOR 2) along the red lines the distance D = 41 km is found in the vertical scale.

## 2.6.2. Distance determination using the instrument „PILOT'S POCKET DME“

Example (see Fig.2.6/3):

magnetic course	010°
magnetic heading	020°
relative bearing	055°
radial	255°
distance destination - beacon	60 NM
bearing destination - beacon	135°
distance D	57.3 NM

The instrument „PILOT'S POCKET DME“ (see Fig.2.6/4) reproduces the situation. The marked point C corresponds to the location of the beacon, the center A to the destination and point B to the aircraft's position.



- 1 The MC mark is set to magnetic course  $MC = 10^\circ$ .
- 2 The green arrow is set to  $MB_{to} (QDM) = MH + RB = 20^\circ + 55^\circ = 75^\circ$  or to radial  $255^\circ$  (VOR).
- 3 A point is marked on the distance ring 60 NM in the direction of  $135^\circ$ .
- 4 A line through the marked point parallel to the dotted lines intersects the distance scale at  $D = 57$  NM.

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The same setting of the instrument is used to explain the course determination for a standard procedure turn (see Fig.2.6/7). After setting the landing direction 300° (1), the outbound course of 120° is read off at 5. Turning to 90° (readable at 6) initiates the procedure turn (120° - 30° = 90°). After one minute flight with heading 090° a right turn is necessary to the intercept heading 255° (read at 7). This crosses the approach course at an angle of 45° (300° - 45° = 255°).

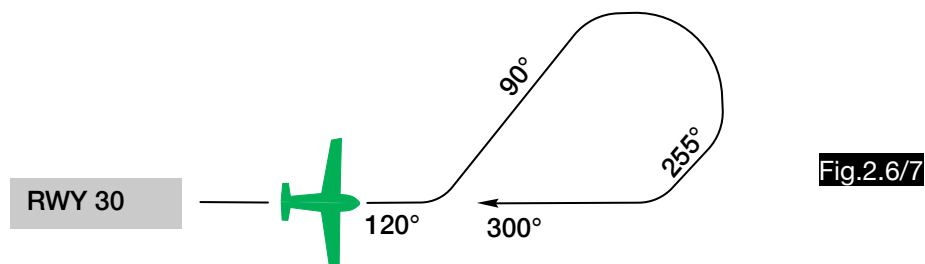


Fig.2.6/7

## 2.6.4. Holding pattern

Form and procedure of standard holding patterns are mandatory. Compliance with the prescribed entry procedures is made easier by technical aids (Fig.2.6/8 shows an example, the handling of which is not described in detail here). Deviations from the flight path due to the wind drift are to be avoided by corrections of the headings and the flight times. It is common practice to use the double wind correction angle of the inbound leg in the outbound leg and to shorten or lengthen the outbound time accordingly. Since the bank angle is also specified, a wind correction is not possible in the curves. Fig. 2.6/9 shows how the shape of the standardised route changes if the wind is corrected in this way. The BAC HOLDING PATTERN COMPUTER is a tool for more precise calculations of outbound heading and time. The device can be used on both sides (for heights below and above 14,000 ft). The following example explains the handling (see Fig. 2.6/10):

altitude	> 14,000 ft
indicated airspeed	280 kt
inbound track	360°
true wind	045° / 60 kt
bank angle	25°

After setting the inbound track to 360° and the wind direction to 45°, the red drift triangle is simulated in the computer. The lower, blue congruent triangle is used to determine the wind correction and the time for the outbound leg. The circular arcs are lines of equal ground speed; however, they are labeled with times in seconds which are directly proportional to the ground speed.

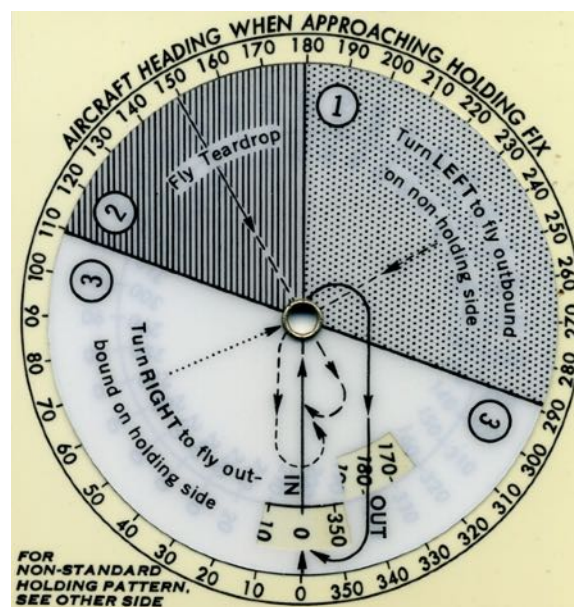


Fig.2.6/8

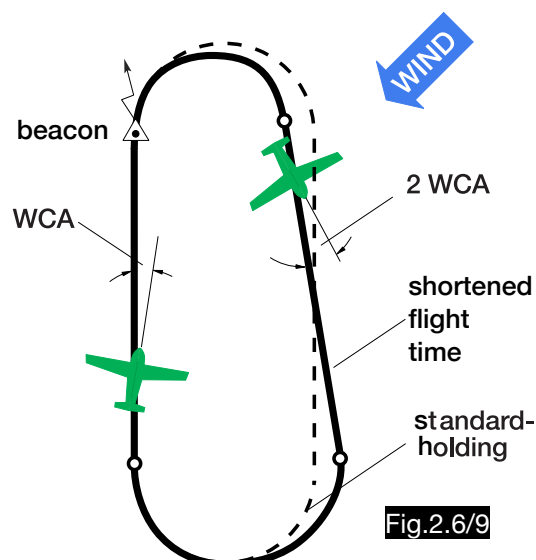


Fig.2.6/9



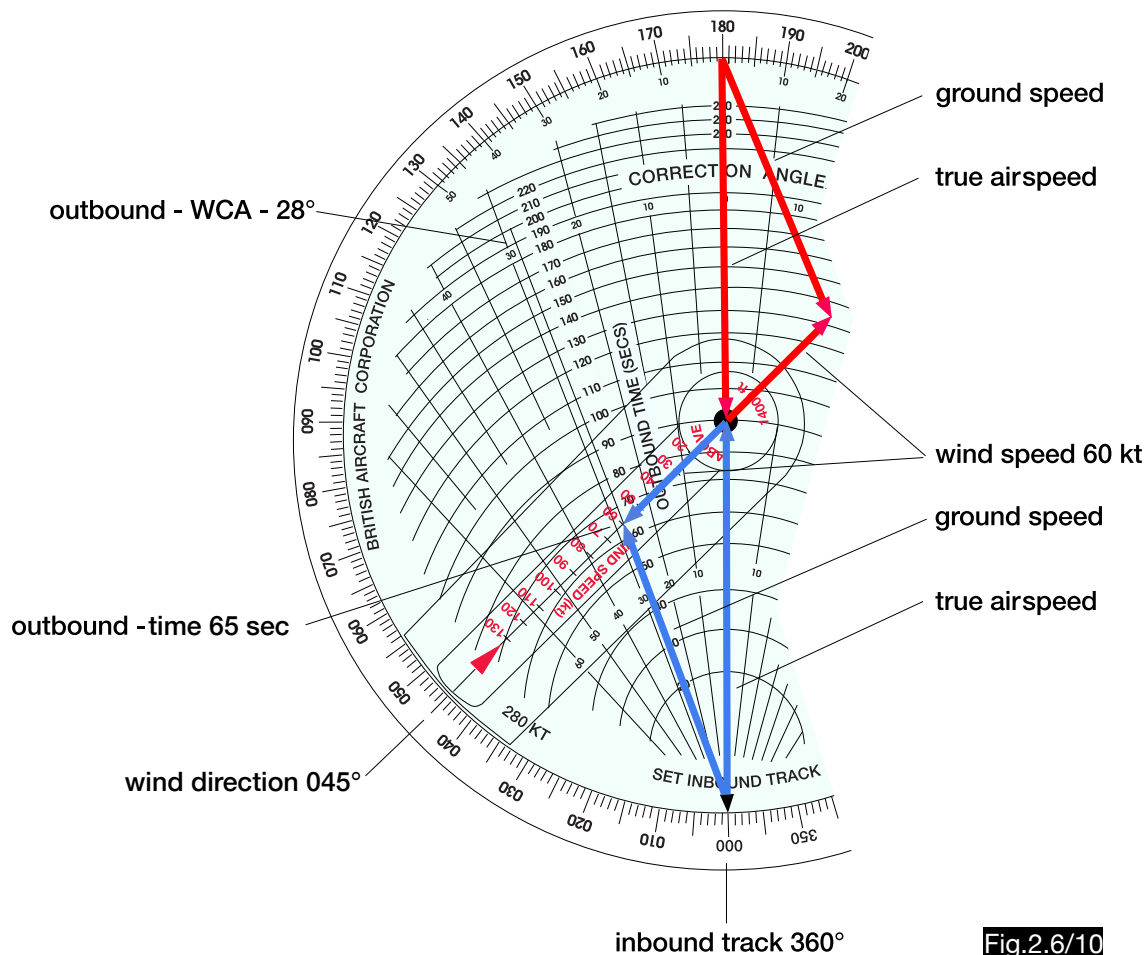


Fig.2.6/10

The accuracy of the outbound heading and the outbound time calculated using the BAC-HOLDING PATTERN COMPUTER is shown in Fig.2.6/11 using the example given here. Under these conditions, a wind correction angle of  $+7^\circ$  is determined in the inbound leg. With the help of the HOLDING PATTERN COMPUTER one will find a wind correction angle of  $-28^\circ$  and a time of 65 seconds in the outbound leg (see Fig.2.6/10). These values result in a flight path which is shown to scale in red in Fig.2.6/11. The black dashed line shows the correct procedure in still air conditions. The calculator generates a correct result, because the displacement of 1117 m at the end of the inbound curve is negligible. It is noteworthy that the fourfold amount of the approach wind correction angle in the outbound leg is somewhat too small in the higher holding patterns.

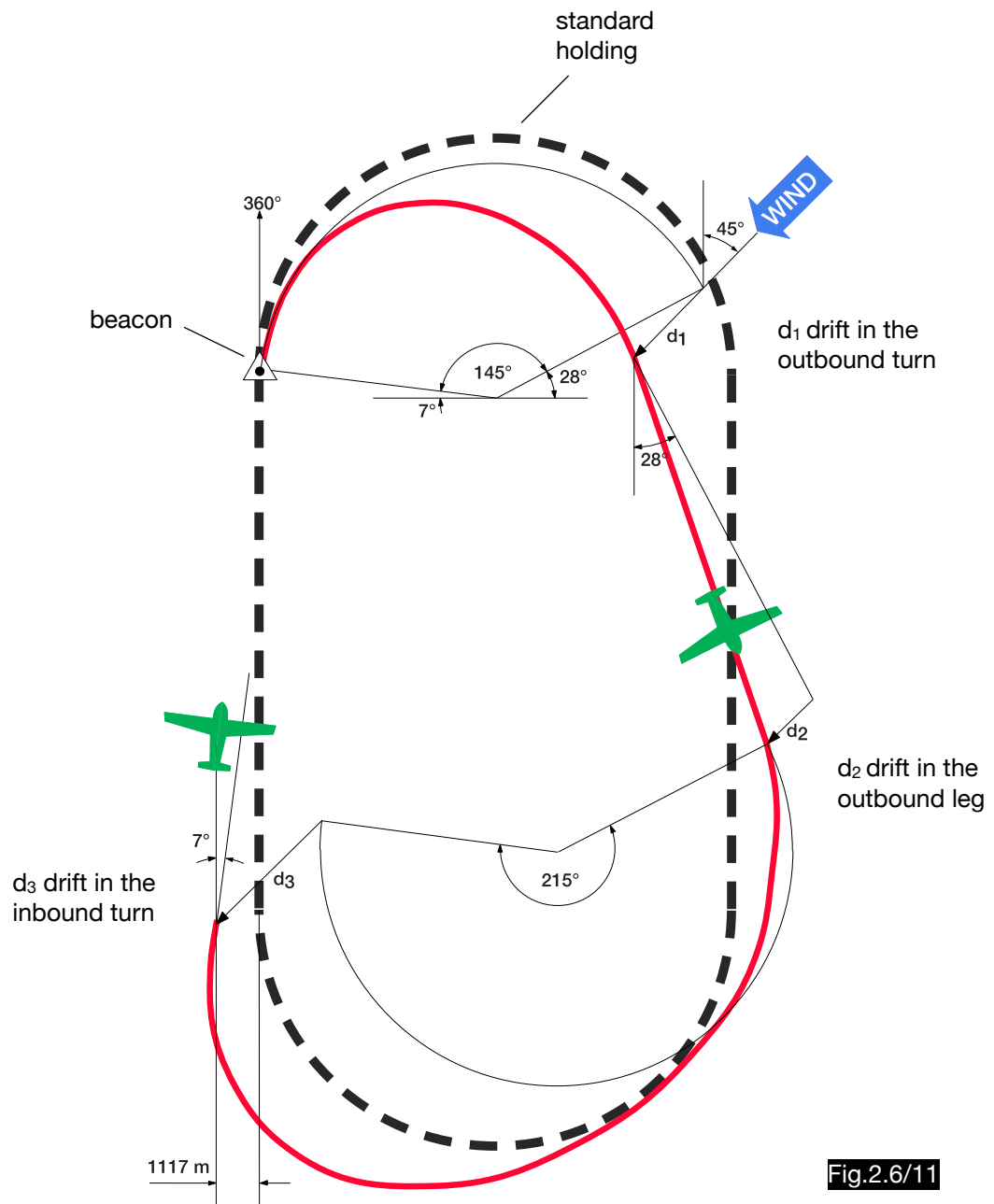


Fig.2.6/11

The calculation of the track (red line) is based on the following parameters:

- $V_{IAS} = 280$  kt
- bank angle =  $25^\circ$
- $H = 16,000$  ft
- ISA
- $V_{TAS} = 353.2$  kt =  $181.7$  m/s
- start heading (over the beacon) =  $007^\circ$
- outbound heading =  $180^\circ - 28^\circ = 152^\circ$
- outbound time = 65 sec

## 2.7. Interception

### 2.7.1. Simmonds-Goudime four vector calculator

The four vector calculator described under 1.8.3. is used to calculate courses for the interception of a moving target (e.g. ship).

The handling of the computer is described in this example:

true course ship	315°
speed ship	30 kt
true bearing (aircraft - ship)	360°
true airspeed aircraft	100 kt
true wind	225° / 40 kt
true intercept heading	330.3°
relative approach speed	90.0 kt

Fig. 2.7/1 shows the three vectors ship speed (CB), wind speed (EC) and aircraft speed (AE). The fourth resulting vector AB is to be found, which stands for the relative speed of the approach, from which the flight time can be calculated from take off to the meeting, provided that the distance between aircraft and ship is known at the beginning of the action. In addition the angle  $\alpha$  is searched, from which the heading of the airplane can be determined. The hatched triangle A'BC consists only of the three vectors A'C (TAS), CB (ship movement) and A'B (relative or approach speed) and applies to the special case of calm. It should be noted that the CB vector of the ship's speed is drawn in opposite to the direction of the ship's movement, since it affects the course of the aircraft in a different way than the wind. Against the wind, which comes from the left, must be corrected to the left. The airplane, which is about to meet a ship moving from east to west (i.e. coming from the right) with a northern course, must also correct to the left. The arm SHIP SPEED 8 in the computer is therefore mounted against the direction of the attached setting element 7 (Fig.1.8/7 and 8). The diagonals of the two parallelograms AFCE and ABCD intersect at point M. For reasons of symmetry, the point M divides the diagonals in the midpoint. The construction of the computer is based on this law. Another graph of the vector diagram shows Fig.2.7/2. This can be transformed once more, as all four vectors are arranged at corners of a square (Fig.2.7/3 schematically shows this computer setting). In both images, the law remains that the intersection point M is halving the diagonals. Fig.2.7/4 shows the calculator with the indicated positions of the speed arms and the rack pairs inside (dashed lines). Since the circular marks on the middle parts are always in the middle between their attachment points, the correct end position of all vectors can be found by overlaying these marks. Of the eight values described by four vectors (direction and amount), two can be calculated with this computer if the other six are known.

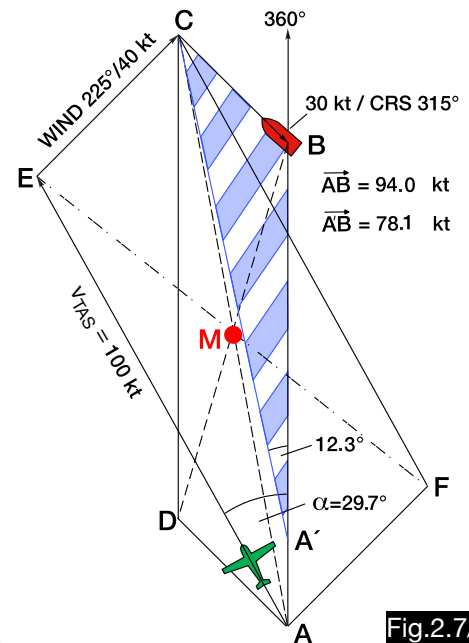


Fig.2.7/1

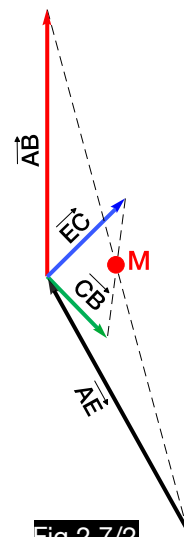


Fig.2.7/2

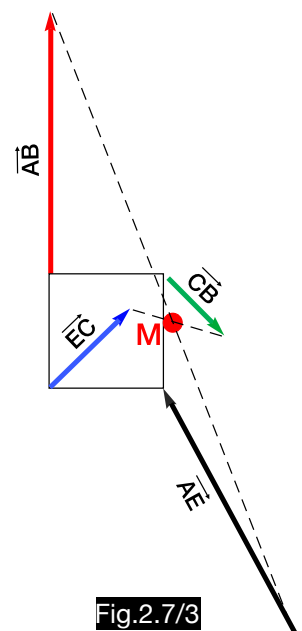


Fig.2.7/3

The two unknowns of this example are determined as follows (see Fig.2.7/4):

1.  $360^\circ$  are adjusted on the bearing element and locked (1).
2. The SHIP SPEED element is set to  $315^\circ$  (2) and 30 kt (3) and both are locked.
3. The airspeed element is set to 100 kt (4) and locked.
4. The wind element is adjusted to  $225^\circ$  (5) and 40 kt (6) and both are locked.
5. At the airspeed element only the speed is fixed and at the bearing element only the direction. The direction of the airspeed (7) and the arm of the relative speed (9) must now be brought into a position where the two marks (8) are exactly above each other in the window. Then the heading of  $330^\circ$  can be read off (7). The relative speed of the approach of 94 kt can be read from the bearing element (9).

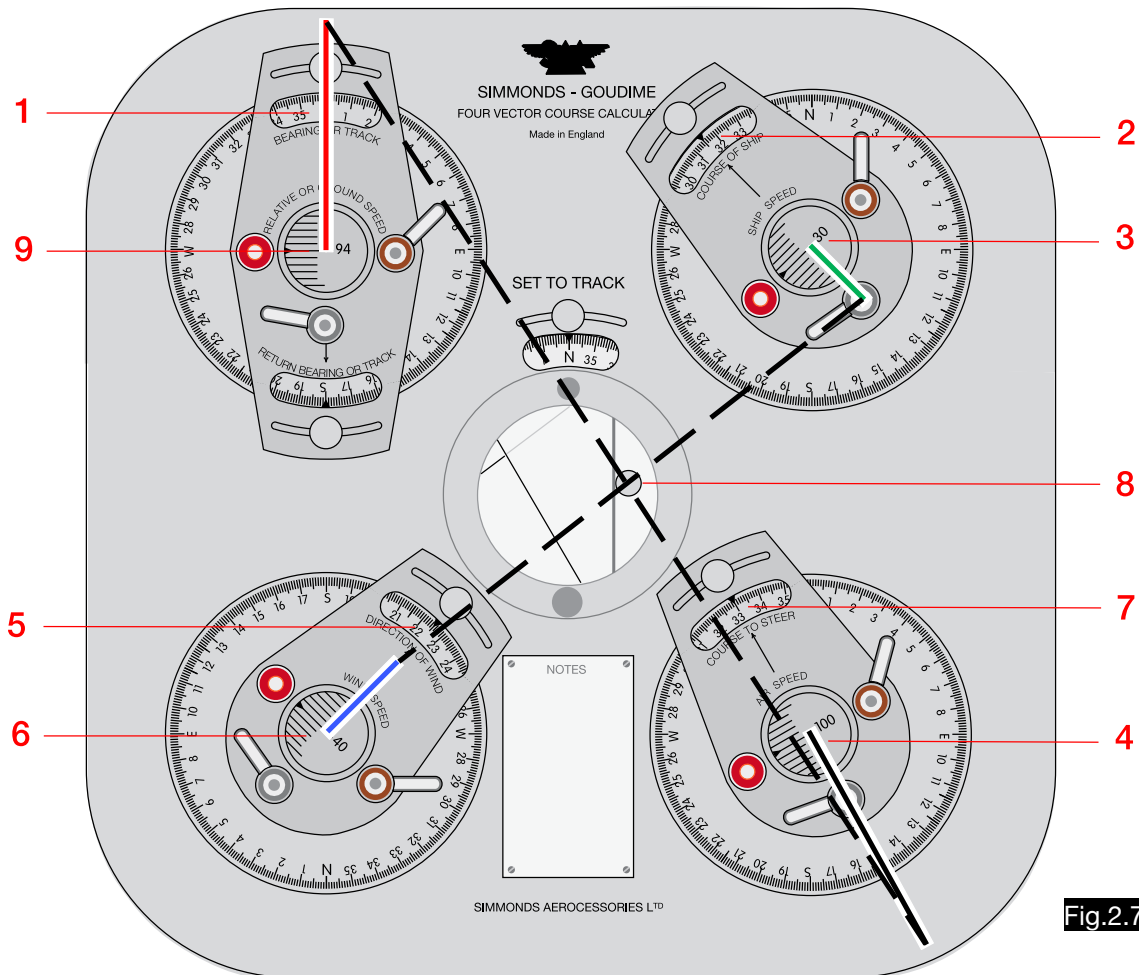


Fig.2.7/4

Fig.2.7/5 schematically illustrates the special situation of calm. After setting a wind speed of 0 kt the heading of  $348^\circ$  and the relative speed of 78 kt are found in the same way. Other special cases in which the SHIP SPEED element does not occur are dealt with under 1.8.3.

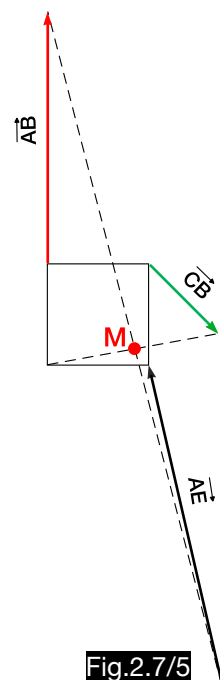


Fig.2.7/5



## 2.7.2. Dalton computer VC-2

The following example describes how the intercept heading to a moving target (ship) can be calculated using the VC-2 computer:

true bearing (aircraft - ship)	360°
true course ship	045°
true wind	315°/25 kt
ship speed $v_{SHIP}$	10 kt
aircraft speed $v_{TAS}$	100 kt
true intercept heading	353.9°

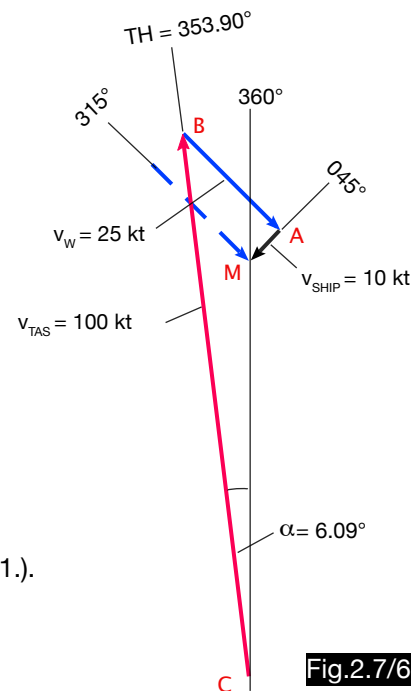


Fig.2.7/6

Note that in Fig.2.7/6 the direction of the vector of the ship's speed is plotted against the direction of movement of the ship (see chapter 2.7.1.).

Solution:

1. The upper disc is aligned to 360° (bearing aircraft - ship).
2. The vector ship speed (point A) is marked on the upper disk with the help of the grid (direction 45° / length 10 kt).
3. The vector wind speed (point B) is entered on the upper disk with the aid of the grid so that it begins at point A, runs in the direction of 315° and has a length of 25 kt (see Fig.2.7/7). The length is determined by counting on the grid.
4. The middle disc is adjusted so that the grid lines run through point B and a point C on the centre line, which has a distance from point B corresponding to 100 kt ( $v_{TAS}$ ).
5. The green course mark indicates the intercept heading 354° (Fig.2.7/8).

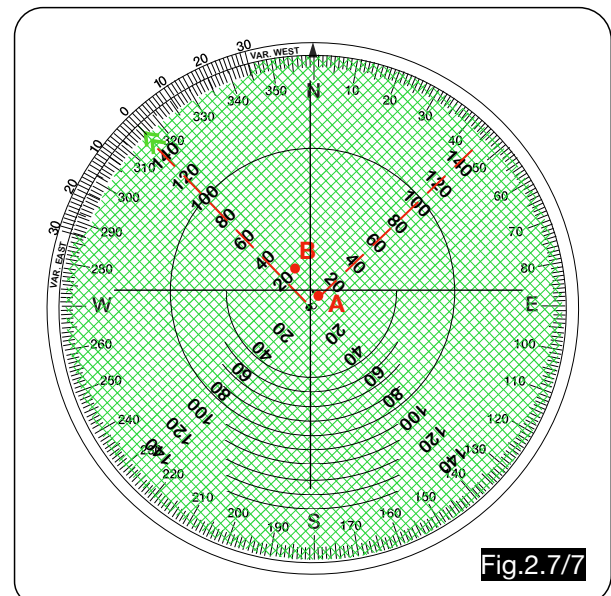


Fig.2.7/7

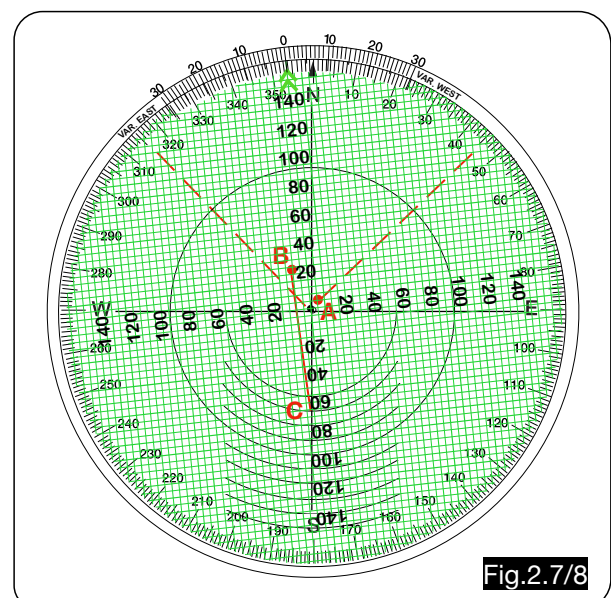


Fig.2.7/8

## 2.8. Descent

The slide rule NL-10 U has an additional scale for the calculation of distance and time of descent from the height to be left and the average values of ground speed and rate of descent. If, for example, one have to descend from an altitude of 9000 m with a vertical speed of 6 m/s, it takes 25 minutes with an average ground speed of 480 km/h and a distance S of 200 km:

$$t = \frac{9000 \text{ m}}{6 \frac{\text{m}}{\text{s}} \times 60 \frac{\text{s}}{\text{min}}} = \frac{9000}{6} 0.0166 = 25 \text{ min}$$

$$S = t v_G = \frac{25}{60} \text{ h} \times 480 \frac{\text{km}}{\text{h}} = \frac{25}{60} \times 480 = 200 \text{ km}$$

Fig.2.8/1 schematically shows the descent scale of the computer NL-10 U and its setting for this example.

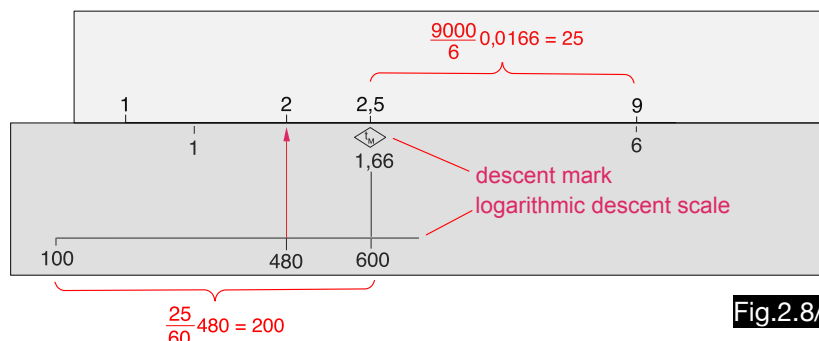


Fig.2.8/1

The slide rule NL-10 MK has an additional scale for the calculation of the rate of descent from the height to be left, the distance and the average ground speed. The following applies to the example used here:

$$v_v = \frac{\Delta H}{t} = \frac{\Delta H}{\frac{S}{v_{GA}}} = \frac{9 \text{ km}}{200 \text{ km}} \times \frac{480 \text{ m}}{3.6 \text{ s}} = \frac{9 \times 480 \times 0.277 \text{ m}}{200} = 6 \frac{\text{m}}{\text{s}}$$

$\Delta H$  height to be left  
 $v_v$  vertical speed (rate of descent)  
 $S$  distance  
 $v_{GA}$  average ground speed during descent

Fig.2.8/2 schematically shows the descent scale of the computer NL-10 MK and its setting for this example.

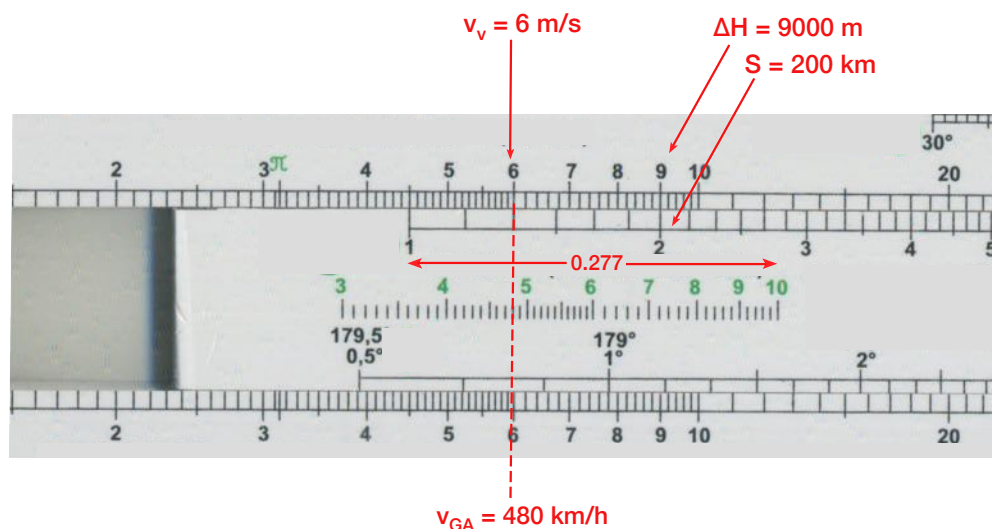


Fig.2.8/2

## 2.9. Dead reckoning

### 2.9.1. Plotting board

Fig.2.9/1 shows a plotting board for position finding by dead reckoning. Two disks are located above a base plate, the lower one is provided on both sides with a rectangular grid of different scales. The upper transparent disk with a compass rose is rotatably arranged above it. The handling is described in an example (see Table 2.9/1), where the location is searched after the flight on three different courses.

	leg		
	A - B	B - C	C - D
magn. heading	045°	360°	320°
distance [NM]	100	50	150
time [h]	0.5	0.25	0.75
true wind	030°/ 50 kt		
true airspeed	200 kt		
variation	10° W		

Table 2.9/1

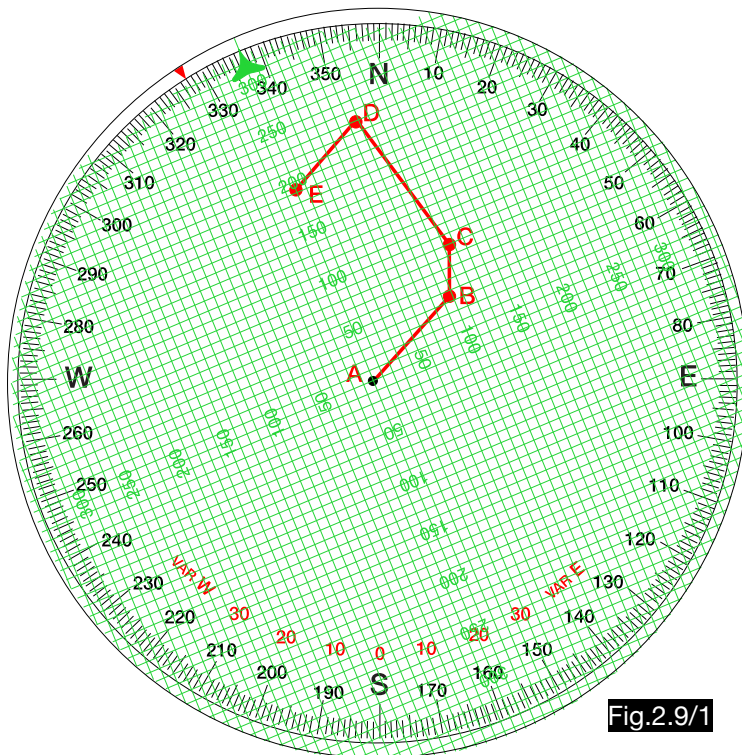


Fig.2.9/1

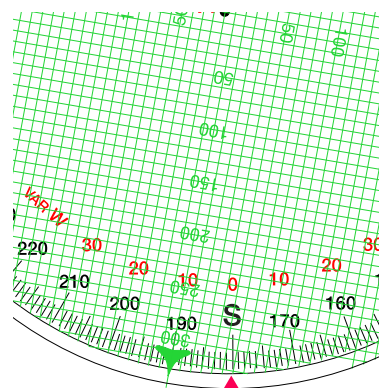


Fig.2.9/2

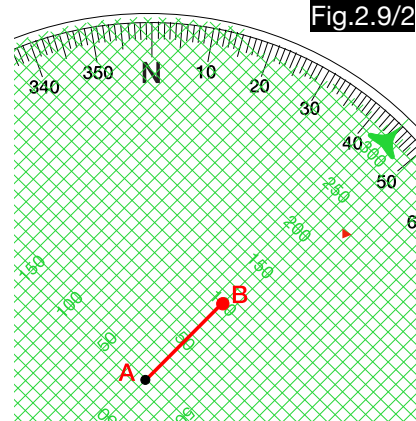


Fig.2.9/3

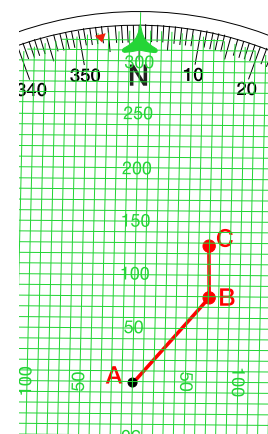


Fig.2.9/4

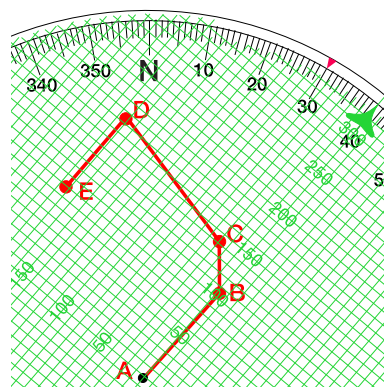


Fig.2.9/5

#### Solution:

1. The green arrow of the grid is set to the variation  $10^\circ \text{ W}$  and an arrow is drawn with a pencil as a "true course mark" on the white border of the grid disc (shown as a red triangle in Fig.2.9/2).
2. The green arrow of the grid is set to the first magnetic heading  $045^\circ$  and the distance of the first leg (100 NM corresponds to 10 divisions) must be marked with a pencil on the upper disc (red point B in Fig.2.9/3).
3. The green arrow of the grid is set to the second magnetic heading  $360^\circ$  and the length of the second leg (50 NM corresponds to 5 divisions) is marked upwards from point B (red point C in Fig.2.9/4).
4. The green arrow of the grid is placed on the third magnetic heading  $320^\circ$  and the length of the third leg (150 NM corresponds to 15 divisions) is marked from point C in the direction of  $320^\circ$  and one wins point D.
5. The arrow marker on the grid, which represents true north (red triangle), is set to the wind direction  $030^\circ$  and the length of the wind path (1.5 h multiplied by  $50 \text{ NM/h} = 75 \text{ NM}$ , corresponding to 7.5 divisions) is marked on the upper disk from point D in wind direction and results in point E (see Fig.2.9/5).
5. Then distance and true course from starting point A to end position E can be determined in order to draw the location on the map. The necessary setting of the computer is shown in Fig.2.9/1. If the central axis of the grid is placed under point E, the green arrow points to magnetic course  $337^\circ$  and the red arrow points to true course  $327^\circ$  (exactly  $327.5^\circ$ ). The distance between the start point A and the end E of the route is 195 NM (exactly 192.9 NM).

#### Note:

If the wind is not known, but the location, the wind path and thus wind direction and velocity can be found in the same way.

2.9.2. Plotter Ko5

The plotter Ko5 (see Fig.2.9/6) is used to draw a flight path on a map of scale 1: 500,000. Its additional use as a triangle calculator is described under 1.9.3. The plotter has the mileage scale 1, the velocity field 2, the pointer 3 and in the zero point of the mileage scale a movable compass point 4, which makes it possible to use the plotter like a drawing triangle and at the same time like a circle. The slot 5 in the center line 6 is used for drawing in course markers. The handling is described using a two-leg route (see Table 2.9/2 and Fig. 2.9/7):

	leg	
	A - B	B - C
true heading	035°	300°
time [min]	10	15
true airspeed	100 kt	

Table 2.9/2

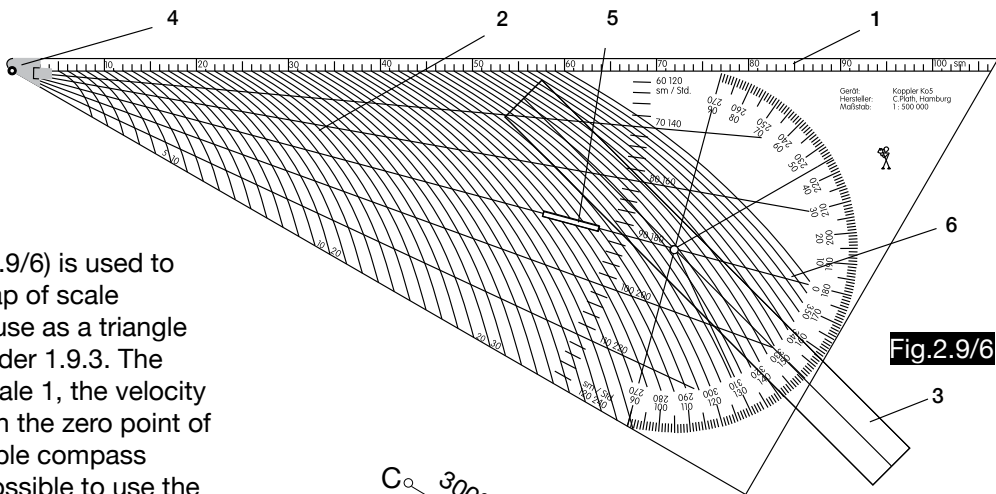


Fig.2.9/6

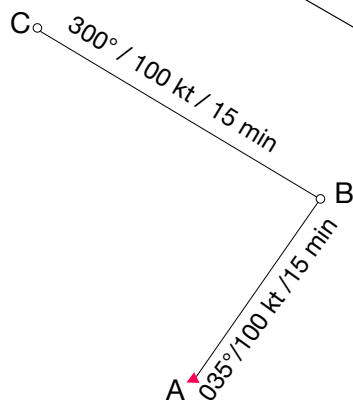


Fig.2.9/7

Solution:

1. In order to draw the first leg on the map, the pointer must be set to the first heading (035°), the center line of the plotter must be placed on the starting point of the leg (red triangle) and the pointer must be aligned parallel to a meridian. Then one draws a mark through the slot into the map (shown red in Fig. 2.9/8). Then the line of the first leg can be drawn in the map (connecting line of the start point A with the course mark).

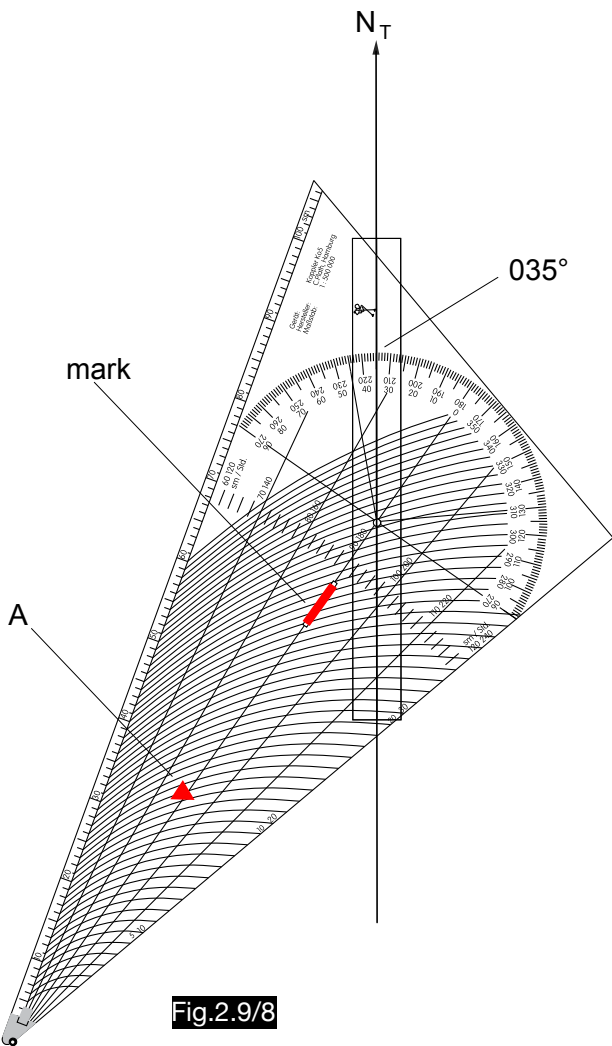
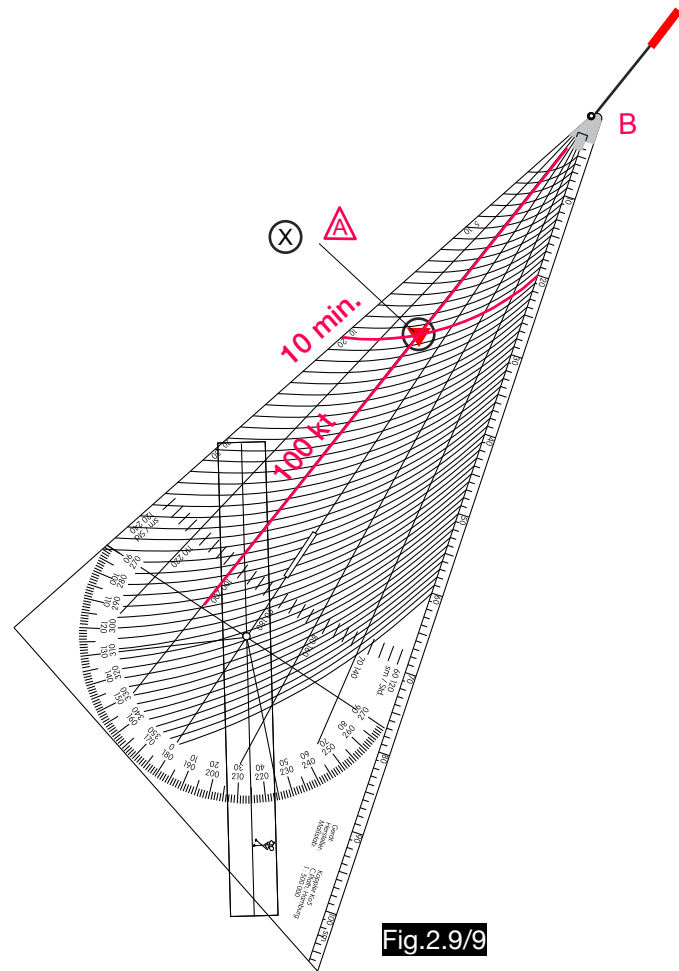


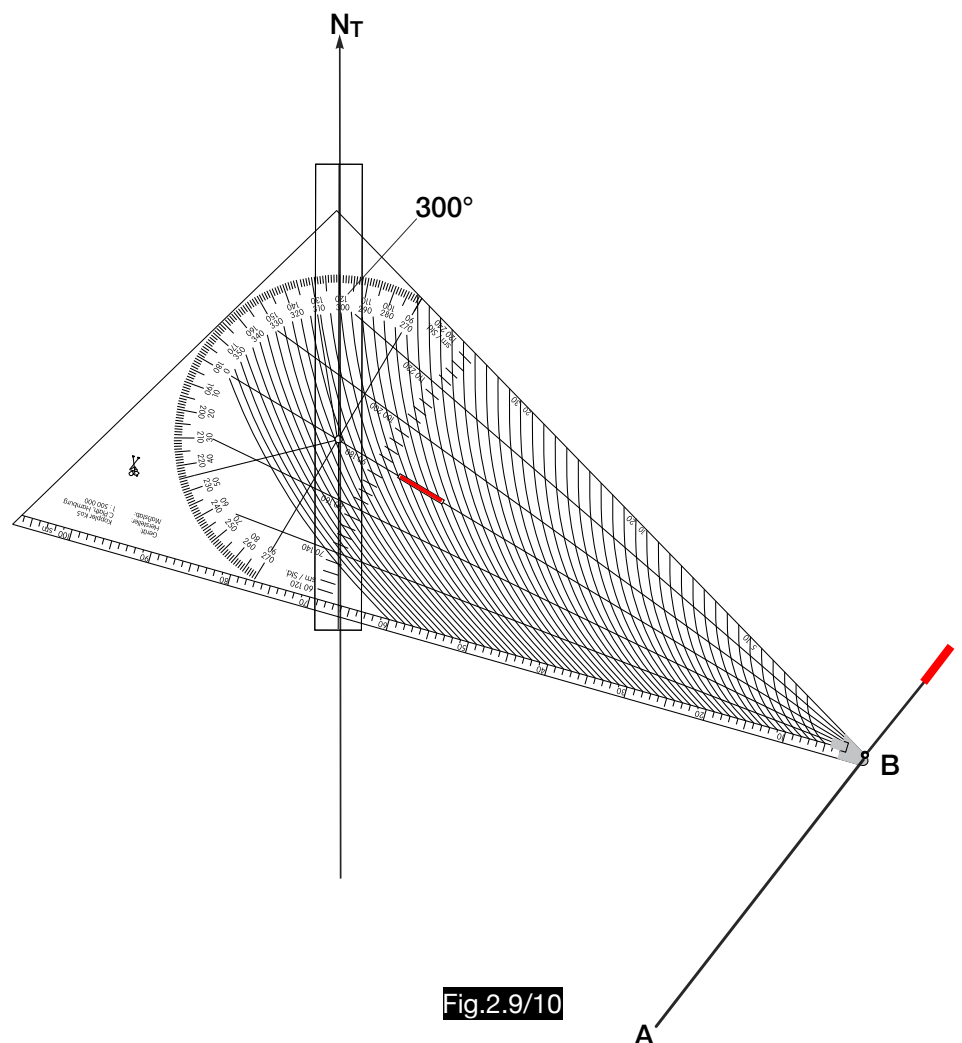
Fig.2.9/8



2. In the velocity field the intersection of the air-speed line 100 kt with the 10 min - line is marked (point X in Fig. 2.9/9). The length from mark X to the compass point corresponds (in a map from scale 1: 500,000) to the distance of the first leg. The plotter is then placed on the map in such a way that point X is on the starting point A (red triangle) and the line 100 kt is on the drawn route. The compass point then marks the route point B. The plotter can now be rotated around point B to draw the second leg and to find point C.



3. After setting the pointer to 300° (course of the second leg), the plotter is rotated around the compass point (route point B) until the center line of the pointer runs parallel to a meridian. Then one draws a line through the slot of the plotter into the map (shown red in Fig. 2.9/10). This marks the direction of the second leg. Its length, i.e. the position of the point C, is again determined by the intersection of the 15-minute line with the air speed line 100 kt.



## 2.10. Rhumb line calculations according to Clemons and Nelson

### 2.10.1. General

The computer A-5 is used to calculate the length of the rhumb line and the loxodrome course. Fig. 2.10/1 shows the abbreviations used in the calculator and the data of an example.

Explanation of symbols:

$\vartheta$	rhumb line (loxodrome) course
ML	mid.latitude
DL	difference of latitude
DLo	difference of longitude
A $\leftrightarrow$ B	rhumb line distance

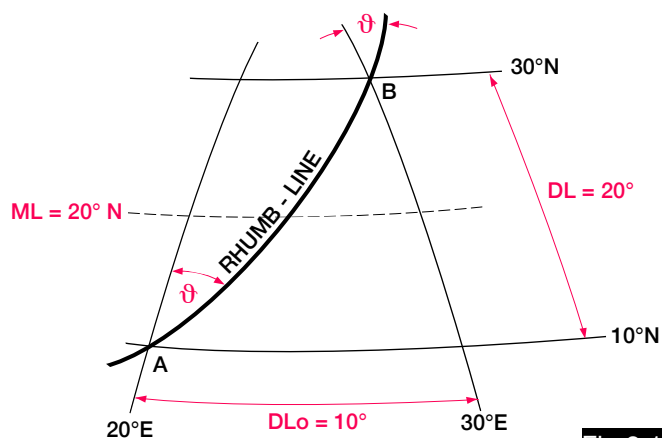
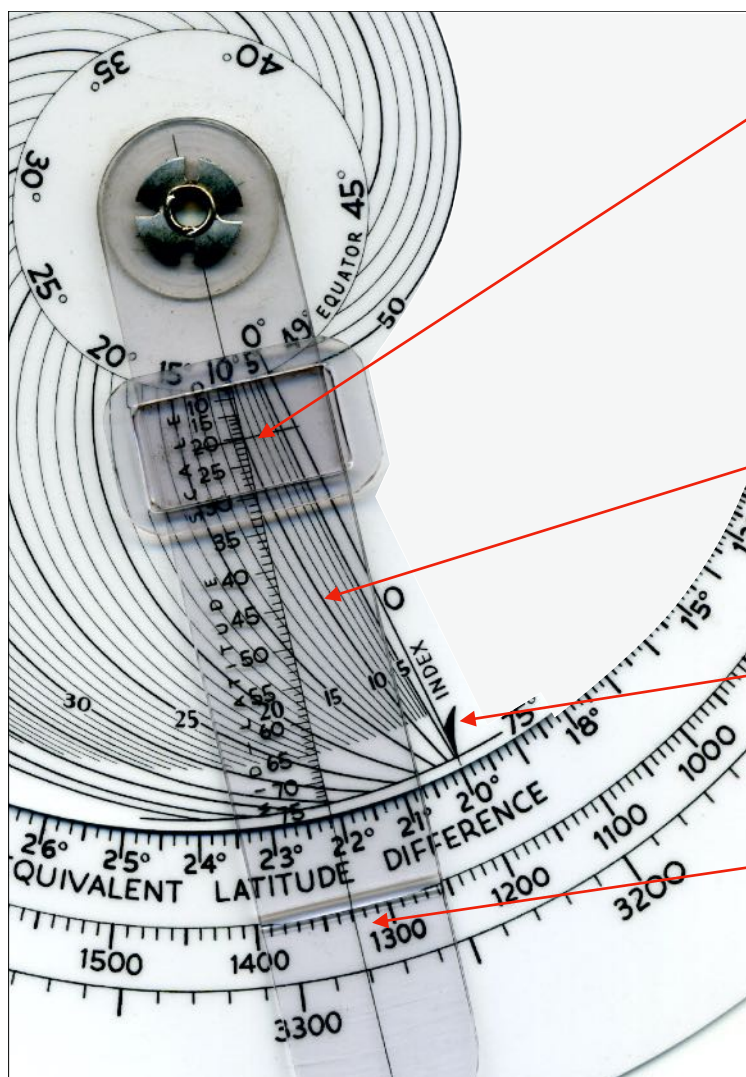


Fig.2.10/1

### 2.10.2. Calculation of the length of the rhumb line



The cursor of the pointer is set to the mid latitude  $ML = 20^\circ$ .

The intersection point of the mid latitude  $ML = 20^\circ$  with the center line of the pointer is set on the line of the length difference  $DLo = 10^\circ$ .

The index arrow is set to the difference of latitude  $DL = 20^\circ$ .

The center line of the pointer indicates on the inner scale the loxodrome distance  $L = 1330$  NM (see note 1).

Fig.2.10/2

#### Note 1:

At the beginning of the calculation, the pointer should be at the distance zero and the INDEX arrow at the difference of latitude DL. Then the pointer is moved clockwise with the cursor set to the mid latitude beyond the index line (longitude difference 0) to the longitude difference line. If the pointer has to be rotated by more than  $360^\circ$ , the outer scale is relevant for the distance indication. In the example calculated here, the pointer from line 0 NM had to be rotated approx.  $90^\circ$  in clockwise direction so that the point ML =  $20^\circ$  on the rotatable pointer line stands above line DLo =  $10^\circ$ . Therefore in this case the distance can be read off the inner scale.

#### Note 2:

The complicated analytical determination of loxodrome distances is avoided in this calculator by the application of a nomogram, which in principle has the form shown in Fig.2.10/3. The shape of the curves leads to the fact that in some cases (e.g. with small latitudinal differences and at the same time large longitudinal differences) intersections at a very small angle occur in the diagram like in the computer, which complicate an exact distance determination. The example calculated here is drawn into the nomogram by red arrows. The calculator A-5 provides a rhumb length between 1325 and 1330 NM. The result can be more accurate by multiplying the differences of longitude and latitude by a factor and dividing the result by this factor.

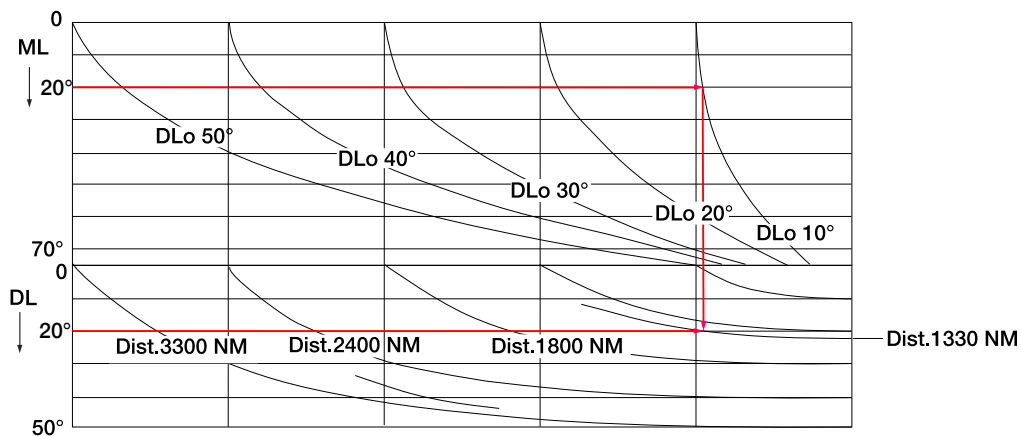


Fig.2.10/3

### 2.10.3. Calculation of the course of the rhumb line

In order to determine the loxodrome course for the example selected here, the latitude difference DL =  $20^\circ$  must be set opposite the rhumb line length 1330 NM (see Fig.2.10/4). Normally, the course angle  $\vartheta$  is now read opposite the index arrow. In this example the arrow is in the sector 0 -  $45^\circ$  (marked by "WARNING"), so that first the "equivalent latitude difference" must be determined on the back of the computer (see Fig.2.10/5).

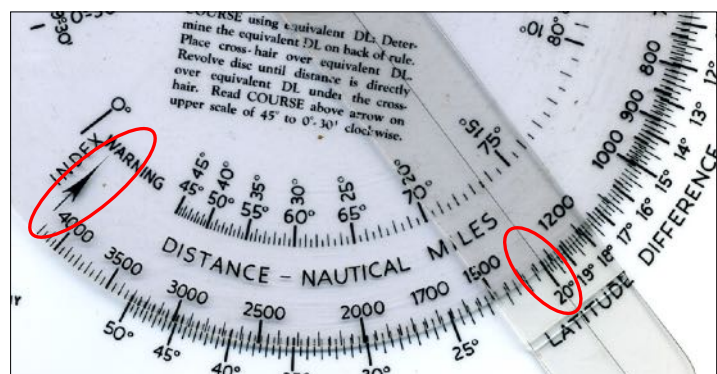


Fig.2.10/4

The equivalent latitude difference is found by setting the INDEX arrow to  $0^\circ$  and the intersection of the mid latitude ML with the pointer line over the line of the longitude difference  $DLo = 10^\circ$ . Then the pointer line will indicate the equivalent latitude difference of  $9.5^\circ$  (see Fig.2.10/5).

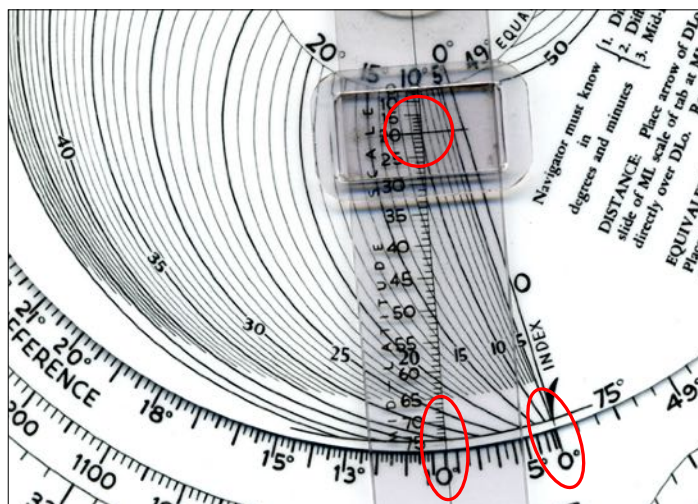


Fig.2.10/5

If the distance of 1330 NM is now set opposite the equivalent latitude difference of  $9.5^\circ$ , the index points on the inner scale to the angle  $\vartheta = 25.5^\circ$  (see Fig.2.10/6). The course of the loxodrome is calculated from the course angle  $\vartheta$  as a function of the quadrant according to Fig.2.10/7. In this example, the rhumb line lies in the northeastern quadrant, where the course angle is equal to the true course. In the north-western quadrant one receives the true course after subtraction of the course angle  $\vartheta$  from  $360^\circ$ . In the south-western quadrant  $180^\circ$  must be added to the course angle. In the southeastern quadrant, the true course is obtained by subtracting the angle  $\vartheta$  from  $180^\circ$ .

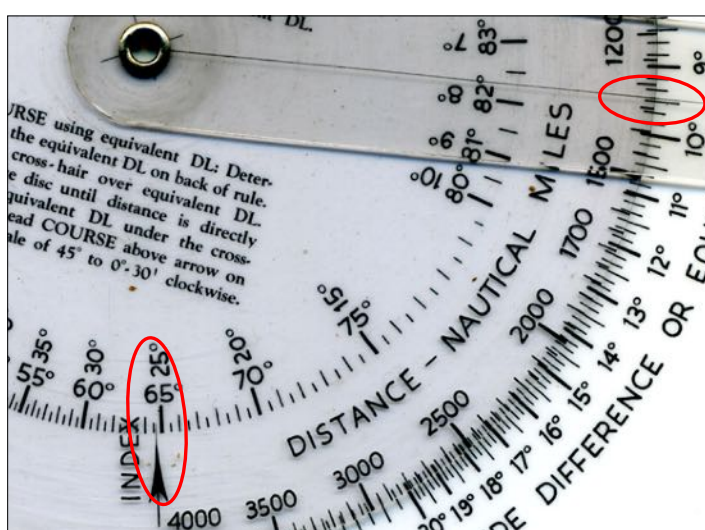


Fig.2.10/6

To simplify the angle calculation, the spherical image is projected into a planar surface. This is possible if the size of the area on the surface of the globe is limited. With the calculator A - 5, loxodrome distances can only be calculated if they lie on a hemisphere (i.e. do not cross the equator), the differences of the longitude and latitude coordinates of the endpoints do not exceed  $50^\circ$  and the mid latitude does not exceed  $75^\circ$ . The calculable area on the globe's surface is therefore limited by great circle arcs of  $50^\circ$  with a length of 3000 NM (one nautical mile corresponds to one angular minute on the great circle:  $50 \times 60 = 3000$ ). The diagonal of such a square therefore has a length of 4242.6 NM ( $3000 \sqrt{2}$ ) and therefore the distance scale of the A-5 computer ends at this value.

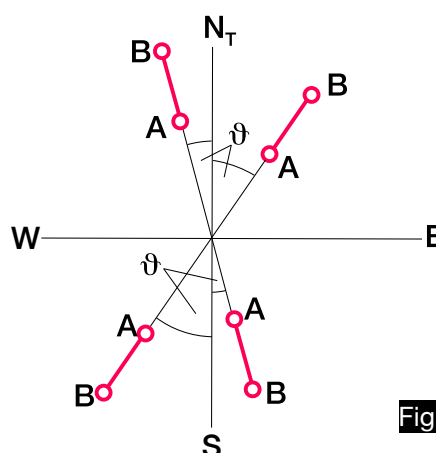


Fig.2.10/7



The calculated rhumb line is located in a spherical triangle, which is projected into a planar triangle for simplification (see Fig. 2.10/8). In this applies to the angle  $\vartheta$ :

$$\cos \vartheta = \frac{\text{arc length of the angle DL (difference of latitude)}}{\text{length of the rhumb line}}$$

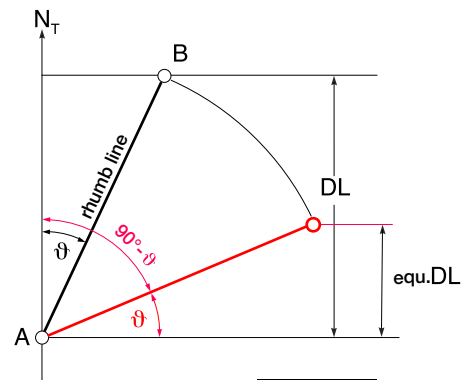


Fig.2.10/8

In the example used here, the length of the rhumb line is 1330 NM. Since the meridians are great circles, the difference of latitude of  $20^\circ$  corresponds to a distance of 1200 NM ( $20 \times 60 = 1200$ ). Thus one would obtain a course angle of:

$$\cos \vartheta = \frac{1200}{1330} = 0.902 \longrightarrow \vartheta = 25.5^\circ$$

This method becomes too inaccurate for angles  $\vartheta \leq 45^\circ$ , which is why in these cases the angle  $90 - \vartheta$  is calculated instead of the angle  $\vartheta$ . This is calculated using the equivalent distance of latitude (equ.DL), which in this example is  $9.5^\circ$  (see Fig.2.10/5). With these  $9.5^\circ$  and the distance 1330 NM one gets  $\vartheta = 25.5^\circ$  (see Fig.2.10/6). In Fig.2.10/9 the logarithmic scales A, B and C of the calculator are shown in a position where the result  $\vartheta = 75^\circ$  is obtained in scale C. These two examples demonstrate that the angles in scale A correspond to the lengths of the great circle arcs and that the course angle is calculated from the division result  $A : B$  using the cosine:

Example 1: LD =  $14^\circ$  ( $\approx 14 \times 60 = 840$  NM)  
distance rhumb line = 3250 NM

$$\cos \vartheta = \frac{840}{3250} = 0.258 \longrightarrow \vartheta = 75.0^\circ$$

Example 2: LD =  $10^\circ$  ( $\approx 10 \times 60 = 600$  NM)  
distance rhumb line = 2350 NM

$$\cos \vartheta = \frac{600}{2350} = 0.255 \longrightarrow \vartheta = 75.2^\circ$$

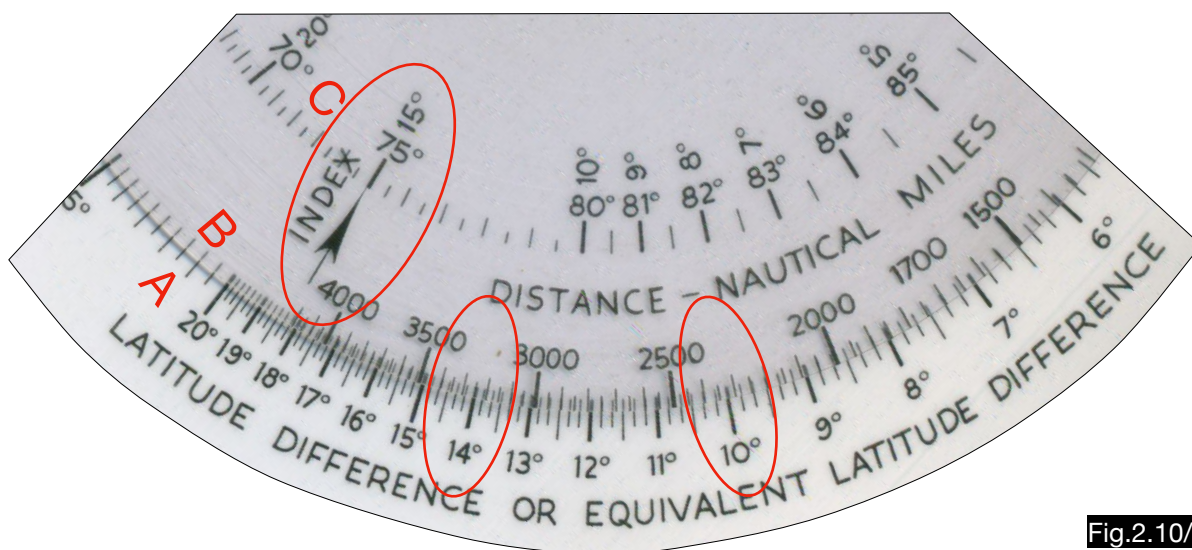


Fig.2.10/9

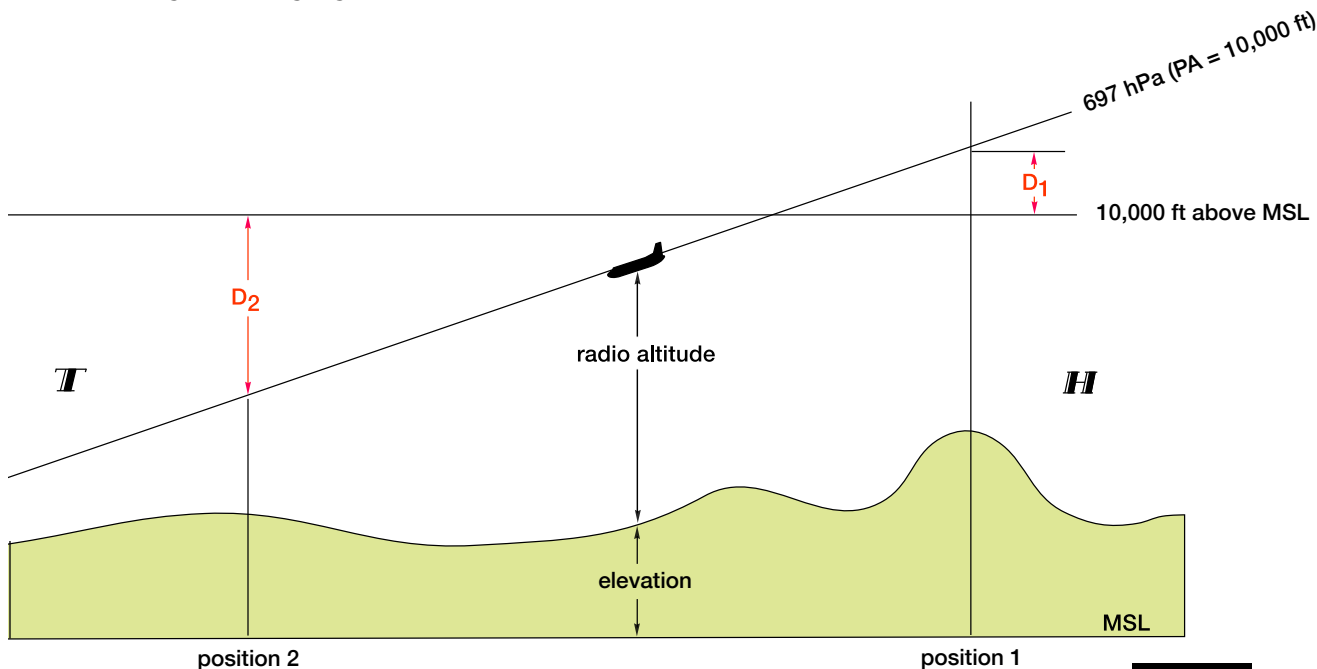
example 1

example 2

## 2.11. Pressure pattern according to Bellamy

### 2.11.1. General

Fig.2.11/1 shows the assumed profile of the pressure level 697 hPa (corresponds in ISA to 10,000 ft) in the vertical section. During the flight in constant pressure altitude of 10,000 ft from an area of high air pressure to an area of lower air pressure, the altitude above main sea level decreases and a drift occurs (on the northern hemisphere to the right). According to Bellamy, the drift speed  $v_n$  is calculated from the difference between true altitude (radio altitude) and pressure altitude. From this one determines the wind correction angle for the so called single heading flight, which is close to the minimum time route.



The Bellamy formula for the drift speed  $v_n$  is:

$$v_n = \frac{21.47 (D_2 - D_1)}{S \sin \varphi} \quad [2.11/1]$$

$v_n$

$D$

21.47

$S$

$\varphi$

$V_{TAS}$

drift speed in kt

difference in feet between true altitude above MSL (radio altitude + elevation) and pressure altitude

constant (the accuracy of this factor is irritating, since the formula can only be an approximation if it fails completely at the equator due to  $\sin 0^\circ = 0$ )

air distance in NM (obtained from the product  $V_{TAS} \times \text{flight time } t$ , which assumes that the wind influence is known)

the average geographical latitude of the route  
true airspeed

### 2.11.2. Calculation of the Bellamy formula with conventional navigation computers

For the calculation of the drift according to the Bellamy formula, various calculators have a special scale for the mean geographical latitude of the flight route, which is marked with LATITUDE FOR PRESSURE PATTERN. As can be seen in Fig.2.11/2, behind the angle of these scales lies the quotient  $21.47 / \sin \varphi$ , as shown by the example  $\varphi = 25^\circ$ :

$$\frac{21.47}{\sin 25^\circ} = \frac{21.47}{0.42} = 50.8$$

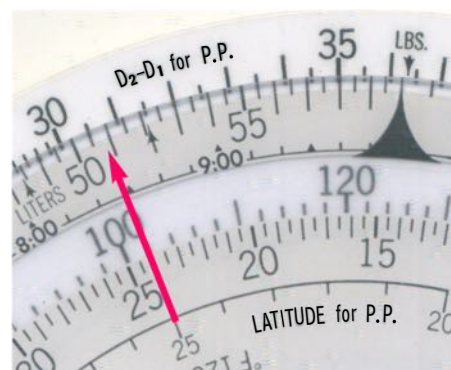


Fig.2.11/2

Fig.2.11/3 shows the settings of a usual navigation calculator for the following example:

D <sub>1</sub>	+10 ft
D <sub>2</sub>	-150 ft
D <sub>2</sub> - D <sub>1</sub>	-140 ft
mid.latitude $\varphi$	30°
VTAS	200 kt
time t	30 min
S	100 NM
drift speed	60.11 kt

$$v_n = \frac{21.47 (-140)}{100 \sin 30^\circ} = -60.11$$

$v_n$	D <sub>2</sub> - D <sub>1</sub>
$\frac{21.47}{\sin \varphi}$	S

The negative sign of the drift speed means a right drift at this procedure.

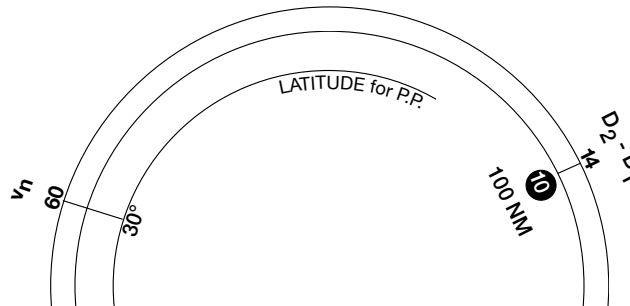


Fig.2.11/3

To calculate the drift angle from the drift speed see ch. 2.4.1.3.

### 2.11.3. Calculation of the Bellamy formula with the computer 1-CH-1

For flights over water in latitudes above 15°, the drift speed and the drift angle can be calculated from the differences D<sub>1</sub> and D<sub>2</sub> (radio altitude minus pressure altitude) using the 1-CH-1 calculator. The following example explains the handling of the computer (see Fig.2.11/4 - 6):

VTAS	200 kt
air distance S	120 NM
mid. latitude $\varphi$	53°N
D <sub>2</sub> - D <sub>1</sub>	- 80 ft

According to the Bellamy formula one gets a drift speed  $v_n = -17.92$  kt:

$$v_n = \frac{21.47 (D_2 - D_1)}{S \sin \varphi} = \frac{21.47 (-80)}{120 \sin 53^\circ} = -17.9 \text{ kt}$$

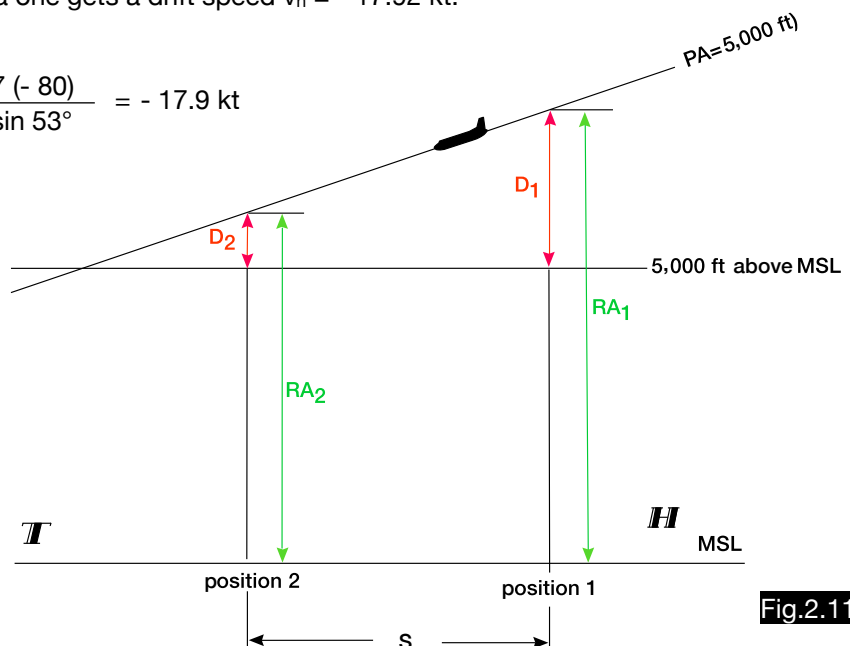


Fig.2.11/4

These settings are required on the logarithmic scales of the computer:

- 1 The difference  $D_2 - D_1 = -80$  ft is set opposite to the air distance 120 NM.

- 2 Opposite to the mid. latitude  $\phi = 53^\circ$  one can read the drift speed  $v_n = 18$  kt on the outer scale.

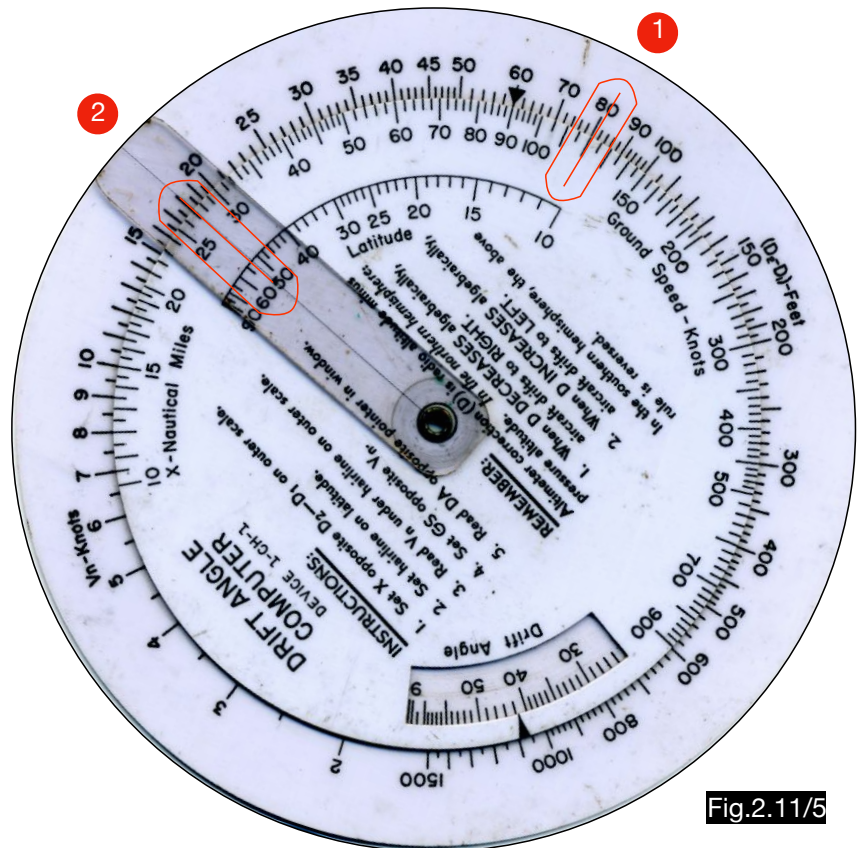


Fig.2.11/5

- 3 The true air speed  $v_{TAS} = 200$  kt (on the inner scale) is set opposite to the drift speed  $v_n = 18$  kt (on the outer scale). This is noted in the printed instructions:

“GS is the estimated ground speed in knots. If unable to estimate GS, use TAS in knots.”

- 4 In the drift angle window, the value  $DA = 5.2^\circ$  is read (the drift angle - scale is a sine scale).

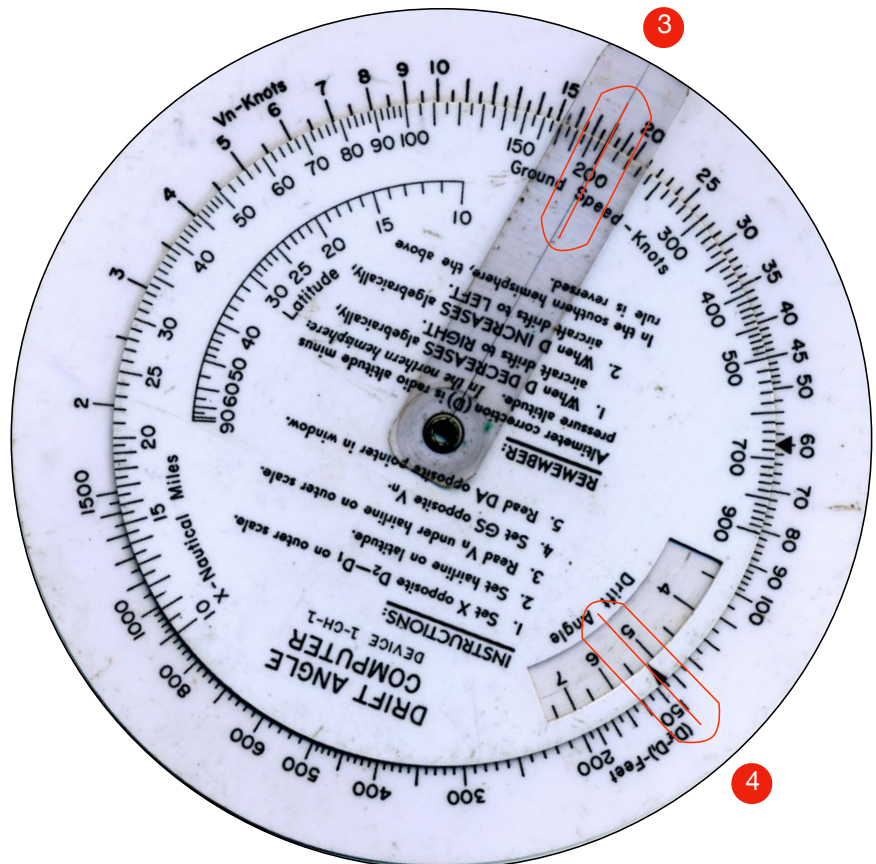


Fig.2.11/6

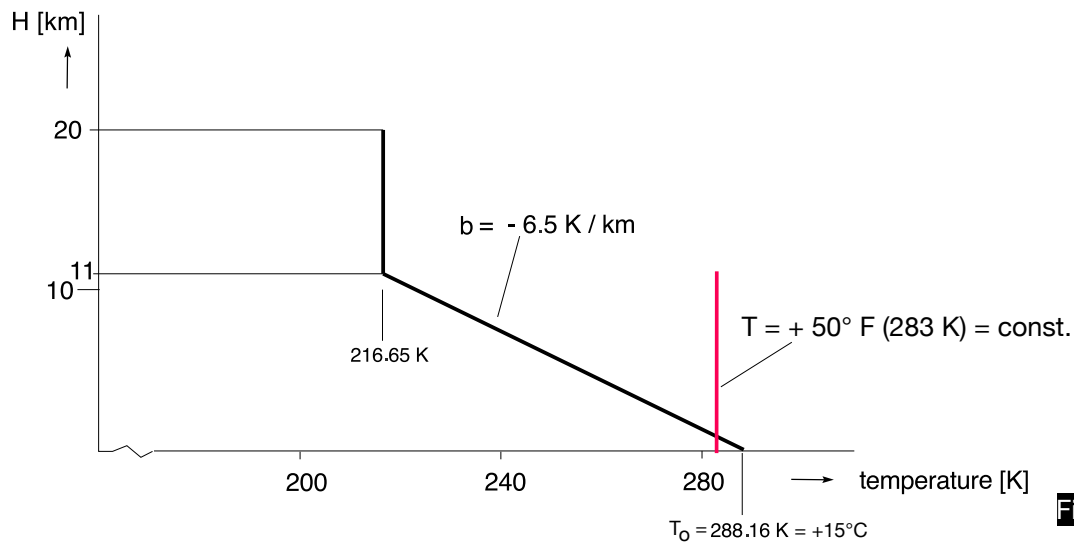


### 3. Altitude and speed calculations

#### 3.1. General

##### 3.1.1. Pressure curve over the height $p = f(H)$

In order for all altimeters to display the same altitude value at a measured air pressure, it was necessary to define the pressure decrease with the altitude as a uniform international value. The curve of the temperature between the altitudes 0 and 80 km in the International Standard Atmosphere (ISA) was uniformly determined (shortened shown as a black line in Fig.3.1/1). A previously used isothermal atmosphere assumes a constant temperature of 50° F over the entire altitude range (red line in Fig.3.1/1).

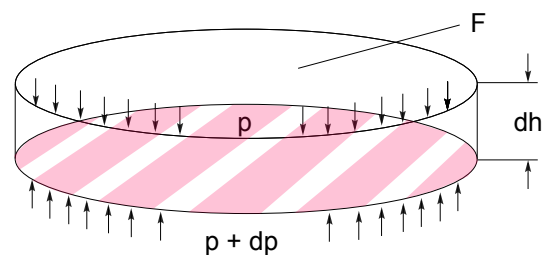


When calculating the pressure curve over the height, both the decrease in temperature and in the acceleration due to gravity  $g$  must be taken into account. To simplify the calculation, the constant acceleration  $g_0 = 9.80665 \text{ m/s}^2$  is used for  $g$  and this atmosphere is called "geopotential". The atmosphere, in which a decreasing acceleration due to gravity is assumed, is called „geometric" (e.g. the counterpart to the geometric height  $Z = 10,000 \text{ m}$  is the geopotential height  $H = 9,984 \text{ m}$ ).

The decrease of pressure in the geopotential atmosphere for  $0 \leq H \leq 11 \text{ km}$  is calculated using the barometric height formula according to Laplace:

$$dp = - \rho \cdot g \cdot dh \quad [3.1/1]$$

At the bottom of the air layer of the height  $dh$  and the area  $F$  (see Fig.3.1/2) the pressure due to the weight  $dm \times g$  is higher by the amount  $dp$  than at the top. From this the equ. 3.1/1 can be derived (the minus sign must be inserted, since the pressure decreases with height):



$$dV = F \cdot dh$$

$$dm = dV \cdot \rho = F \cdot dh \cdot \rho$$

$$dp = \frac{dm \cdot g}{F} = \frac{F \cdot dh \cdot \rho \cdot g}{F}$$

With the equation for air density (derived from the general gas equation according to Boyle and Mariotte and Gay Lussac)

$$\rho = \frac{p}{R T} \quad [3.1/2]$$

one obtains from equ.3.1/1 the law of pressure decrease with altitude for  $0 \leq H \leq 11$  km:

$$\begin{aligned} \frac{dp}{p} &= - \frac{g_0}{R T} dh \longrightarrow \frac{dp}{p} = - \frac{g_0}{R (T_0 + bH)} dh \quad (b = -6,5 \text{ K / km}) \\ \int_{p_0}^{p_H} \frac{dp}{p} &= - \frac{g_0}{R} \int_0^H \frac{dh}{T_0 + bH} \quad \left| \int \frac{dx}{x} = \ln x \right| \quad \left| \int \frac{dx}{a + bx} = \frac{1}{b} \ln(a + bx) \right| \\ \ln p_H - \ln p_0 &= - \frac{g_0}{R} \left\{ \frac{1}{b} \left[ \ln (T_0 + bH) \right]_0^H \right\} \longrightarrow \ln \frac{p_H}{p_0} = - \frac{g_0}{R \cdot b} \left[ \ln (T_0 + bH) - \ln T_0 \right] \\ \ln \frac{p_H}{p_0} &= \ln \left( \frac{T_0 + bH}{T_0} \right)^{-\frac{g_0}{Rb}} \longrightarrow \boxed{\frac{p_H}{p_0} = \left( \frac{T_H}{T_0} \right)^{-\frac{g_0}{Rb}}} \quad [3.1/3] \end{aligned}$$

With  $R = 287 \text{ Nm / kg K}$ ,  $g_0 = 9.81 \text{ m/s}^2$  and  $b = -0.0065 \text{ K/m}$  one gets this formula for the pressure curve in the geopotential standard atmosphere for  $0 \leq H \leq 11$  km:

$$\frac{p_H}{p_0} = \left( \frac{T_H}{T_0} \right)^{\frac{9.81}{(287) 0.0065}} \longrightarrow \boxed{\frac{p_H}{p_0} = \left( \frac{T_H}{T_0} \right)^{5.255}} \quad [3.1/4]$$

The formula 3.1/3a for the pressure altitude can be derived from equ. 3.1/3:

$$\begin{aligned} \frac{p_H}{p_0} &= \left( \frac{T_0 + bH}{T_0} \right)^{-\frac{g_0}{Rb}} \\ \left( \frac{p_H}{p_0} \right)^{-\frac{Rb}{g_0}} &= 1 + \frac{bH}{T_0} \\ H &= \frac{T_0}{b} \left[ \left( \frac{p_H}{p_0} \right)^{-\frac{Rb}{g_0}} - 1 \right] \quad [3.1/3a] \end{aligned}$$

In the isothermal section of the geopotential ISA for  $11 < H \leq 20$  km applies accordingly:

$$\frac{dp}{p} = - \frac{g_o}{R T} dh$$

$$\int_{p_{11}}^{p_H} \frac{dp}{p} = - \frac{g_o}{R T} \int_{H_1}^{H_2} dh \quad \left| \quad \int \frac{dx}{x} = \ln x \right.$$

$$\ln p_H - \ln p_{11} = - \frac{g_o}{R T} h \Big|_{H_1}^{H_2} \longrightarrow \ln \frac{p_H}{p_{11}} = - \frac{g_o}{R T} (H_2 - H_1) \quad [3.1/5]$$

$$\boxed{\frac{p_H}{p_{11}} = e^{-\frac{g_o}{R T} (H_2 - H_1)}} \quad H_1 = 11,000 \text{ m} \quad [3.1/5a]$$

For the pressure curve in the isothermal atmosphere is valid according to equ. 3.1/5:

$$\boxed{p_{\text{Hiso}} = p_o e^{-\frac{H g_o}{283 R}}} \quad [3.1/6]$$

Fig.3.1/3 compares the pressure curves of the ISA and the isothermal atmosphere for  $0 \leq H \leq 11$  km. Both models lead to almost identical results below 3500 m altitude (assuming the initial value  $p_o = 1013$  hPa). The simultaneous use of different calibrated altimeters was therefore unproblematic at low altitudes. However, the temperature error of the altimeters had to be eliminated using different slide rules. Fig.3.1/4 shows two examples of such computers (I.C.A.N. stands for „International Commission for Air Navigation“, a precursor of the ICAO).

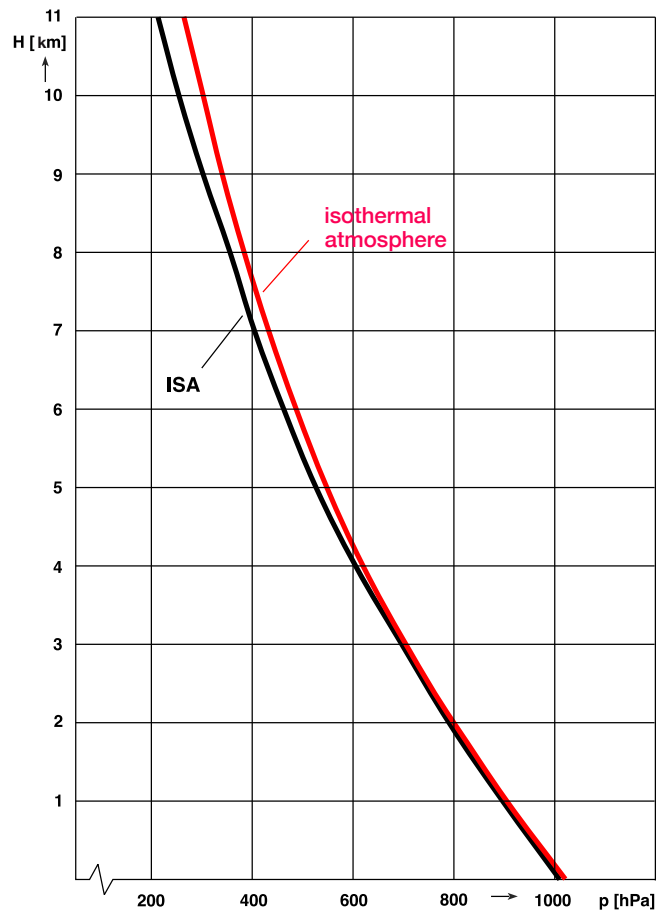


Fig.3.1/3

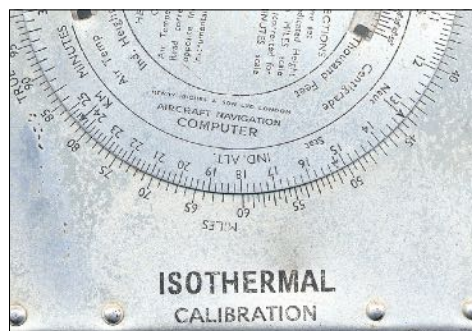


Fig.3.1/4

### 3.1.2. Air density

#### 3.1.2.1 Derivation of the air density curve over height $\rho = f(H)$

The flight performances calculated for the standard atmosphere and dependent on the air density can be converted to the actual prevailing density either via the deviation of the temperature from the normal atmosphere (i.e.  $ISA \pm \Delta t$ ) or via the density altitude. The density altitude is the altitude that belongs to the actual density in the standard atmosphere. Its calculation is possible on many navigation computers using a special scale. The shape of the density curve in the standard atmosphere is subject to the following laws:

According to equ.3.1/2 and 3.1/4 applies to the curve of air density in the standard atmosphere for  $0 \leq H \leq 11$  km:

$$\rho_H = \rho_0 \left( \frac{T_H}{T_0} \right)^{5.255}$$

$$\rho_H = \frac{p_H}{R T_H}$$

$$\rho_H = \rho_0 \frac{T_0}{T_H} \left( \frac{T_H}{T_0} \right)^{5.255}$$

$$\rho_H = \rho_0 \left( \frac{T_H}{T_0} \right)^{4.255}$$

[3.1/7]

According to equ.3.1/5 one gets this curve of air density in the standard atmosphere for  $11 \leq H \leq 20$  km:

$$\frac{\rho_H}{\rho_{11}} = e^{-\frac{g_0}{R T} (h_2 - h_1)} \quad \left| \quad \begin{array}{l} h_1 = 11,000 \text{ m} \\ \rho_H = \rho_{11} R T_{11} \end{array} \right.$$

$$\frac{\rho_H}{\rho_{11}} = e^{-\frac{g_0}{R T} (h_2 - h_1)} \quad [3.1/8]$$



From equ. 3.1/7 the following formula can be derived for the density altitude for  $0 \leq H \leq 11$  km:

$$\rho_H = \rho_o \left( \frac{T_H}{T_o} \right)^{4.255} \quad \left| \quad T_H = T_o + b H \right.$$

$$\frac{\rho_H}{\rho_o} = \left( \frac{T_H + b H}{T_o} \right)^{4.255} \longrightarrow \left( \frac{\rho_H}{\rho_o} \right)^{\frac{1}{4.255}} = \frac{T_H + b H}{T_o} = 1 + \frac{b H}{T_o} \longrightarrow \left( \frac{\rho_H}{\rho_o} \right)^{0.235} - 1 = \frac{b H}{T_o}$$

$$\frac{T_o}{b} \left[ \left( \frac{\rho_H}{\rho_o} \right)^{0.235} - 1 \right] = H \quad \left| \quad \rho_H = \frac{p_H}{R T_H} \longrightarrow \rho_H^{0.235} = \left( \frac{p_H}{T_H} \right)^{0.235} \frac{1}{R^{0.235}} \right.$$

$$H_p = \frac{T_o}{b} \left[ \frac{\left( \frac{p_H}{T_H} \right)^{0.235}}{R^{0.235} \rho_o^{0.235}} - 1 \right] = \frac{T_o}{b} \left[ \left( \frac{p_H}{R \rho_o} \right)^{0.235} - 1 \right]$$

This formula can be simplified by inserting the constants. The density altitude in meters is obtained by using the pressure  $p_H$  in N/m<sup>2</sup> and the temperature in Kelvin:

$$H_{p[m]} = 44307 - 11173 \left( \frac{p_H}{T_H} \right)^{0.235} \quad [3.1/9]$$

From equ. 3.1/5 the following formula can be derived for the density altitude for  $11\text{km} \leq H \leq 20$  km:

$$\ln \frac{\rho_H}{\rho_{11}} = - \frac{g_o}{R T_{11}} (H - 11000) \longrightarrow H_p = 11000 - \frac{R T_{11}}{g_o} \ln \frac{\rho_H}{\rho_{11}} \quad \left| \quad \rho_H = \frac{p_H}{R T_H} \right.$$

$$H_{p[m]} = 11000 - \frac{R T_{11}}{g_o} \ln \frac{p_H}{R T_H \rho_{11}} \longrightarrow H_{p[m]} = 11000 - 6334 \ln \frac{p_H}{R T_H \rho_{11}} \quad [3.1/10]$$

The scales of the individual computers are constructed with the help of the standard atmosphere table in such a way that the density altitude can be read in a window after setting the temperature and pressure values opposite each other. Fig.3.1/5 shows the following example ( equ. 3.1/9 for  $H < 11$  km):

press. altitude 30,000 ft ( $p_H = 30,093$ Pa)	temperature ISA+20° (-24.44°C = 248.71 K)
--	---

$$H_p = 44,307 \text{ m} - 11,173 \text{ m} \left( \frac{p_H}{T_H} \right)^{0.235} = 44,307 \text{ m} - 11,173 \text{ m} \left( \frac{30,093}{248.71} \right)^{0.235} = 9,822 \text{ m} = 32,226 \text{ ft}$$

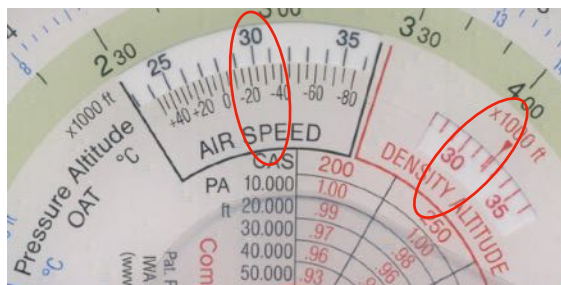


Fig.3.1/5

Fig.3.1/5a shows the following example ( equ. 3.1/10 for  $11\text{ km} \leq H \leq 20\text{ km}$ ):

press. altitude 11,500 m ( $p_H = 20916\text{ Pa}$ )	temperature ISA+30° (-26.5°C = 246 K)	$\rho_{11} = 0.3639\text{ kg/m}^3$
--	---------------------------------------	------------------------------------

$$H_{p_{[m]}} = 11000 - 6334 \ln \frac{p_H}{R T_H \rho_{11}} = 11000 - 6334 \ln \frac{20916}{(287) (246) (0.3639)} = 12299.6\text{ m}$$



Fig.3.1/5a

Since the temperature gradient in the standard atmosphere changes at the altitude  $H = 11\text{ km}$  (see Fig.3.1/1), the air pressure and density curves at this altitude have a discontinuity point. Fig. 3.1/6 shows the shape of the density curve. The discontinuity point, however, is much less significant than shown. For the construction of the density height scale of mechanical calculators, this special feature of the curve is no problem, since each density (i.e. each quotient  $p/T$ ) can only be allocated a single "density altitude". For electronic calculators which calculate the density height using the formulas 3.1/9 and 10, difficulties occur in the transition area (altitudes around  $11\text{ km}$ ), as it is not immediately clear from the initial conditions which formula is applicable. Therefore, density altitudes above  $11\text{ km}$  altitude cannot be determined with all electronic calculators. The Techstar calculator uses the formula for altitudes below  $11\text{ km}$  over the entire altitude range. The resulting errors are shown in Tab.3.1/2 for some computers.

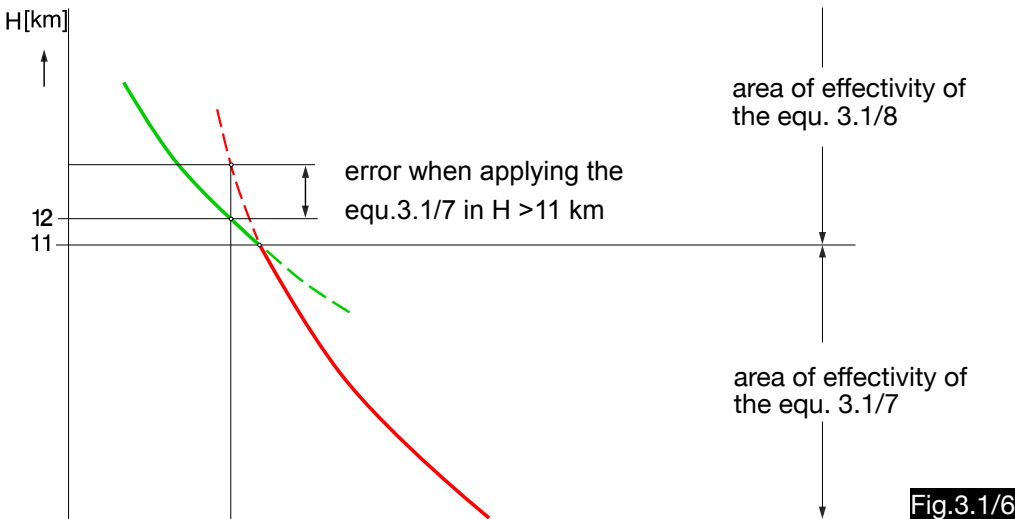


Fig.3.1/6

The altitude error  $\Delta H$ , which results from the application of the formula for altitudes below  $11\text{ km}$  at a pressure altitude of  $12\text{ km}$ , is only  $205\text{ m}$ , which corresponds to  $1.7\%$  (see Tab.3.1/1):

PA = 12,000 m	$p_{H_{ISA}} = 19330.4\text{ N/m}^2$	$T_{H_{ISA}} = 216.65\text{ K}$
$H_{p_{[m]}} = 44307 - 11173 \left( \frac{p_H}{T_H} \right)^{0.235} = 44307 - 11173 \left( \frac{19330.4}{216.65} \right)^{0.235} = 12205\text{ m} \rightarrow \frac{12205}{12000} = 1.017$		

Tab. 3.1/1

Comparison of the analytically calculated density altitude with the results of different computers

pressure altitude (PA)		ft	10000		20000		30000		35000	37000	40000	50000			
		m	3048		6096		9144		10668	11278	12192	15240			
		Pa	69689		46569		30093		23843	21645	18738	11584			
temperature	T <sub>ISA</sub>	K /°C	268.34 / -4.81		248.53 / -24.62		228.71 / -44.44		218.8 / -54.35	216.65 / -56.5					
	ISA ±	K	-20	+20	-20	+20	-20	+20	+20	-20	-20	+20	-20	+20	
	T <sub>act.</sub>	K	248.34	288.34	228.53	268.53	208.71	248.71	238.8	196.65		236.65	196.65	236.65	
	t <sub>act.</sub>	°C	-24.81	+15.19	-44.62	-4.62	-64.44	-24.44	-34.35	-76.5		-36.5	-76.5	-36.5	
air density ρ		kg/m³							0.3479	0.3835					
Density altitude determined with different computers and error in % compared to the analytically determined value	AVIAT 615	ft	7800	12200	17700	22220	27500	32100	37000	35000	38000	41900	48000	–	
		%	+4	–	+1.2	–	–	--	--	--	--	--	--	--	
	CSG - 6 P	ft	7500	12000	17500	22200	27200	32000	--	34600	--	--	--	--	
		%	–	- 2.1	–	–	-1	--	--	--	--	--	--	--	
	IWA 11092	ft	7600	12200	17250	22100	27200	32000	36600	34700	37800	41800	47800	--	
		%	+1.4	–	-1.3	–	-1	--	-1.2	--	--	--	--	--	
	CR - 3	ft	7500	12200	17500	22200	27500	32200	37000	35000	38000	41800	48000	51800	
		%	–	–	–	–	--	–	–	--	--	--	–	--	
	asa E-6 B	ft	7800	12400	17600	22500	27700	32200	37200	--	--	43100	--	--	
		%	+4	+1.2	+0.7	+1.2	+0.8	--	+0.5	--	--	+3	--	--	
	ARC - 2	ft	7800	12400	17300	22300	27500	32200	36800	34900	37900	41800	47900	--	
		%	+4	+1.2	-1	--	--	--	--	--	--	--	--	--	
	Felsenthal PT	ft	7600	12200	17200	22200	27500	32200	37000	35000	38000	41800	48000	51800	
		%	+1.4	--	-1.6	--	--	--	--	--	--	--	--	--	
	Techstar	ft	7513	12269	17503	22261	27490	32251	37246	34717	38405	42962	49836	53906	
		%	--	--	--	--	--	--	+0.5	--	+1.15	+2.7	+3.9	+4.1	
	navtronic NAVIGATOR	ft	7517	12271	17510	22266	27501	32259	37254	not to be determined with this calculator					
		%	--	--	--	--	--	--	+0.5						
analytically calculated density altitude	ft	7497	12251	17483	22240	27467	32226	37221 <sup>1)/37024<sup>2)</sup></sup>	34997 <sup>3)/34767<sup>2)</sup></sup>	37986	41825	47957	51796		
	m	2285	3734	5329	6779	8372	9822	11285 <sup>1)/11345<sup>2)</sup></sup>	10594 <sup>3)/10667<sup>2)</sup></sup>	11578	12748	14617	15787		
	calculated with equ. 3.1/9								1) calculated with equ. 3.1/10 2) determined by linear interpolation in the ISA 3) calculated with equ. 3.1/9		calculated with equ. 3.1/10				

Tab. 3.1/2

### 3.1.2.2 Calculator for the consideration of the density altitude

To calculate the influence of air density on flight performance, so-called DENSITY ALTITUDE performance computers were developed, which specify the factors with which the parameters

- vertical speed or rate of climb (ROC)
- take off distance

are to be multiplied if the air density (i.e. the density altitude) deviates from the values of the standard atmosphere. Fig.3.1/7 shows such a calculator and Fig. 3.1/8 shows an enlarged section of it.

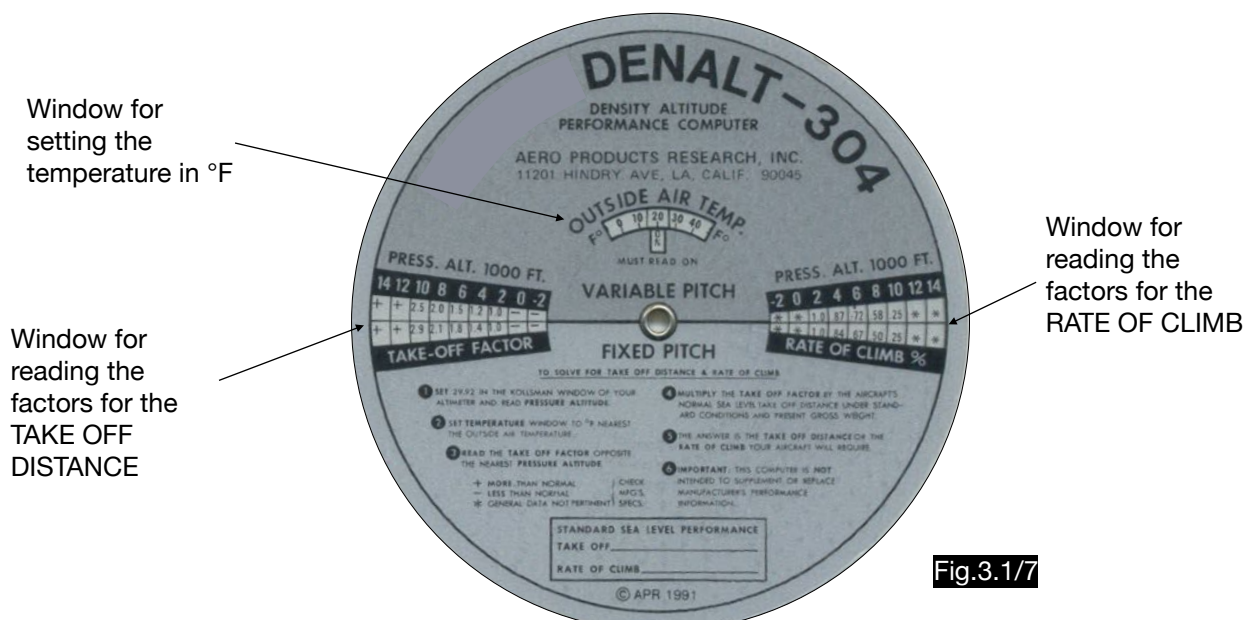


Fig.3.1/7



Fig.3.1/8

The setting selected here provides at 10,000 ft (approx. 3000 m) and +20°F (-7°C) the factor 0.25 for the reduction of the rate of climb compared to the rate under ISA conditions for aircraft with variable pitch propellers. So the rate of climb decreases to a quarter. These factors are shown in Table 3.1/3. The various types of DENALT computers have the same figures, i.e. they probably come from one source.

The accuracy of these factors is checked using an example. For this purpose, the vertical speed of a variant of the single-engine aircraft Jak-18 is calculated for the altitudes 0 and 3 km, for the flight mass of 1000 kg and for ISA conditions. The necessary curves for  $P_a$  (power available) and  $P_r$  (power required) are shown in Fig.3.1/9. Doubts about the authenticity of these curves are out of the question, as one could well design an aircraft with these characteristics. For the rate of climb  $w$  in the altitudes 0 m and 3000 m and for the factor  $w_{3000} / w_0$  one gets:

constant speed propeller airplanes (variable pitch)									
		Pressure Altitude [ft]							
[°C]	[°F]	-2000	0	2000	4000	6000	8000	10000	
-40	-40					0.98	0.83	0.68	Percent ROC
-29	-20					0.93	0.78	0.64	
-23	-10					0.89	0.74	0.60	
-18	0				1.00	0.84	0.70	0.55	
-12	+10				0.94	0.80	0.66	0.52	
-7	+20				0.90	0.75	0.61	0.28	
-1	+30			1.00	0.87	0.72	0.58	0.25	
+5	+40			0.97	0.83	0.68	0.54	0.22	
+10	+50			0.93	0.78	0.64	0.49	0.18	
+16	+60			0.90	0.75	0.61	0.27		
+21	+70		0.99	0.86	0.71	0.57	0.23		
+27	+80		0.95	0.81	0.68	0.54	0.22		
+32	+90		0.91	0.78	0.64	0.50	0.18		
+38	+100		0.87	0.74	0.61	0.27			
+43	+110	1.00	0.83	0.71	0.57	0.24			
+49	+120	0.95	0.81	0.68	0.54	0.22	0.13		
		0.93	0.78	0.64	0.50	0.20			

Tab. 3.1/3

$$w = \frac{P_a - P_r}{m g}$$

$$w_0 = \frac{63,600 - 28,600}{1000 \times 9.81} \left[ \frac{\text{N m}}{\text{s N}} \right] = 3.56 \text{ m/s}$$

$$w_{3000} = \frac{48,400 - 32,400}{1000 \times 9.81} \left[ \frac{\text{N m}}{\text{s N}} \right] = 1.63 \text{ m/s}$$

$$\frac{w_{3000}}{w_0} = \frac{1.63}{3.56} = 0.458$$

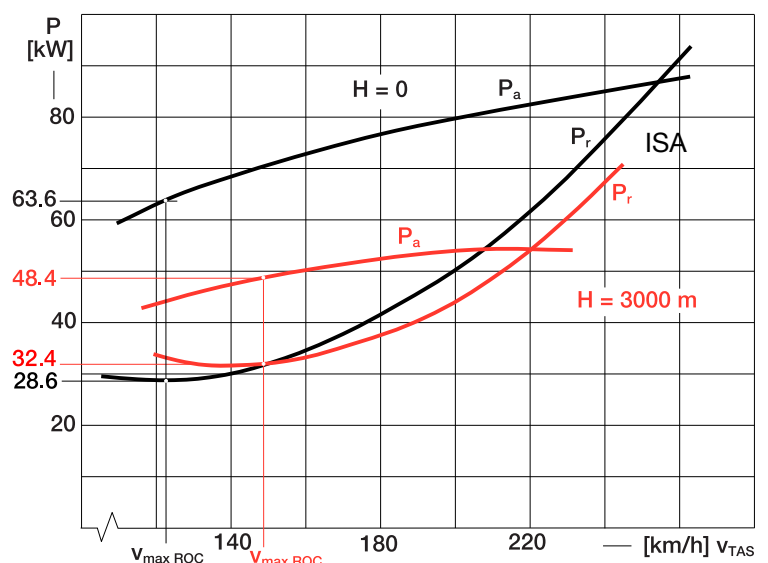


Fig.3.1/9

DENALT computers provide a factor of 0.25 for these conditions (see Tab.3.1/3). An even greater difference results if one assumes that the otherwise unchanged aircraft is equipped with a more powerful engine, so that the available power can be multiplied by 1.5:



$$w_0 = \frac{1.5 \times 63,600 - 28,600}{1000 \times 9.81} \left[ \frac{\text{N m}}{\text{s N}} \right] = 6.8 \text{ m/s}$$

$$w_{3000} = \frac{1.5 \times 48,400 - 32,400}{1000 \times 9.81} \left[ \frac{\text{N m}}{\text{s N}} \right] = 4.1 \text{ m/s}$$

$$\frac{w_{3000}}{w_0} = \frac{4.1}{6.8} = 0.603$$

The reduction "PERCENT ROC", which should remain the same (DENALT computers only make a difference with regard to the type of propeller), becomes even bigger. This makes these factors somewhat questionable; after all, it has to be said that they are deviations to the safe side.

### 3.1.3. Temperature measurement

#### 3.1.3.1. Definitions

The temperature rise (temperature increase due to internal friction and compressibility of the air) causes the temperature measured at the aircraft is falsified depending on the airspeed. A classification is made between the following three temperatures:

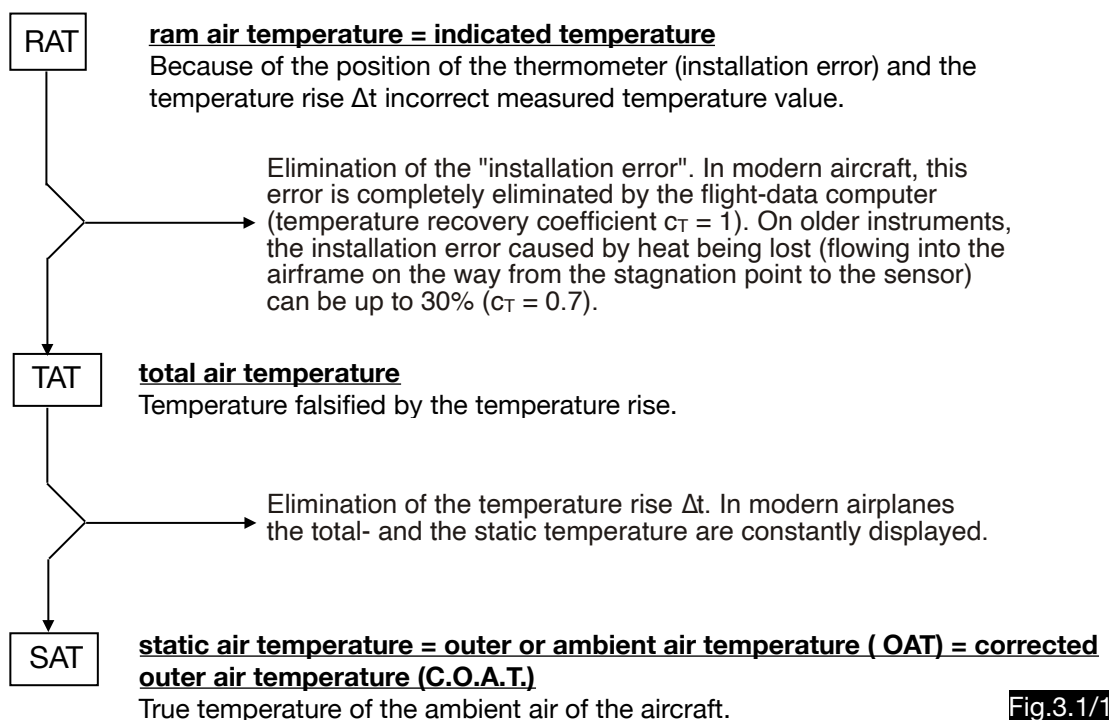


Fig.3.1/10

### 3.1.3.2. Temperature rise

These forms of energy can be contained in a flowing air mass:

Kinetic energy:  $\frac{m v^2}{2}$

Pressure energy:  $m R T$  or  $m \frac{p}{\rho}$   $\left| \begin{array}{l} R = 287 \frac{\text{Nm}}{\text{kg K}} \end{array} \right.$

Potential energy:  $m g h$

Thermal energy:  $m c T$   $\left| \begin{array}{l} c = 715 \frac{\text{Nm}}{\text{kg K}} \end{array} \right.$

The sum of these energies must be constant (Bernoulli's theorem):

$$\frac{m v^2}{2} + m R T + m g h + m c T = \text{const.}$$

With the assumption that the mass  $m$  and the height  $h$  are also constant, one obtains this formula:

$$\frac{(v_{[\text{m/s}]})^2}{2} + 287 T_{[\text{K}]} + 715 T_{[\text{K}]} = \text{const.} \quad \boxed{(v_{[\text{m/s}]})^2 + 2000 T_{[\text{K}]} = \text{const.}} \quad [3.1/11]$$

In exceptional cases of low velocities, the flow may be considered incompressible, so that density, height, mass and temperature are constant and the Bernoulli formula is simplified as follows:

$$\boxed{\frac{\rho v^2}{2} + p = \text{const.}} \quad [3.1/12] \quad \left| \begin{array}{l} \frac{\rho v^2}{2} = q = \text{dynamic pressure} \\ p = \text{static pressure} \end{array} \right.$$

When the air particles hit an obstacle, their speed decreases. The associated decrease in kinetic energy must lead to an increase in pressure and thermal energy.

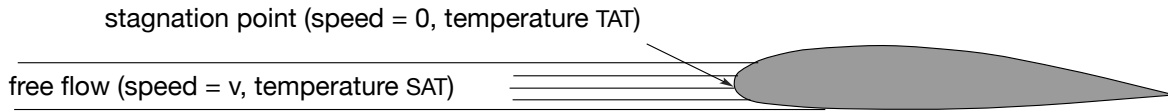


Fig.3.1/11

The temperature increase  $\Delta t$  in a stream that is decelerated at an obstacle to zero can be calculated by equating the equ.3.1/11 for the free flow and for the stagnation point:

$$(v_{[m/s]})^2 + 2000 T_{SAT[K]} = 0 + 2000 T_{TAT[K]}$$

$$T_{TAT[K]} = \frac{(v_{[m/s]})^2}{2000} + T_{SAT[K]}$$

$$\left. \begin{array}{l} v_{TAS}^2 = M^2 a^2 \\ a \approx 20 \sqrt{T_{SAT}} \\ v_{TAS}^2 \approx M^2 400 T_{SAT} \end{array} \right| \begin{array}{l} \text{sound speed } a = \sqrt{1.4 R T} \\ a_{[m/s]} = 20.04 \sqrt{T_{SAT[K]}} \\ \text{Mach number } M = \frac{v_{TAS}}{a} \end{array}$$

$$T_{TAT[K]} = \frac{M^2 400 T_{SAT[K]}}{2000} + T_{SAT[K]}$$

$T_{TAT[K]} = T_{SAT[K]} (1 + 0.2 M^2)$

[3.1/13]

The temperature rise (temperature increase at the stagnation point) is equal to the difference between the total air temperature and the static air temperature:

$$\Delta t = T_{TAT} - T_{SAT}$$

$$T_{TAT} = T_{SAT} (1 + 0.2 M^2)$$

$$\longrightarrow T_{TAT} = T_{SAT} \left( 1 + 0.2 \frac{v_{TAS}^2}{400 T_{SAT}} \right)$$

$$\longrightarrow T_{TAT} = T_{SAT} + 0.2 \frac{v_{TAS}^2}{400}$$

$$\left. \begin{array}{l} M^2 = \frac{v_{TAS}^2}{a^2} \\ M^2 = \frac{v_{TAS}^2}{400 T_{SAT}} \end{array} \right| a_{[m/s]} \approx 20 \sqrt{T_{SAT}}$$

$\Delta t [K] = T_{TAT[K]} - T_{SAT[K]} = \frac{v_{TAS}^2 [m/s]}{2000}$

[3.1/14]

For other speed dimensions these relations are derived:

$\Delta t [K] = \frac{v_{TAS}^2 [kt]}{7600}$

$\Delta t [K] = \frac{v_{TAS}^2 [km/h]}{26050}$

$\Delta t [K] = 0.0000384 v_{TAS}^2 [km/h]$

[3.1/15]
[3.1/16]
[3.1/17]

The temperature rise can be calculated with flight computers in the following way:

### 1. Calculation of the temperature rise according to Huber

After equ.3.1/15 one gets this result for the example  $v_{TAS} = 300$  kt and  $c_T = 1$  (see Fig.3.1/12):

$$\Delta t = \frac{300^2}{7600} = 11.8^\circ$$

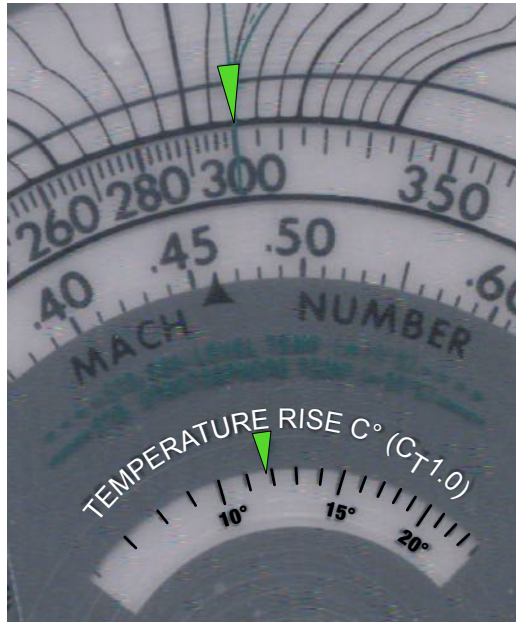


Fig.3.1/12

### 2. Calculation of the temperature rise with computers of the type SN-3m

The slide rule SN-3m (an NL copy) has an additional scale for calculating the temperature rise. This scale, consisting of the temperature recovery coefficients  $c_T$  equal to .5, .6, .7, .8, .9 and 1, was inserted into the tangent scale in such a way that the temperature rise in degrees Celsius can be read directly from it. For a true airspeed of 720 km/h, the equ.3.1/17 will give a temperature rise of  $20^\circ\text{C}$  (valid for  $c_T = 1$ ).

$$\Delta t = 0.0000384 v_{TAS}^2 = 0.0000384 \times 720^2 = 19.9^\circ$$

The calculation for the selected example consists of the following settings (see Fig.3.1/13):

- The cursor is set to  $v_{TAS} = 720$  km/h on the  $v:100$  scale.
- The slider is moved so that the mark  $40^\circ/140^\circ$  of the sine scale is below the cursor line.
- Under the red mark 1 on the tangent scale, one reads a temperature rise of  $20^\circ\text{C}$  (applies to  $c_T = 1$ ; for  $c_T = 0.8$ , for example, one gets  $16^\circ$ ).

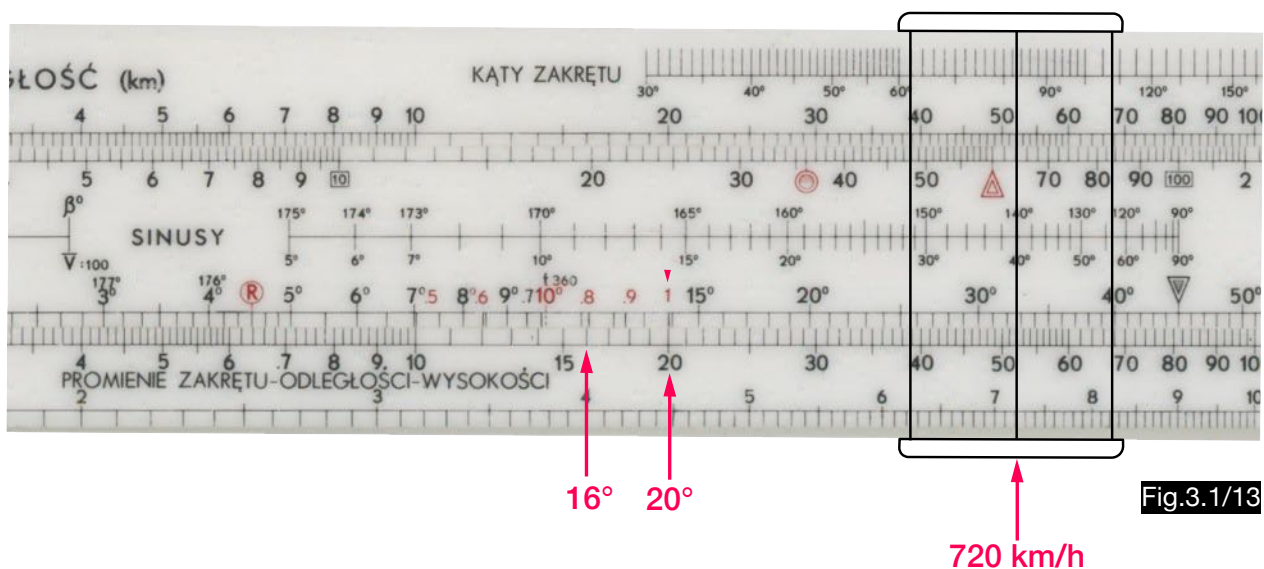


Fig.3.1/13



As the computer setting in Fig.3.1/14 shows, the square of the true air speed is multiplied by 0.0000384 or divided by 26050 in order to find the temperature rise of 20°C for  $c_T = 1$  and 18°C for  $c_T = 0.9$

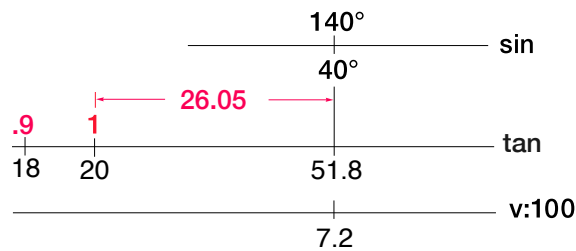


Fig.3.1/14

### 3. Calculation of temperature rise with AVIAT computers and computers of type NL

Fig.3.1/15 shows the "TEMPERATURE CORRECTION" scale of an AVIAT computer. Neither the calculator nor the manual contains information about the "temperature recovery coefficient"  $c_T$  on which these scales are based. Comparisons with analytically calculated values suggest a  $c_T$  - value of about 0.7 (see Tab. 3.1/4).



Fig.3.1/15

Fig.3.1/16 shows the  $\Delta t$  - scale of a computer of type NL. With these calculators sometimes the measuring element is named, but no information about its  $c_T$ -value is given. This should also be 0.7 (see Tab.3.1/4).

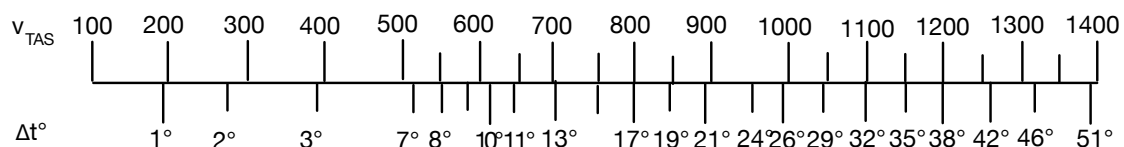
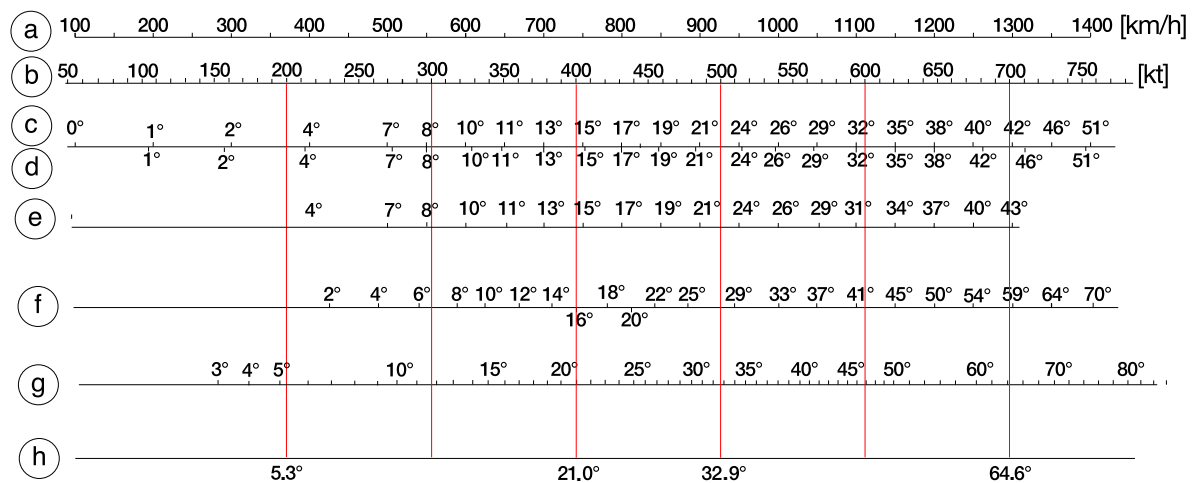


Fig.3.1/16

### Comparison of different computers regarding the TEMPERATURE RISE calculation



#### Explanation:

- (a) true airspeed [km/h]
- (b) true airspeed [NM/h]
- (c) scale  $\Delta t^\circ$  of the computers of the NL - series
- (d) scale  $\Delta t^\circ$  or TEMP. CORR of duplicates of the NL - computers and of AVIAT-Computers (except AVIATJET)
- (e) Chinese computers (deviations from the scales c and d only at high speeds)
- (f) scale  $\Delta t^\circ$  of the computers NL - 10 U and NRK - 2
- (g) computers working with the Huber patent (clearly labelled with  $c_T = 1$ ), IWA 11092 and NL-derivative SN-3m (Fig.3.1/13)
- (h) the values of this scale were calculated according to equ.3.1/15

### 3.1.4. Flight envelope

In Fig.3.1/17 the hatched area shows the flight envelope (range of possible speeds and flight altitudes), which is limited by

- the minimum dynamic pressure required for lift ( $V_{CAS\ min}$ ),
- the maximum permissible dynamic pressure (max. operating  $V_{CAS}$ ) for reasons of structural strength, and
- the maximum Mach number permissible for aerodynamic reasons (max. operating  $M$ ).

At the intersection of the lines of the maximum permissible dynamic pressure ( $V_{CAS}$ ) and the maximum permissible Mach number, the greatest possible true airspeed  $V_{TAS\ max}$  is determined. The corresponding altitude is called crossover altitude. The intersection of the lines of the minimum dynamic pressure ( $V_{CAS\ min}$ ) and the max. operating Mach number determines the aerodynamic ceiling, the theoretically highest possible altitude (in the slang called coffin corner).

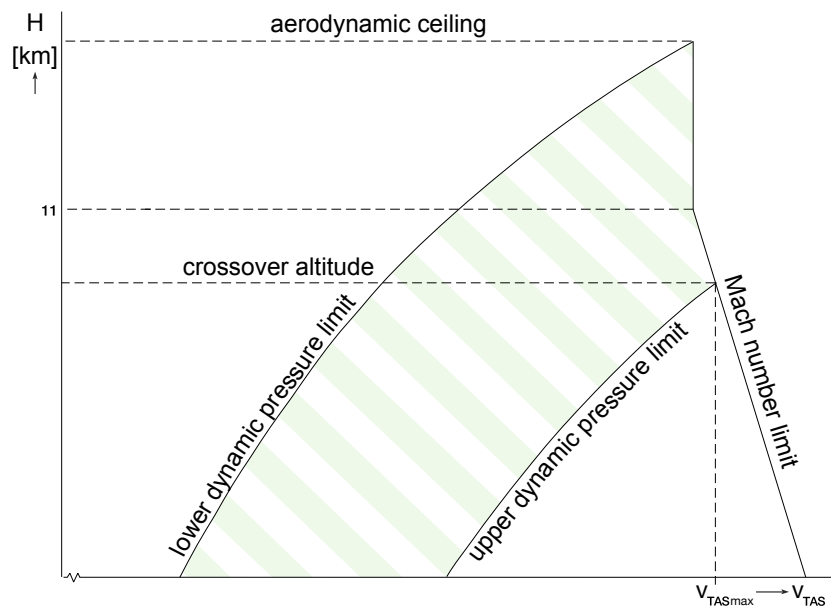


Fig.3.1/17

This formula applies to the true airspeed in the subsonic compressible flow (see John D. Anderson „Introduction to flight“):

$$V_{TAS} = \sqrt{\frac{2a_H^2}{\kappa - 1} \left[ \left( \frac{q}{p_H} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]} \quad \left| \begin{array}{l} a_H \text{ sound speed} \\ \text{in alt. } H \\ p_H \text{ static pressure} \\ \text{in alt. } H \\ q \text{ dynamic pressure} \end{array} \right. \quad [3.1/18]$$

With  $V_{TAS} = a M$  this relation for the Mach number can be derived from equ. 3.1/18:

$$M = \sqrt{\frac{2}{\kappa - 1} \left[ \left( \frac{q}{p_H} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]} \quad [3.1/19]$$

For the pressure  $p_H$  is obtained:  $M^2 = \frac{2}{\kappa - 1} \left[ \left( \frac{q}{p_H} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right] \rightarrow 1 + \frac{\kappa - 1}{2} M^2 = \left( \frac{q}{p_H} + 1 \right)^{\frac{\kappa - 1}{\kappa}}$

$$\frac{q}{p_H} + 1 = (1 + 0.2 M^2)^{\frac{\kappa}{\kappa - 1}} \rightarrow p_H = \frac{q}{(1 + 0.2 M^2)^{3.5} - 1} \quad [3.1/20]$$

After inserting the minimum dynamic pressure  $q$  ( $V_{CAS\ min}$ ) and the critical Mach number  $M_{krit}$  into this formula, the pressure  $p_H$  at the aerodynamic ceiling is obtained. After inserting the maximum dynamic pressure  $q$  ( $V_{CAS\ max}$ ) and the maximum operating Mach number ( $M_{MO}$ ) into this formula, the pressure  $p_H$  at the crossover altitude is obtained. Note that these altitudes are not temperature-dependent.

The maximum true air speed that can be achieved by respecting  $V_{CAS\ max}$  (that means maximum dynamic pressure  $q$ ) and maximum operating Mach number ( $M_{MO}$ ) is then:

$$V_{TAS\ max} = a M_{MO} = M_{MO} \sqrt{1.4 R T_{CR.OV.ALT.}} \quad [3.1/21] \quad \left| \begin{array}{l} (T_{CR.OV.ALT.} - \text{ISA-temperature in} \\ \text{the crossover altitude}) \end{array} \right.$$

According to equ.3.1/19, the Mach number is a function of the static pressure  $p_H$  (and thus the altitude) and the dynamic pressure  $q$  (and thus the  $v_{CAS}$ ).

The setting of a computer of type CR using the Huber method is described by an example where the  $v_{CAS}$  is 300 kt and the pressure altitude 30,000 ft and where the Mach number  $M = 0.79$  is determined (see Fig. 3.1/18). In the logarithmic scales for the dynamic pressure ( $v_{CAS}$ ) and the static pressure (altitude), the quotient  $q/p_H$  is calculated, to each of which a Mach number belongs. The same setting of the calculator should be used if the crossover altitude or the aerodynamic ceiling is to be calculated from  $v_{CAS}$  and Mach number.

300 kt  
30,000 ft



Fig.3.1/18

The calculation of the Mach number with Huber computers is also possible via the true temperature  $t_{SAT}$  and the true airspeed  $v_{TAS}$ . For an example where the true air speed  $v_{TAS} = 450$  kt (232 m/s) and the true temperature  $t_{SAT} = -61^\circ\text{C}$  (212 K), the Mach number  $M = 0.795$  is obtained.

$$M = \frac{v_{TAS}}{a} = \frac{v_{TAS}}{\sqrt{\kappa R T_{SAT}}} = \frac{232}{\sqrt{1.4 \times 287 \times 212}} = 0.795$$

In the interest of a more exact and simpler calculation, the Huber computer works with the indicated temperature  $t_{RAT}$ . The temperature rise at  $v_{TAS} = 450$  kt is  $\Delta t = 27^\circ\text{C}$  (according to equation 3.1/15). An accurate thermometer ( $c_T = 1$ ) would therefore indicate  $-34^\circ\text{C}$  ( $-61^\circ + 27^\circ = -34^\circ$ ). Assuming a temperature recovery coefficient of  $c_T = 0.8$ , only  $21^\circ$  ( $27 \times 0.8 = 21$ ) of the temperature rise would be displayed, i.e. the indicated temperature  $t_{RAT}$  is  $-40^\circ\text{C}$  ( $-61 + 21$ ). Fig. 3.1/19 and 3.1/20 show the settings and results in two such computers for this example.

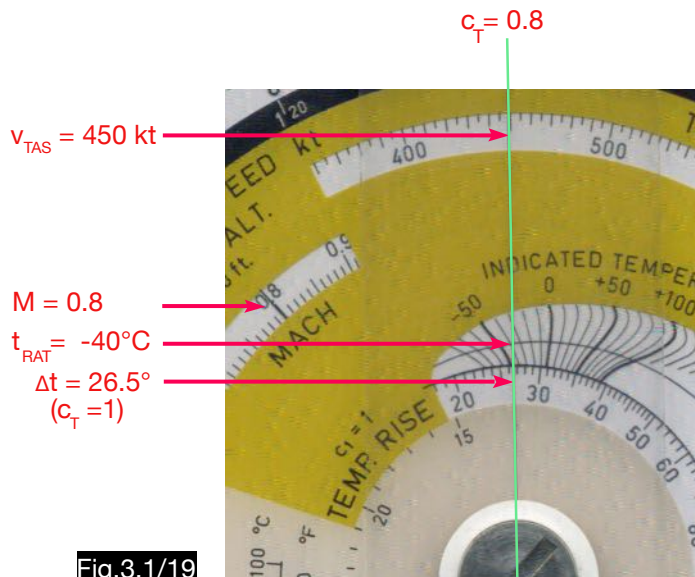


Fig.3.1/19

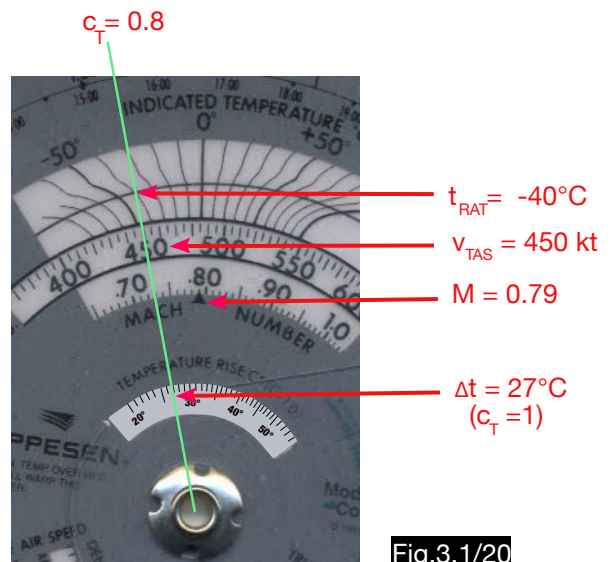


Fig.3.1/20



## 3.2. The systematic errors of barometric altimeters and their correction

### 3.2.1. General

Barometric altimeters always display the altitude value in meters or feet standardized to the measured pressure in the standard atmosphere. They are therefore not capable to determine the height above the relief and thus the obstacle clearance. The unavoidable deviations from the ISA result in further systematic errors. The pressure at altitude depends on the pressure at ground as well as on the temperature at ground and at altitude (see equ.3.1/4 and 5). The more the air temperature on the ground and the more the mean layer temperature deviate from the standard value, the greater the systematic errors.

### 3.2.2. Correcting the altimeter indication for the temperature error

The temperature error can only be eliminated if the true temperature curve is known. This formula is derived from the Law of Laplace (see chapter 3.1.1):

$$\frac{dp}{p} = - \frac{g_0}{R T} dh$$

If the constant mean layer temperature  $T_m$  is used for  $T$ , integration is simplified and this result is obtained:

$$\frac{dp}{p} = - \frac{g}{R T_m} dh$$

$$\int_{p_0}^{p_H} \frac{dp}{p} = - \frac{g}{R T_m} \int_0^H dh$$

$$\ln p_H - \ln p_0 = - \frac{g}{R T_m} [H]_0^H \longrightarrow \ln \frac{p_H}{p_0} = - \frac{g}{R T_m} H$$

$$H = - \frac{R T_m}{g} \ln \frac{p_H}{p_0}$$

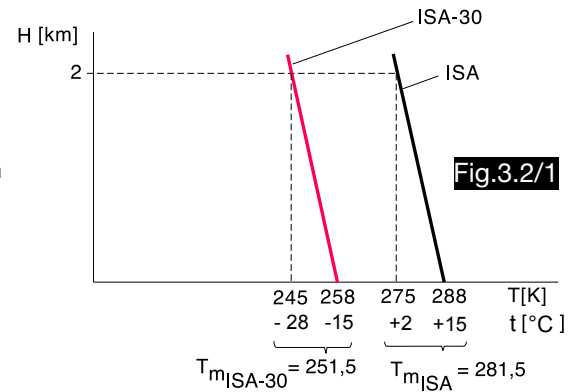
$$H_{\text{true}} = - \frac{R T_{m \text{ act.}}}{g} \ln \frac{p_H}{p_0} \quad \left| \quad H_{\text{indic.}} = - \frac{R T_{m \text{ ISA}}}{g} \ln \frac{p_H}{p_0} \right.$$

$$\frac{H_{\text{true}}}{H_{\text{indic.}}} = \frac{T_{m \text{ act.}}}{T_{m \text{ ISA}}} \longrightarrow \boxed{H_{\text{true}} = H_{\text{indic.}} \frac{T_{m \text{ act.}}}{T_{m \text{ ISA}}}} \quad [3.2/1]$$

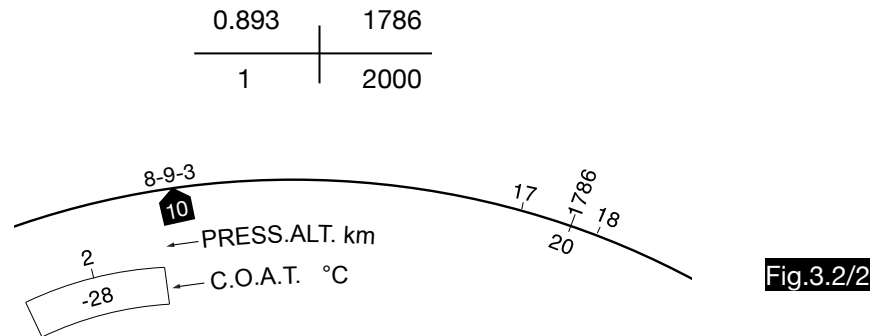
**Example 1 (temperature lower than in ISA at normal gradient - 6.5 K/km):**

With an altimeter indication from  $H_{\text{indic.}} = 2000 \text{ m}$  in an atmosphere whose temperature has the normal gradient of - 6.5 K per km altitude, but which is 30 K below the ISA (i.e. on the ground  $-15^\circ\text{C}$  instead of  $+15^\circ\text{C}$  and at 2 km altitude  $-28^\circ\text{C}$  instead of  $+2^\circ\text{C}$ ), then according to equ.3.2/1 a corrected altitude of  $H_{\text{true}} = 1786 \text{ m}$  results:

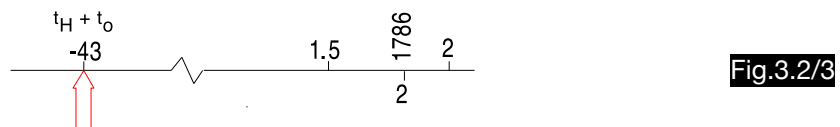
$$H_{\text{true}} = H_{\text{indic.}} \cdot \frac{T_{\text{m act.}}}{T_{\text{m ISA}}} = 2000 \text{ m} \cdot \frac{251.5}{281.5} = 0.893 \times 2000 \text{ m} = 1786 \text{ m}$$



On computers with logarithmic scales, this correction is realized by multiplying the indicated altitude by a factor, in this case 0.893. On a computer of type AVIAT the setting according to Fig.3.2/2 is required for example 1 and the exact result  $H_{\text{true}} = 1786 \text{ m}$  is obtained. In this figure the calculation scheme for the true altitude according to equ.3.2/1 using logarithmic scales is easily visible.



On computers of type NL, for which the temperature at zero altitude ( $t_0$ ) must also be known, the setting according to Fig.3.2/3 is required for example 1 and the exact result  $H_{\text{true}} = 1786 \text{ m}$  is also obtained.



The correction of altitudes below 11 km is carried out on computers of type NL-10 using separate logarithmic scales according to the formula derived from equ.3.2/1:

$$H_{\text{true}} = \frac{H_{\text{indic.}}}{T_{\text{m ISA}}} T_{\text{m act}} \longrightarrow \log H_{\text{true}} = \log \frac{H_{\text{indic.}}}{T_{\text{m ISA}}} + \log T_{\text{m act}}$$

In Fig.3.2/4 the calculation method is shown schematically. The scale of the medium absolute temperature  $T_{\text{mact}}$  was labeled  $t_0 + t_H$  for easier handling. This is possible if the division of the scale is halved, since with an increase of  $t_H$  or  $t_0$  by  $2^\circ\text{C}$  the mean layer temperature  $T_m$  only increases by one degree.

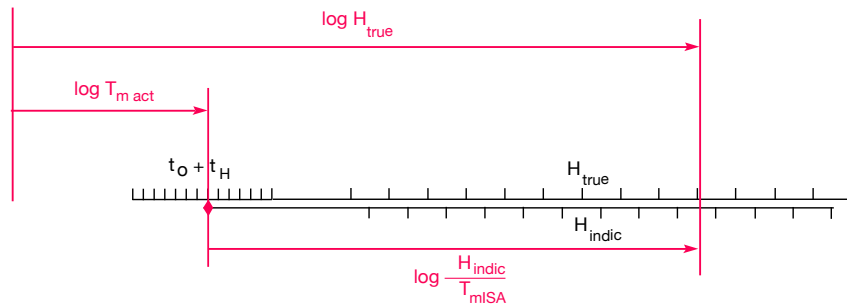


Fig.3.2/4

The correction of altitudes over 11 km is done with computers of type NL-10 using separate logarithmic scales according to the formula derived from equ.3.2/1. In Fig.3.2/5 the calculation method is shown schematically.

$$H_{\text{true}} - 11 \text{ km} = \frac{H_{\text{indic}} - 11 \text{ km}}{216.5} T_{\text{act}} \longrightarrow \log H_{\text{true}} - 11 \text{ km} = \log \frac{H_{\text{indic}} - 11 \text{ km}}{216.5} + \log T_{\text{act}}$$

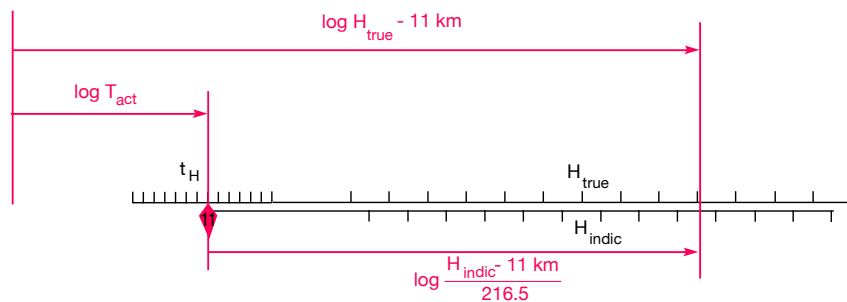


Fig.3.2/5

Both types of computers achieve incorrect results if the actual temperature curve does not have the same difference to the ISA in all altitudes due to inversions. From the direct comparison of the errors of the two computer types (see example 2 and 3 and Fig. 3.2/6 and 7) no conclusions can be made regarding the accuracy of the two methods. To determine the indicated altitude, the altimeter must be set to QNH.

#### Example 2:

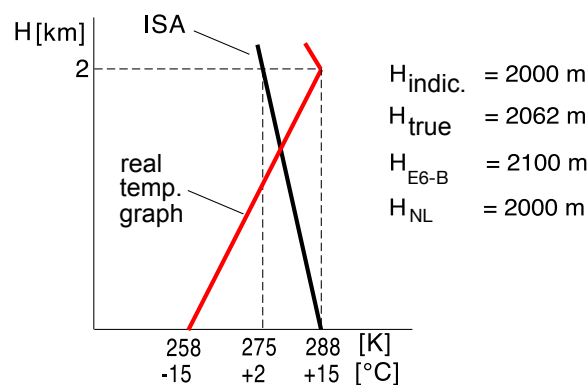


Fig.3.2/6

#### Example 3:

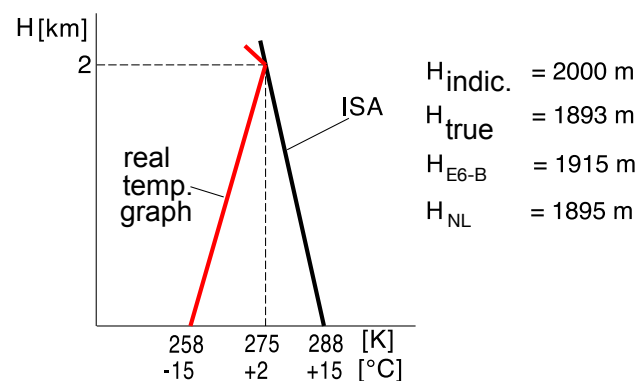


Fig.3.2/7

### 3.2.3. Correcting the altimeter indication by the air pressure error

Using the adjusting knob on the altimeter, the pressure error is eliminated by setting the air pressure measured on the ground (QFE or QNH). When the QFE is set, the altimeter at the touchdown point shows zero altitude, when the QNH is set, elevation. Exact conversion of the QFE into QNH is only possible by means of an electronic computer.

The calculation of the pressure decrease with the altitude over a constant factor is only possible as a first approximation and for low elevations. According to equ.3.1/1 applies:

$$\frac{dh}{dp} = - \frac{1}{\rho g}$$

With  $g_0 = 9.81 \text{ m/s}^2$  and  $\rho_0 = 1.225 \text{ kg/m}^3$ , for the height zero applies:

$$\frac{dh}{dp} = - 8.32 \frac{\text{m}}{\text{hPa}} = - 27.3 \frac{\text{ft}}{\text{hPa}}$$

With  $\rho_{1000} = 1.111 \text{ kg/m}^3$  one gets in 1000 ft elevation:

$$\frac{dh}{dp} = - 9.17 \frac{\text{m}}{\text{hPa}} = - 30.1 \frac{\text{ft}}{\text{hPa}}$$

However, these values are only suitable for rough calculations of the pressure decrease with elevation. An exact formula for the calculation of the QFE from the QNH can be derived from equ.3.1/4:

$$\frac{p_H}{p_0} = \left( \frac{T_H}{T_0} \right)^{5.255}$$

The QFE corresponds to the pressure  $p_H$  and the QNH to the pressure  $p_0$ , so that this formula results:

$$p_{\text{QFE}} = p_{\text{QNH}} \left( \frac{T_{\text{elev.}}}{T_0} \right)_{\text{ISA}}^{5.255}$$

The ISA temperature gradient  $- 6.5^\circ/\text{km}$  is used to calculate the temperature  $T_{\text{elev.}}$  and this formula is obtained:

$$p_{\text{QFE}} = p_{\text{QNH}} \left( 1 - \frac{H_{\text{elev. [ft]}}}{145366} \right)^{5.255} \quad [3.2/4]$$



### 3.3. The errors of airspeed indicators and their correction

#### 3.3.1. Airspeed definitions

Airspeed indicators are measuring instruments of dynamic pressure which have instrumental errors. If the indication of the instrument (airspeed indicator reading - ASIR) has been corrected for its instrument error by means of the error table of the corresponding instrument, the indicated airspeed (IAS) is obtained. The dynamic pressure sensors are dependent on the flow around them, which leads to the position error. After correcting the IAS for this error using the aircraft manufacturer's table, the calibrated airspeed CAS (also called rectified airspeed RAS) is obtained. For altitudes above zero, the determination of the TAS from the CAS shall take into account the systematic error resulting from the conversion of the dynamic pressure with the standard value of air density at sea level  $\rho_0 = 1.225 \text{ kg/m}^3$ . This error occurs at any altitude above zero (at 12 km altitude the TAS is twice as large as the CAS). At higher speeds the air may no longer be considered as an incompressible medium. The compression of the air in front of the pitot tube creates a higher dynamic pressure which results in too high speed values. Therefore, at high speeds, the remaining error due to compressibility must also be taken into account, since when calibrating the airspeed indicator, this error is eliminated by suitable construction of the indicator for the altitude zero and for normal atmospheric conditions so that a remaining error exists. After correcting the CAS for the remaining error one gets the EAS (equivalent airspeed). At the altitude zero is in the standard atmosphere  $\text{CAS} = \text{EAS} = \text{TAS}$ .

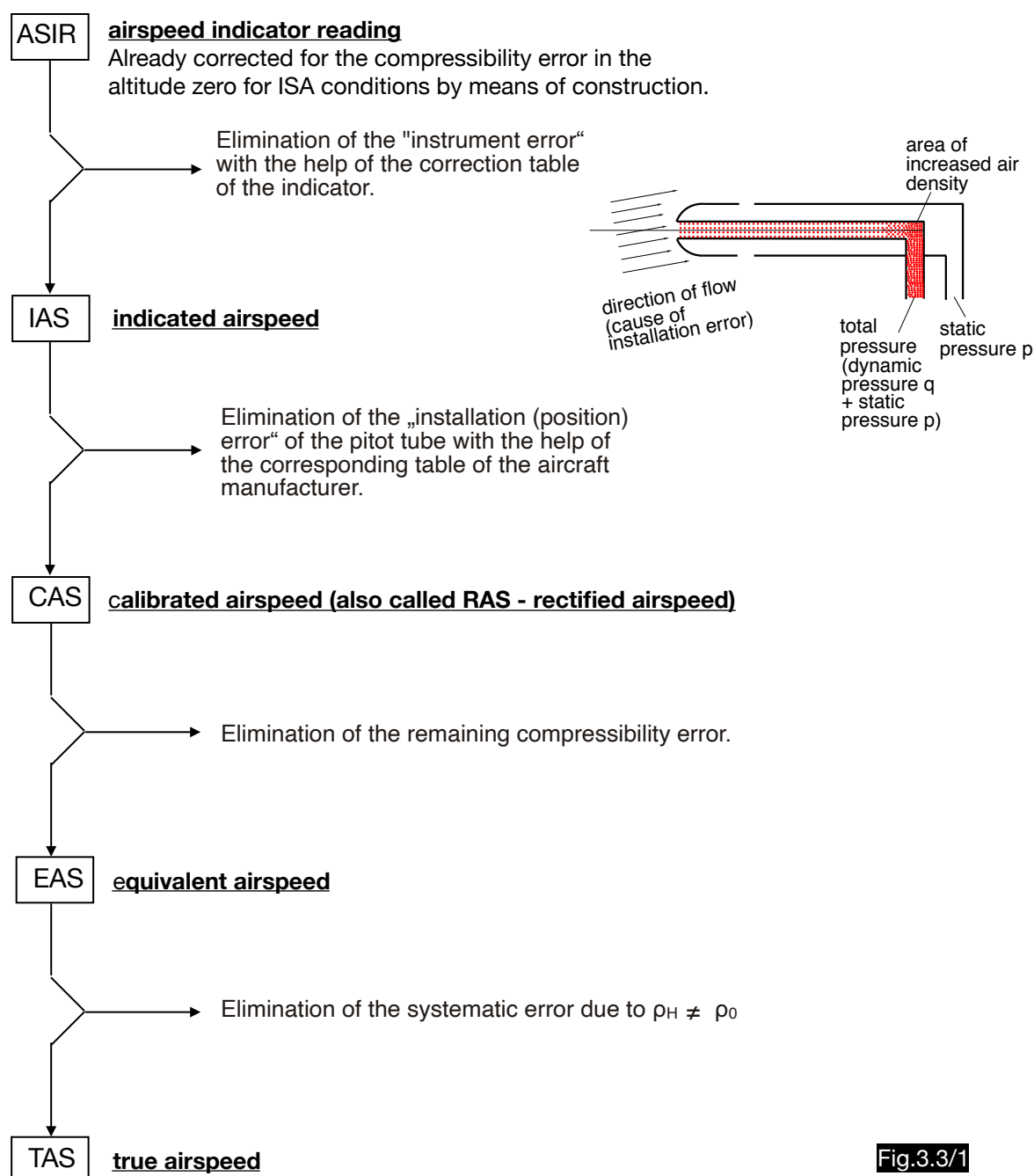


Fig.3.3/1

According to Bernoulli, for the incompressible flow, the sum of dynamic pressure  $q$  and static pressure  $p$  is constant (see equ.3.1/12):

$$\frac{\rho v^2}{2} + p = q + p = \text{const.}$$

Analogous to the formula 3.1/18 for the  $v_{TAS}$ , this relationship results for the  $v_{CAS}$  for the subsonic compressible flow:

$$v_{CAS} = \sqrt{\frac{2a_o^2}{\kappa - 1} \left[ \left( \frac{q}{p_o} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]} \quad [3.3/1]$$

From this, this formula for dynamic pressure  $q$  can be derived:

$$q = p_o \left[ \left( \frac{v_{CAS}^2 (\kappa - 1)}{2 a_o^2} + 1 \right)^{\frac{\kappa}{\kappa - 1}} - 1 \right] \quad [3.3/2]$$

At low CAS (e.g. 100 kt = 51.44 m/s),  $\rho_o = 1.225 \text{ kg/m}^3$  and  $p_o = 101325 \text{ N/m}^2$  as well as  $a_o = 340 \text{ m/s}$  with both dynamic pressure formulae result, as expected, in almost the same dynamic pressure values:

$$q = \frac{1.225 (51.44)^2}{2} = 1621 \text{ N/m}^2 \quad q = 101325 \left[ \left( \frac{51.44^2}{5 (340)^2} + 1 \right)^{3.5} - 1 \right] = 1633 \text{ N/m}^2$$

At a CAS of 600 km/h (166.7 m/s), the simple formula delivers a dynamic pressure of 17020 N/m<sup>2</sup>, while the exact formula delivers a pressure of 18099 N/m<sup>2</sup>:

$$q = \frac{1.225 (166.7)^2}{2} = 17020 \text{ N/m}^2 \quad q = 101325 \left[ \left( \frac{166.7^2}{5 (340)^2} + 1 \right)^{3.5} - 1 \right] = 18099 \text{ N/m}^2$$

Both the systematic error (due to  $p_H \neq p_o$ ) and the remaining error due to compressibility are eliminated and the error-free  $v_{TAS}$  is found if the formula after 3.1/18 is applied:

$$v_{TAS} = \sqrt{\frac{2 a_H^2}{\kappa - 1} \left[ \left( \frac{q}{p_H} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]}$$

The speeds  $v_{TAS}$  and  $v_{EAS}$  differ by the systematic error so that applies to  $v_{EAS}$ :

$$v_{EAS} = v_{TAS} \sqrt{\frac{\rho_H}{\rho_o}} \quad [3.3/3]$$

### 3.3.2. Sound speed and Mach number

The formula for the sound speed is:  $a = \sqrt{\kappa \frac{p_H}{\rho_H}}$  [3.3/4]

The formula can be brought into this form with the equ.3.1/2:  $a = \sqrt{\kappa \cdot R \cdot T_H}$  [3.3/5]

According to equ. 3.3/3 and 3.1/2 applies:

$$v_{TAS} = v_{EAS} \sqrt{\frac{\rho_o}{\rho_H}} \quad \left| \quad \rho = \frac{p}{R T} \rightarrow \rho_H = \frac{p_H}{R T_H} \quad \right| \quad \rho_o = \frac{P_o}{R T_o}$$

It therefore follows:  $\frac{v_{TAS}}{v_{EAS}} = \sqrt{\frac{p_o T_H}{p_H T_o}}$  [3.3/6]

Most navigation computers have a special scale for calculating the  $v_{TAS}$  from  $v_{EAS}$ ,  $T_H$  and  $p_H$  using equ. 3.3/6, which can be used to calculate the sound speed. Since this is only dependent on the temperature, a value  $x$  must be searched in the pressure scale, opposite which the temperature is set, so that the sound speed can be read opposite the speed value 1000. With the help of equ.3.3/6 this pressure altitude  $x$  can be calculated (since the sound speed is to be calculated in km/h, 3.6  $a$  must be used):

$$\frac{3.6 a}{1000} = \sqrt{\frac{p_o T_H}{x T_o}} \quad \left| \quad a = \sqrt{R \cdot 1.4 \cdot T_H} \right.$$

$$x = \frac{p_o 1000^2}{T_o 1.4 R 3.6^2} = \frac{(101325) 1000^2}{3.6^2 (288) 1.4 (287)} = 67563 \text{ N/m}^2$$

With the help of equ.3.1/3a the corresponding altitude can be calculated from the pressure:

$$H_p = \frac{288}{0.0065} \left[ 1 - \left( \frac{67563}{101325} \right)^{0.1903} \right] = 3288 \text{ m}$$

For a temperature (SAT) of e.g.  $t = -20^\circ\text{C}$  (253 K) the sound speed  $a = 318.75 \text{ m/s}$  ( $= 1147.5 \text{ km/h} = 619.6 \text{ kt}$ ) is obtained according to equ.3.3/5. For this example the slide rule type NL must be set according to Fig.3.3/2 (not all computers of this type have the mark  $M$  - but newer instruction manuals mention a mark at 3.25 km). To obtain the speed of sound in NM/h, a second mark at 1.25 km can be found in the same way.

The Mach number is defined as:  $M = \frac{v_{TAS}}{a}$

Fig.3.3/2 also shows the possibility of Mach number calculations. Since the mark for the sound speed was chosen at 1000 the multiplication with the Mach number in the logarithmic speed scale is simply possible. As an example it is shown that at the sound speed  $a = 1147 \text{ km/h}$  (619 kt) and a Mach number of  $M = 0.8$  a  $v_{TAS}$  of 917 km/h (495 kt) results. Conversely, the Mach number can of course also be determined from the true airspeed and the sound speed.

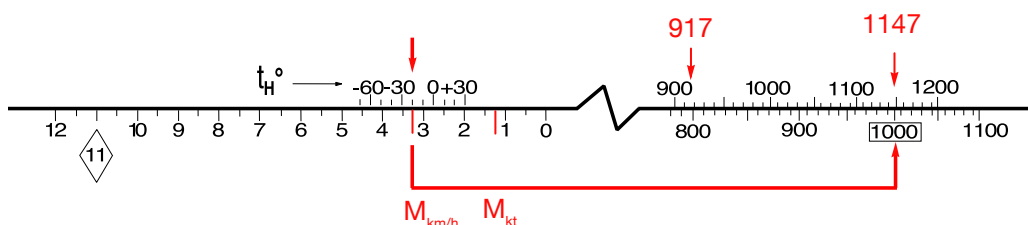
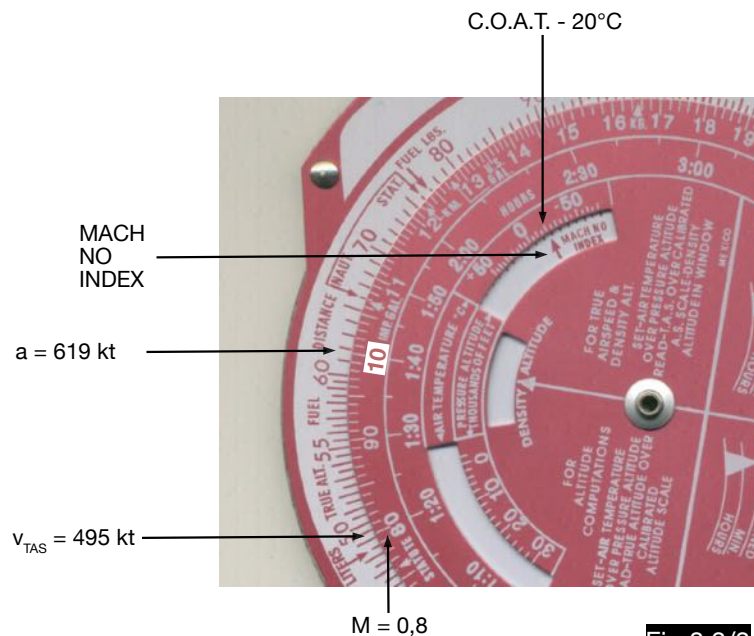


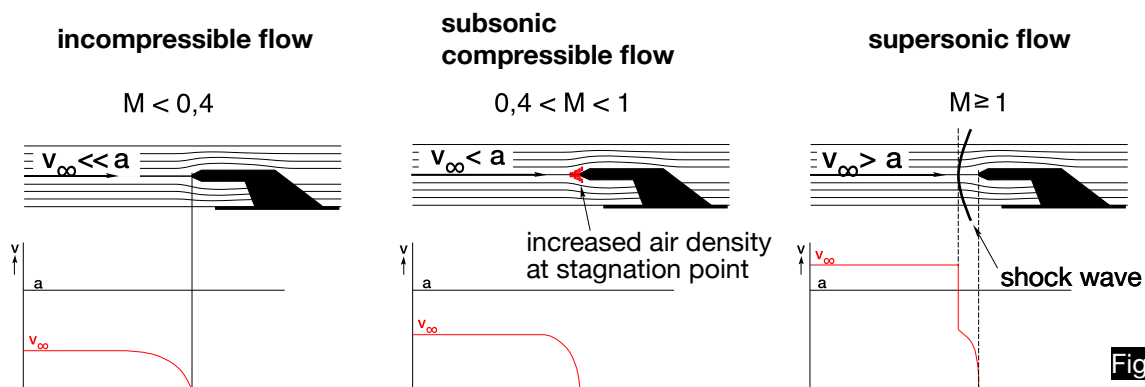
Fig.3.3/2

Most circular computers have a window in which a MACH-INDEX is placed opposite the temperature (SAT) in order to be able to read at the mark 10 in the logarithmic scale the speed of sound.

Fig.3.3/3 shows the setting of the same example on a computer of type E-6B.



The calculation of the Mach number from calibrated speed and pressure altitude according to Huber must be carried out in different ways due to the effect of the compressibility of the air. A differentiation is made between these flow conditions on a pitot tube:



### Incompressible flow

In the incompressible flow the equation for dynamic pressure is valid  $q = \frac{\rho}{2} v^2$

For the Mach number  $M = \frac{v_{TAS}}{a}$  you get with  $\rho_H = \frac{p_H}{R T_H}$  and  $a = \sqrt{\kappa R T}$

$$v_{CAS} = \sqrt{\frac{2q}{\rho_0}} \quad \left| \quad v_{TAS} = \sqrt{\frac{2q}{\rho_H}} \quad \right| \quad v_{TAS} = v_{CAS} \sqrt{\frac{\rho_0}{\rho_H}} \longrightarrow M = v_{CAS} \sqrt{\frac{\rho_0}{\kappa p_H}} \quad [3.3/8]$$

The Mach number is therefore a function of the dynamic pressure ( $v_{CAS}$ ) and the pressure altitude ( $p_H$ ). The Mach number  $M = 0.25$  results for the following example:

pressure altitude PA	25400 ft (7742 m)
pressure $p_H$	36906 N/m <sup>2</sup>
$v_{CAS}$	100 kt (51.4 m/s)

Analytical solution:

$$M = v_{CAS} \sqrt{\frac{\rho_0}{\kappa p_H}} = 51.4 \sqrt{\frac{1.225}{(1.4) 36906}} = 0.2502$$

### Subsonic compressible flow

In the compressible subsonic flow, equ. 3.1/19 must be applied for the Mach number M:

$$M = \sqrt{\frac{2}{\kappa - 1} \left[ \left( \frac{q}{p_H} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]}$$

The Mach number  $M = 0.8$  results for the following example:

pressure altitude PA	30600 ft (9327 m)
pressure $p_H$	29285 N/m <sup>2</sup>
VCAS	300 kt (154.3 m/s)

### Analytical solution:

For the dynamic pressure  $q$  equ.3.3/2 applies:

$$q = p_0 \left[ \left( \frac{v_{CAS}^2 (\kappa - 1)}{2 a_0^2} + 1 \right)^{\frac{\kappa}{\kappa - 1}} - 1 \right] \quad q = 101325 \left[ \left( \frac{154.3^2}{5 \times 340^2} + 1 \right)^{3.5} - 1 \right] = 15375 \text{ N/m}^2$$

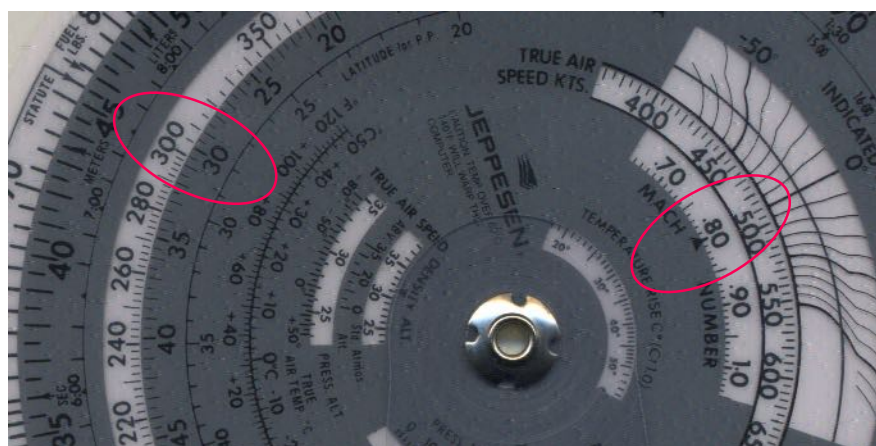
For the Mach number, it results:

$$M = \sqrt{5 \left[ \left( \frac{15375}{29285} + 1 \right)^{0.286} - 1 \right]} = 0.80$$

The fact that the formulas of the compressible flow are also applicable for the incompressible is proven by this recalculation of the example used above:

$$q = 101325 \left[ \left( \frac{51.4^2}{5 \times 340^2} + 1 \right)^{3.5} - 1 \right] = 1630 \text{ N/m}^2 \quad M = \sqrt{5 \left[ \left( \frac{1630}{36905} + 1 \right)^{0.286} - 1 \right]} = 0.2493$$

### Solution by means of computer CR:



**Fig.3.3/5**



## Supersonic flow

In the supersonic flow the Rayleigh pitot tube formula equ.3.3/9 applies:

$$\frac{q}{p_H} = \left[ \frac{(\kappa + 1)^2 M^2}{4 \kappa M^2 - 2(\kappa - 1)} \right]^{\frac{\kappa}{(\kappa - 1)}} \left[ \frac{1 - \kappa + 2 \kappa M^2}{(\kappa + 1)} \right]^{-1} \quad [3.3/9]$$

For  $M = 1.5$  one gets:

$$\frac{q}{p_H} = \left[ \frac{2.4^2 \times 1.5^2}{4 \times 1.4 \times 1.5^2 - 0.8} \right]^{3.5} \left[ \frac{1 - 1.4 + 2 \times 1.4 \times 1.5^2}{2.4} \right]^{-1} = 2.412$$

Using equ.3.3/19 valid for  $M < 1$  would lead to the following result (error 3.2 %):

$$M = \sqrt{\frac{2}{\kappa - 1} \left[ \left( \frac{q}{p_H} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]} = 5 \left[ \left( 2.42 + 1 \right)^{0.286} - 1 \right] = 1.45$$

This discrepancy confirms that equation 3.3/19 is not applicable to Mach numbers  $M > 1$ . Since the Mach number scale of the Huber calculators reaches up to  $M = 1.8$ , it can be assumed that it is based on equ. 3.3/9 (see Fig. 3.3/5a).



**Fig.3.3/5a**

### 3.3.3. The systematic error of airspeed indicators

#### 3.3.3.1. General

The systematic error of the airspeed indicator is eliminated in navigation computers by multiplication with a correction factor  $f$  in a logarithmic speed scale. As a result of the decrease in air density, the error, which is dependent on pressure  $p_H$  and temperature  $T_H$ , occurs at all altitudes above zero. However, the indication of the temperature is falsified as a result of the temperature rise, which itself depends on the true airspeed. Most computers therefore require at least two operations. The first one uses an estimated  $v_{TAS}$  to determine an approximate temperature. This is then used to correct the  $v_{CAS}$ . With the now calculated  $v_{TAS}$  a nearly exact temperature rise can be found and with this an even more exact true airspeed can be determined. This is not necessary for computers that use the Huber method, since it works with the indicated temperature. The altitude correction scale is constructed by considering the temperature as constant and the temperature correction scale is constructed by considering the altitude as constant.

Influence of pressure (temperature is considered constant):

According to equ.3.3/8 applies:

$$\left. \begin{aligned} v_{TAS} &= \sqrt{\frac{2q}{\rho_H}} \\ v_{EAS} &= \sqrt{\frac{2q}{\rho_0}} \end{aligned} \right\} v_{TAS} = v_{EAS} \sqrt{\frac{\rho_0}{\rho_H}} \quad \left| \quad \rho_H = \frac{p_H}{R T_H} \right.$$

$$v_{TAS} = v_{EAS} \sqrt{\frac{\rho_0 R T_H}{p_H}}$$

For the altitudes 0 and 10 km applies then:

$$v_{TAS 0} = v_{EAS} \sqrt{\frac{\rho_0 R T_0}{p_0}} \quad v_{TAS 10} = v_{EAS} \sqrt{\frac{\rho_0 R T_{10}}{p_{10}}}$$

From  $T$ ,  $\rho_0$  and  $R = \text{const.}$  follows:

$$\frac{v_{TAS 10}}{v_{TAS 0}} = \sqrt{\frac{p_0}{p_{10}}} \longrightarrow v_{TAS 10} = v_{TAS 0} \sqrt{\frac{p_0}{p_{10}}}$$

$$f_{10} = \sqrt{\frac{p_0}{p_{10}}} \quad \left| \begin{array}{l} p_0 = 101325 \text{ N / m}^2 \\ p_{10} = 26436 \text{ N / m}^2 \\ v_{TAS 0} = v_{EAS} \end{array} \right| f_{10 \text{ km}} = \sqrt{\frac{101325}{26436}} = 1.957$$

For the influence of 10 km altitude one receives at constant temperature:

$$v_{TAS 10} = 1.957 v_{EAS}$$

In the same way, the factor  $f_{5 \text{ km}} = 1.369$  is found for the point 5 km in the altitude scale.

Fig.3.3/6 illustrates in a detail of a NL- calculator how in the logarithmic speed scale the  $v_{EAS}$  is multiplied by a factor of 1.957 when the pressure altitude is increased by 10 km and by a factor of 1.369 when the pressure altitude is increased by 5 km.

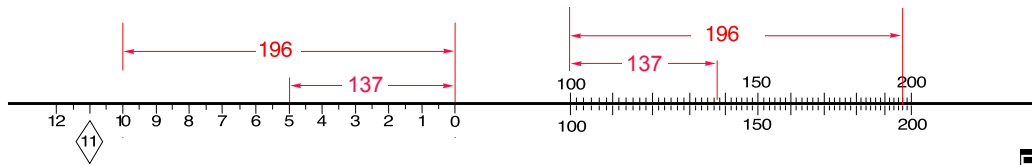


Fig.3.3/6

Influence of temperature (pressure is considered constant):

$$v_{TAS} = v_{EAS} \sqrt{\frac{\rho_0 R T_H}{p_H}}$$

From  $p_H$ ,  $\rho_0$  and  $R = \text{const.}$  follows:

$$v_{TAS} = v_{EAS} \sqrt{T_H}$$

For the temperatures  $+15^\circ\text{C}$  (288 K) and  $-60^\circ\text{C}$  (213 K) applies then:

$$v_{TAS +15} = v_{EAS} \sqrt{288} \quad \left| \quad v_{TAS -60} = v_{EAS} \sqrt{213} \right.$$

For the influence of  $75^\circ$  temperature one receives at constant pressure:

$$\frac{v_{TAS +15}}{v_{TAS -60}} = \sqrt{\frac{288}{213}} = 1.163$$

### 3.3.3.2. Correction of the airspeed indicators for the systematic error with different computers

#### 3.3.3.2.1. Computer DR-2

Fig.3.3/7 illustrates in a detail of a DR-2 calculator how, in the logarithmic speed scale, the  $v_{EAS}$  is multiplied by a factor of 1.16 when the temperature is increased by  $75^\circ\text{C}$  and by a factor of 1.96 when the altitude is increased by 10 km.

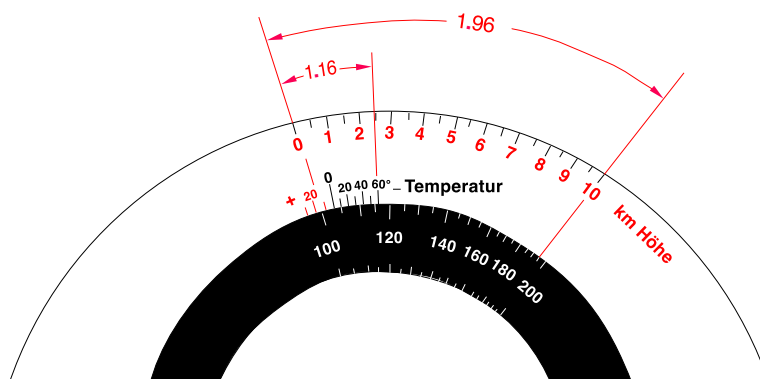


Fig.3.3/7

### 3.3.3.2.2. Computer Junkers ADJUVATOR

The correction of the  $v_{EAS}$  for the systematic error can also be done via diagrams, which are used with the ADJUVATOR computer. This example is shown in the figures:

$v_{EAS}$  420 km/h  
pressure altitude PA 6 km  
C.O.A.T. 0°C

Analytically was calculated the air density according to equ. 3.1/2, the density altitude according to equ. 3.1/10 and the  $v_{TAS}$  according to equ. 3.3/18:

air density  $\rho$  0.602 kg/m<sup>3</sup>  
density altitude DA 6.825 km  
 $v_{TAS}$  599.1 km/h

The calculation starts with the determination of the air density resp. the density altitude with the help of Fig.3.3/9 (at the intersection of the temperature line 0°C with the line of the pressure altitude 6 km point A is found). At the intersection of the vertical line with the speed line 420 km/h in Fig.3.3/8 the true airspeed  $v_{TAS} = 600$  km/h is determined.

In the computer (see Fig.3.3/10) both diagrams are shown in a circle. The application or reading is analogous to the use of the diagrams. For easier handling the computer has a rotatable pointer with altitude scale.

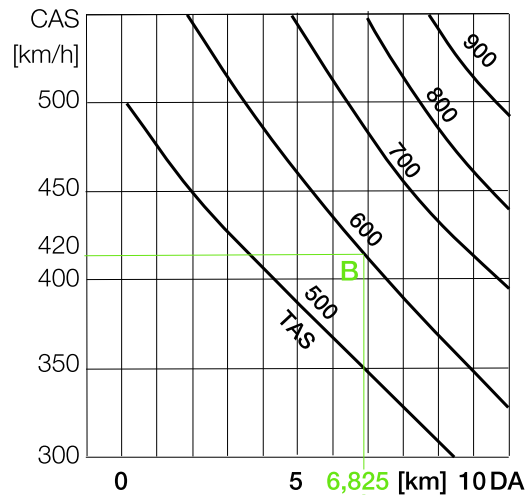


Fig.3.3/8

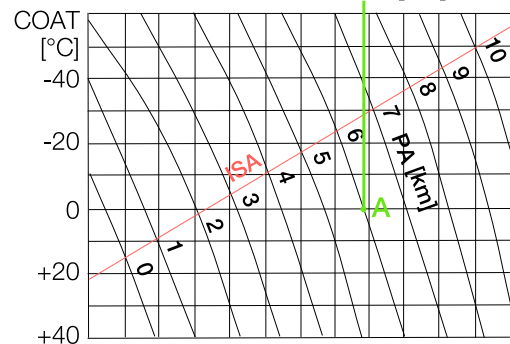


Fig.3.3/9

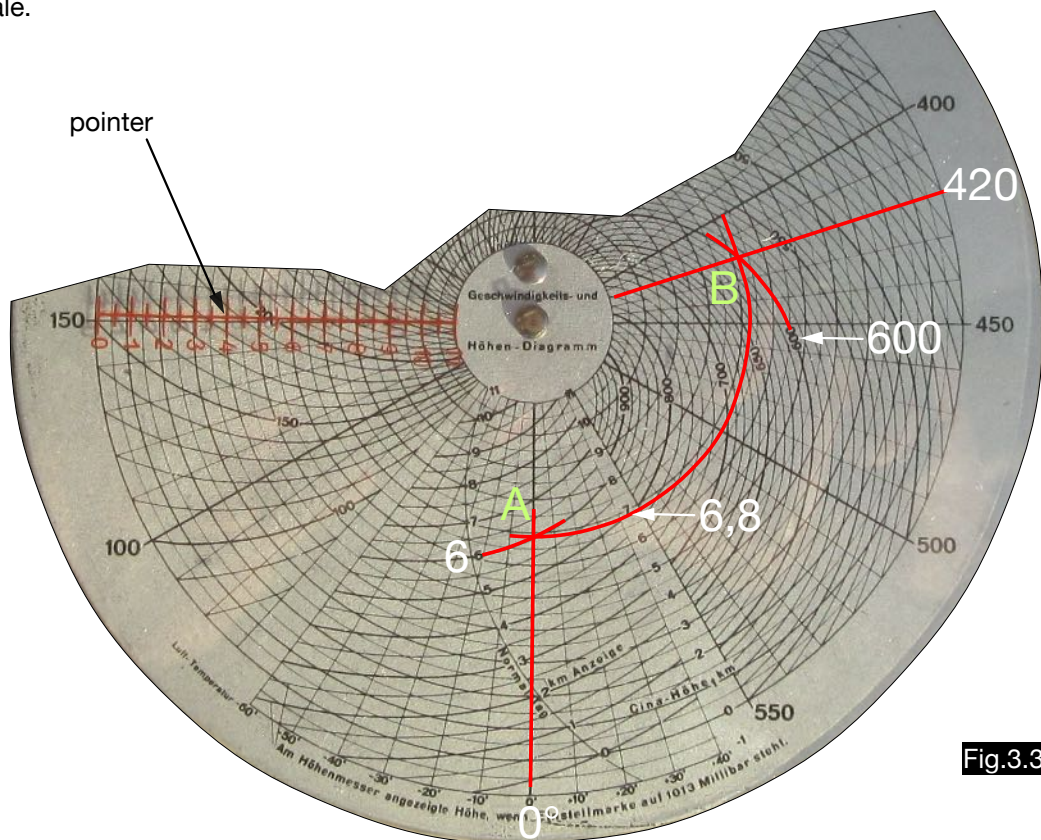


Fig.3.3/10

#### Note:

The computer provides no means to consider the error due to compressibility of the air. For example, at a  $v_{CAS} = 550$  km/h and a pressure altitude of 8 km (both values for which the computer is intended), this already leads to an error of + 25 km/h. A further source of error is the difficulty in determining the exact temperature, since the temperature rise is a function of the still unknown  $v_{TAS}$ .

### 3.3.3.2.3. Circular computer

Fig.3.3/11 shows the following example:

VEAS	100 kt
pressure altitude	10 km = 32808 ft
C.O.A.T.	- 50°C
VTAS	172.8 kt
density altitude	9988 m = 32769 ft

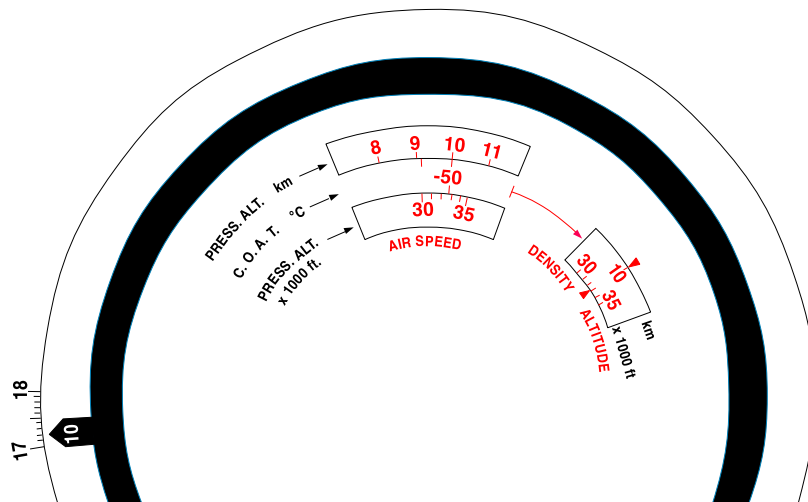


Fig.3.3/11

### 3.3.3.2.4. Airspeed indicator with correction scale

Fig.3.3/12 shows the following example:

VIAS	160 m.p.h.
pressure altitude	20000 ft
C.O.A.T.	+ 10°C
VTAS	233.9 m.p.h.

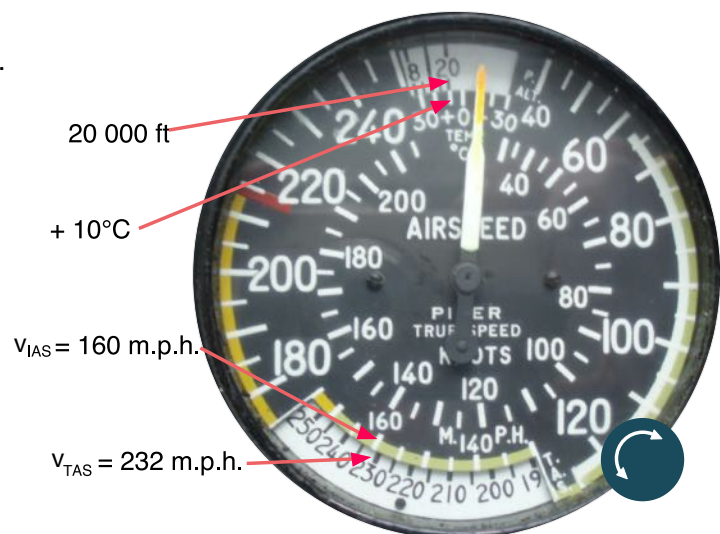


Fig.3.3/12



### 3.3.3.2.5. Computer using the procedure according to Huber

The procedure in which the true airspeed can be calculated with the indicated temperature is explained using a type CR calculator in the following example:

pressure altitude	10000 ft
V <sub>CAS</sub>	356 kt
indicated temperature t <sub>TAT</sub>	0°C (273 K)
temp. recovery coeff. c <sub>T</sub>	0.8
Mach number*)	0.637
V <sub>TAS</sub>	397.5 kt

According to equ.3.1/13 applies:

$$T_{SAT} = \frac{T_{TAT}}{(1 + 0.2 M^2)} = \frac{273}{(1 + 0.2 \times 0.637^2)} = 252.5 \text{ K}$$

The temperature rise  $\Delta t$  is therefore:  $\Delta t = T_{TAT} - T_{SAT} = 273 - 252.5 = 20.5^\circ\text{C}$

At a c<sub>T</sub> value of 0.8, only 16.4°C of the temperature rise is displayed (20.5 × 0.8). The actual temperature T<sub>SAT</sub> is therefore 4.1°C higher, i.e. 256.6 K (252.5 + 4.1). This results in a sound speed of 321.1 m/s:

$$a = \sqrt{\kappa R T_H} = \sqrt{1.4 \times 287 \times 256.6} = 321.1 \text{ m/s}$$

The v<sub>TAS</sub> is achieved by the formula v<sub>TAS</sub> = a M = 321.1 × 0.637 = 204.5 m/s = 397.5 kt

Fig.3.3/13 shows the calculation of this example.

After PA = 10,000 ft opposite CAS = 356 kt has been set, the Mach number M = 0.64 and the temperature rise for c<sub>T</sub> = 1 of 20.5°C are indicated. Then place the line c<sub>T</sub> = 0.8 of the rotatable pointer over the intersection of the line 0°C (INDICATED TEMPERATURE) with the spiral line and then read the v<sub>TAS</sub> = 399 kt below.

#### Note:

With the Huber method, both errors, the systematic and the remaining error due to compressibility, are eliminated in one operation.

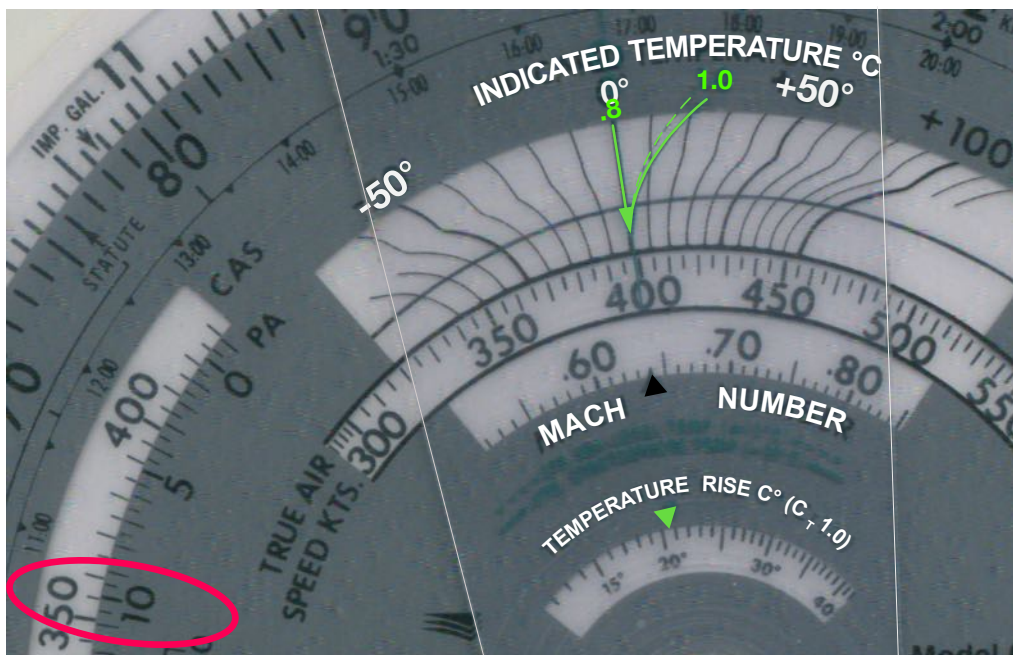


Fig.3.3/13

\*) Calculated according to equ.3.1/19.

### 3.3.3.2.6. Quasi density-compensated airspeed indicator

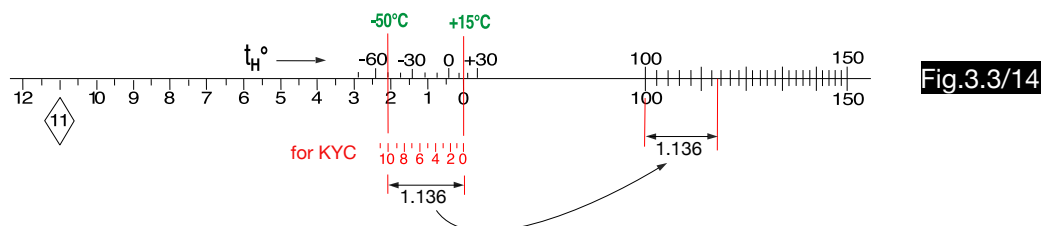
Since the systematic error of the airspeed indicator at high altitudes assumes considerable values airspeed indicators have been developed that have a second pointer whose systematic error is significantly lower (in ISA even zero). These airspeed indicators therefore additionally require the input of the air density (i.e. temperature and pressure) in the flight altitude. While the input of the parameter  $p_H$  via an additional measuring box for the static pressure can be solved constructively with relatively simple means, the input of the temperature into mechanical measuring instruments is difficult. The temperature is therefore considered to be normal atmospheric in the design of the device, i.e. it is replaced by the value that belongs to the ISA pressure altitude. Under ISA conditions, therefore, no systematic error of the true airspeed pointer occurs. The NL slide rules therefore have a second pressure scale marked with the Cyrillic letters KYC (which stands for комбинированный указатель скорости = dual airspeed indicator), which is used to correct the indication of the additional pointer when the temperature deviates from the ISA. These relationships apply to the indication of the additional pointer  $v_{KYC}$  and the actual speed  $v_{TAS}$ :

$$v_{KYC} = \sqrt{\frac{\rho_0 \cdot R \cdot T_{H_{ISA}}}{p_H}} \quad \left| \quad v_{TAS} = \sqrt{\frac{\rho_0 \cdot R \cdot T_H}{p_H}} \right| \longrightarrow v_{TAS} = v_{KYC} \sqrt{\frac{T_H}{T_{H_{ISA}}}} \quad [3.3/8]$$

As described under 3.3.3, the correction factor  $f$  is constructed for the 10 km mark by using the value 288 (starting point of the additional KYC scale) for  $T_H$  and the value 223 (ISA temperature at 10 km) for  $T_{H_{ISA}}$ :

$$v_{TAS} = v_{KYC} \sqrt{\frac{288}{223}} = 1.136 = f$$

Fig.3.3/14 illustrates in a detail of an NL calculator how in the logarithmic speed scale the  $v_{KYC}$  is multiplied by a factor of 1.136 when the pressure altitude is increased from 0 to 10 km in the KYC scale.



In Fig.3.3/14 it can be seen that the additional scale KYC also indicates (although relatively difficult to read) the normal atmospheric temperature for each altitude (at 10 km altitude e.g.  $-50^{\circ}\text{C}$ ) when the  $15^{\circ}$ -degree-mark is set opposite to zero.

#### Note:

It is not necessary to correct the  $v_{KYC}$  indication for the error due to the compressibility of the air, as this only depends on the dynamic pressure  $q$  and the pressure  $p_H$ . As these values are available to the instrument, the correction for this error is already carried out during its calibration resp. construction.

### 3.3.4. The elimination of the compressibility error of the airspeed indicator

#### 3.3.4.1. General

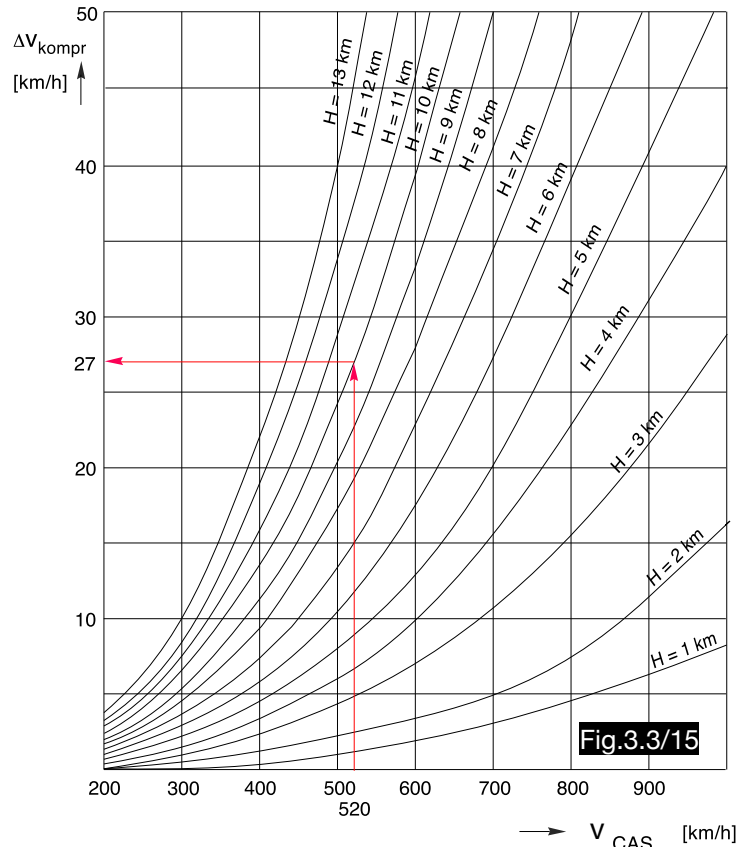
According to 3.3.1, the compressibility error is the difference between  $v_{CAS}$  and  $v_{EAS}$ . This error can be eliminated by an additive correction ( $v_{EAS} = v_{CAS} - \Delta v_{compr}$ ) or by a factor ( $v_{EAS} = v_{CAS} \times f_{compr}$ ). The additive correction must be made at  $v_{CAS}$ . One can correct multiplicatively the  $v_{CAS}$  as well as the systematic-error-corrected  $v_{CAS}$ , which is called  $v_{DAS}$  in the computer IWA 11092.

#### 3.3.4.2 Determination of the error $\Delta v_{compr}$ using a diagram

The error  $\Delta v_{compr}$  is shown in Fig.3.3/15, in which the following example was drawn:

$v_{CAS}$  520 km/h  
pressure altitude 10 km  
 $\Delta v_{compr}$  27.36 km/h

This method is used, for example, with the computers of the NL series. The disadvantage is that the diagram is part of the manual. On the NL-10 MK computer it is at least shown on the cover.



The additive correction  $\Delta v_{compr}$  can be calculated as follows using equ.3.3/2 and 3.1/18:

$v_{CAS} = 520 \text{ km/h} = 144.4 \text{ m/s}$	$H = 10 \text{ km}$	$\rho_0 = 1.225 \text{ kg/m}^3$
$p_{HISA} = 26436 \text{ N/m}^2$	$T_{HISA} = 223.15 \text{ K}$	$a_H = 20.04 \sqrt{223.15} = 299.36 \text{ m/s}$
		$\rho_{HISA} = 0.413 \text{ kg/m}^3$

$$q_{520} = p_0 \left[ \left( \frac{v_{CAS}^2 (\kappa - 1)}{2a_0^2} + 1 \right)^{\frac{\kappa}{\kappa - 1}} - 1 \right] \longrightarrow q_{520} = 101325 \left[ \left( \frac{144.4^2}{5 \cdot 340^2} + 1 \right)^{3.5} - 1 \right] = 13381 \text{ N/m}^2$$

$$v_{TAS} = \sqrt{\frac{2a^2}{\kappa - 1} \left[ \left( \frac{q}{\rho_H} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]} \longrightarrow v_{TAS} = \sqrt{5 \cdot 299^2 \left[ \left( \frac{13381}{26436} + 1 \right)^{0.286} - 1 \right]} = 235.6 \text{ m/s}$$

$$v_{EAS} = v_{TAS} \sqrt{\frac{\rho_H}{\rho_0}} = 235.6 \sqrt{\frac{0.413}{1.225}} = 136.8 \text{ m/s}$$

$$\Delta v_{kompr} = v_{CAS} - v_{EAS} = 144.4 - 136.8 = 7.6 \text{ m/s} = 27.36 \text{ km/h}$$

**Note:**

The remaining error due to compressibility  $\Delta v_{\text{compr}}$  is therefore determined via the  $v_{\text{EAS}}$ . This depends only on pressure altitude  $p$  and dynamic pressure  $q$ :

$$\Delta v_{\text{compr}} = v_{\text{CAS}} - v_{\text{EAS}}$$

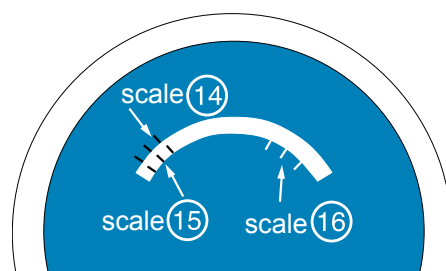
$$v_{\text{EAS}} = v_{\text{TAS}} \sqrt{\frac{\rho_H}{\rho_0}}$$

$$v_{\text{EAS}} = \sqrt{\frac{\rho_H}{\rho_0} \frac{2 a^2}{(\kappa - 1)} \left[ \left( \frac{q}{p_H} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]} \quad \left| \quad a = \sqrt{\kappa \frac{p_H}{\rho_H}} \rightarrow v_{\text{EAS}} = \sqrt{\frac{2 \kappa p_H}{\rho_0 (\kappa - 1)} \left[ \left( \frac{q}{p_H} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]} \right.$$

However, this does not generally apply to the compressibility correction, which of course depends on the character of the flow (see Fig.3.3/4) and thus on the Mach number. This is included in the calculation via  $v_{\text{CAS}}$ , because the error due to compressibility for the altitude zero and the temperature 288 K is already eliminated during the construction resp. calibration of the airspeed indicator.

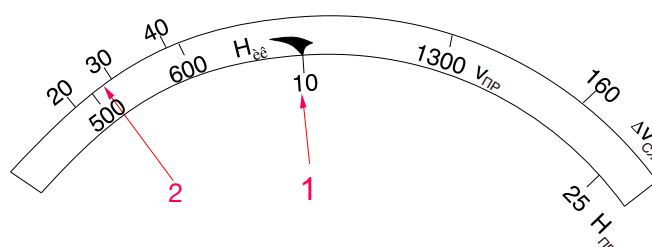
### 3.3.4.3. Determining the error $\Delta v_{\text{compr}}$ with the computer NRK-2

The diagram (Fig.3.3/15) for the additive correction  $\Delta v_{\text{compr}}$  is shown in the manual of the NRK-2 calculator and additionally incorporated into the calculator. The example discussed under 3.3.4.2. is calculated with the aid of scales 14 - 16 (see Fig.3.3/16 and for an enlarged view Fig.3.3/17).



**Fig.3.3/16**

1. The triangle symbol in scale 15 is placed opposite the pressure altitude 10 km in scale 16.
2. Above the  $v_{\text{CAS}} = 520$  km/h in scale 15 the value  $\Delta v_{\text{compr}} = 27$  km/h is read in scale 16.



**Fig.3.3/17**

3.3.4.4. Determination of the correction factor  $f_{\text{comp}}$

Computers that are designed for higher flight speeds often have a table with the correction factors  $f_{\text{comp}}$  on the diagram slide. In Fig.3.3/18 - 21 some examples are given. The table of Fig.3.3/21 is printed in the manual and Fig.3.3/22 is located in the centre of the calculator. It is remarkable that the tables of the different manufacturers make the same selection for the values for pressure altitude and  $v_{\text{CAS}}$ . Of course the factors are identical, unless there are two minor exceptions in Fig.3.3/21 and 22 (see marking).

PRESS. ALT FEET	F CORRECTION FACTORS FOR TAS							
	CALIBRATED AIRSPEED KNOTS							
	200	250	300	350	400	450	500	550
10.000	1.0	1.0	.99	.99	.98	.98	.97	.97
20.000	.99	.98	.97	.97	.96	.95	.94	.93
30.000	.97	.96	.95	.94	.92	.91	.90	.89
40.000	.96	.94	.92	.90	.88	.87	.87	.86
50.000	.93	.90	.87	.86	.84	.84	.84	.84

DIRECTIONS  
USE CALIBRATED AIRSPEED AND PRESS. ALT. TO OBTAIN F FACTOR. MULTIPLY F FACTOR BY TAS OBTAINED WITH COMPUTER TO OBTAIN TAS CORRECTED FOR COMPRESSIBILITY.

Fig.3.3/18

COMPRESSIBILITY CORRECTION								
Pressure Altitude	F correction factors for TAS calibrated airspeed in knots							
	200	250	300	350	400	450	500	550
10,000 Feet	1.00	1.00	.99	.99	.98	.98	.97	.97
20,000 Feet	.99	.98	.97	.97	.96	.95	.94	.93
30,000 Feet	.97	.96	.95	.94	.92	.91	.90	.89
40,000 Feet	.96	.94	.92	.90	.88	.87	.87	.86
50,000 Feet	.93	.90	.87	.86	.84	.84	.84	.84

Use calibrated airspeed and Pressure Altitude to obtain F factor.  
Multiply F factor by TAS to obtain the TAS corrected for compressibility.

Fig.3.3/19

PRESS. ALT.	F CORRECTION FACTORS FOR TAS							
	CALIBRATED AIRSPEED KNOTS							
	200	250	300	350	400	450	500	550
10,000	1.0	1.0	.99	.99	.98	.98	.97	.97
20,000	.99	.98	.97	.97	.96	.95	.94	.93
30,000	.97	.96	.95	.94	.92	.91	.90	.89
40,000	.96	.94	.92	.90	.88	.87	.87	.86
50,000	.93	.90	.87	.86	.84	.84	.84	.84

DIRECTIONS  
USE CALIBRATED AIRSPEED AND PRESS. ALT. TO OBTAIN F FACTOR.  
MULTIPLY F FACTOR BY TAS OBTAINED WITH COMPUTER  
TO OBTAIN TAS CORRECTED FOR COMPRESSIBILITY.

Fig.3.3/20

Pressure altitude in ft.	Rectified air speed in Kt.							
	200	250	300	350	400	450	500	550
10.000	1.0	1.0	0.99	0.99	0.98	0.98	0.97	0.97
20.000	0.99	0.98	0.97	0.97	0.96	0.95	0.94	0.93
30.000	0.97	0.96	0.95	0.94	0.94	0.91	0.90	0.89
40.000	0.96	0.94	0.92	0.90	0.90	0.87	0.87	0.86
50.000	0.93	0.90	0.87	0.86	0.86	0.84	0.84	0.84

Fig.3.3/21

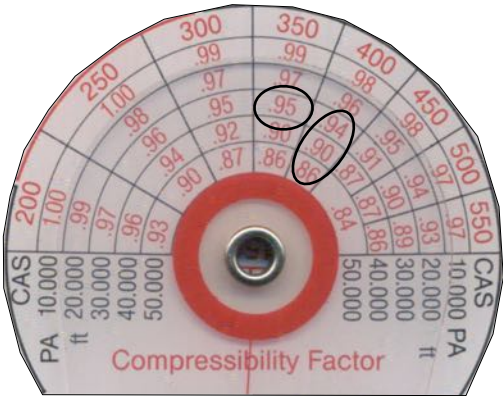


Fig.3.3/22



The calculation of the factors is done in the same way as described under 3.3.4.2. For a pressure altitude of 40,000 ft and a  $v_{CAS}$  of 400 kt (206 m/s) one obtains a dynamic pressure of 28,515 N/m<sup>2</sup> and then with this the  $v_{EAS}$  = 180.06 m/s. The factor  $f_{compr}$  is calculated as follows:

$$v_{CAS} = \sqrt{\frac{2a_o^2}{\kappa - 1} \left[ \left( \frac{q}{p_o} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]} \longrightarrow q = p_o \left[ \left( \frac{v_{CAS}^2 (\kappa - 1)}{2a_o^2} + 1 \right)^{\frac{\kappa}{\kappa - 1}} - 1 \right] \quad \left| \begin{array}{l} a_o = 340 \text{ m/s} \\ p_o = 101325 \text{ N/m}^2 \end{array} \right.$$

$$\longrightarrow q_{300} = 101325 \left[ \left( \frac{206^2}{5 \times 340^2} + 1 \right)^{3.5} - 1 \right] = 28515 \text{ N/m}^2$$

$$\longrightarrow v_{EAS} = \sqrt{\frac{2 \kappa p_H}{\rho_o (\kappa - 1)} \left[ \left( \frac{q}{p_H} + 1 \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]} \quad \left| \begin{array}{l} p_H = 18730 \text{ N/m}^2 \\ \rho_o = 1.225 \text{ kg/m}^3 \end{array} \right.$$

$$\longrightarrow v_{EAS} = \sqrt{\frac{2 \times 1.4 \times 18730}{1.225 \times 0.4} \left[ \left( \frac{28515}{18730} + 1 \right)^{0.286} - 1 \right]} = 180.06 \text{ m/s}$$

$$f_{compr} = \frac{v_{EAS}}{v_{CAS}} = \frac{180}{206} = 0.874$$

Tables 3.3/18 - 20 show the rounded value 0.88.

### 3.3.4.5. Determining of the factor $f_{compr}$ with the computer Nr. 611

Fig.3.3/23 shows the principle of the procedure of the computer 611 (here called „Kompr. Berichtg.“ = compressibility correction).

The speed corrected for temperature and pressure altitude (in the example set here 1000 km/h) is placed opposite the red arrow and the  $v_{TAS}$  = 930 km/h is shown under the 1000 km/h mark. This is equivalent to multiplication by the factor  $f_{compr}$  = 0.93. The other speeds displayed on the computer are listed in Tab.3.3/1.

v	[km/h]	400	600	800	1000
	[kt]	216	324	432	540
$f_{compr}$		0.99	0.97	0.95	0.93

Tab.3.3/1



Fig.3.3/23

Tab.3.3/2 shows the correction factors  $f_{\text{compr}}$  for selected altitudes and speeds in km and km/h, in which the factors of the calculator 611 are highlighted.

Pressure altitude [km]	Calibrated airspeed [km/h]						
	400	500	600	700	800	900	1000
3	1.0	0.99	0.99	0.98	0.98	0.97	0.96
6	0.99	0.98	0.97	0.96	0.95	0.94	0.93
9	0.97	0.96	0.95	0.93	0.92	0.90	0.89
12	0.95	0.93	0.90	0.88	0.87	0.86	0.85
15	0.92	0.89	0.87	0.86	0.84	0.84	0.84

Tab.3.3/2

This clearly indicates that the correction factors of this calculator are only valid for the pressure altitude 6000 m. It is possible that the errors were neglected or that other altitudes were taken into account approximately by a rule of thumb.

### 3.3.4.6. Determining of the factor $f_{\text{compr}}$ with the calculator IWA 11092

The IWA 11092 calculator has a rotatable pointer with a logarithmic scale with the factors 0.84.....1.0 which is used to allow multiplication by the factor  $f_{\text{compr}}$  in the speed scale. The table with these factors (see Fig.3.3/22) is part of the calculator. This example explains the handling with the aid of Fig.3.3/24 and 25 (enlarged picture):

$V_{\text{CAS}}$  350 kt  
 pressure altitude 20,000 ft  
 C.O.A.T. - 40°C

The correction of the  $V_{\text{CAS}}$  by the systematic error leads to an intermediate result which is called density airspeed (DAS) in the manual. The calculation is done in the usual way by setting temperature and altitude in the AIRSPEED window opposite each other. In the speed scale, the value  $V_{\text{DAS}} = 464$  kt is determined opposite  $V_{\text{CAS}} = 350$  kt (analytically calculated: 464.3 kt). This speed multiplied by the factor  $f_{\text{compr}} = 0.97$  results in  $V_{\text{TAS}} = 450$  kt (analytically calculated: 448.8 kt). In addition,  $V_{\text{EAS}} = V_{\text{CAS}} \times 0.97 = 339.5$  kt can be read on the same scale. In the scale TEMPERATURE RISE one reads for  $V_{\text{TAS}} = 450$  kt a value around 26°, which speaks for the coefficient  $C_T = 1$  (see equ.3.1/15).

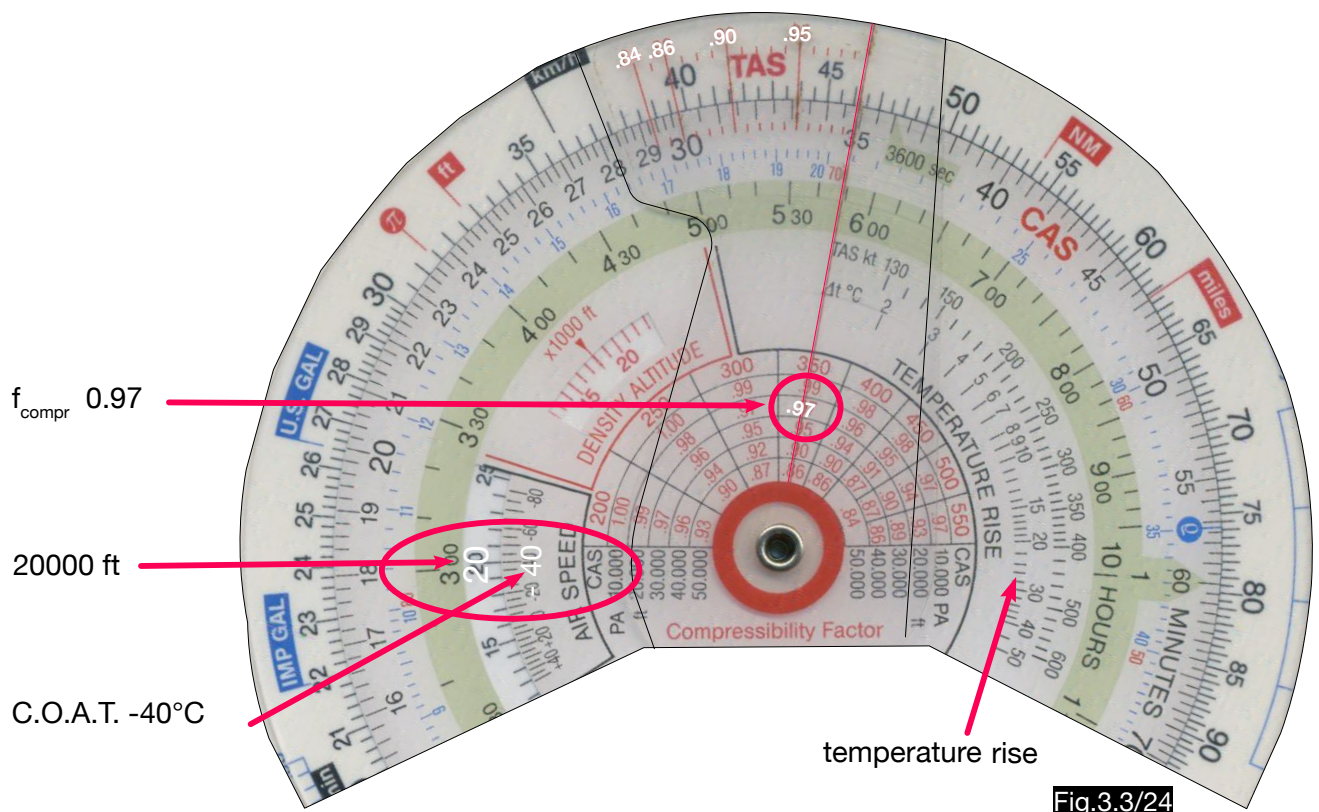
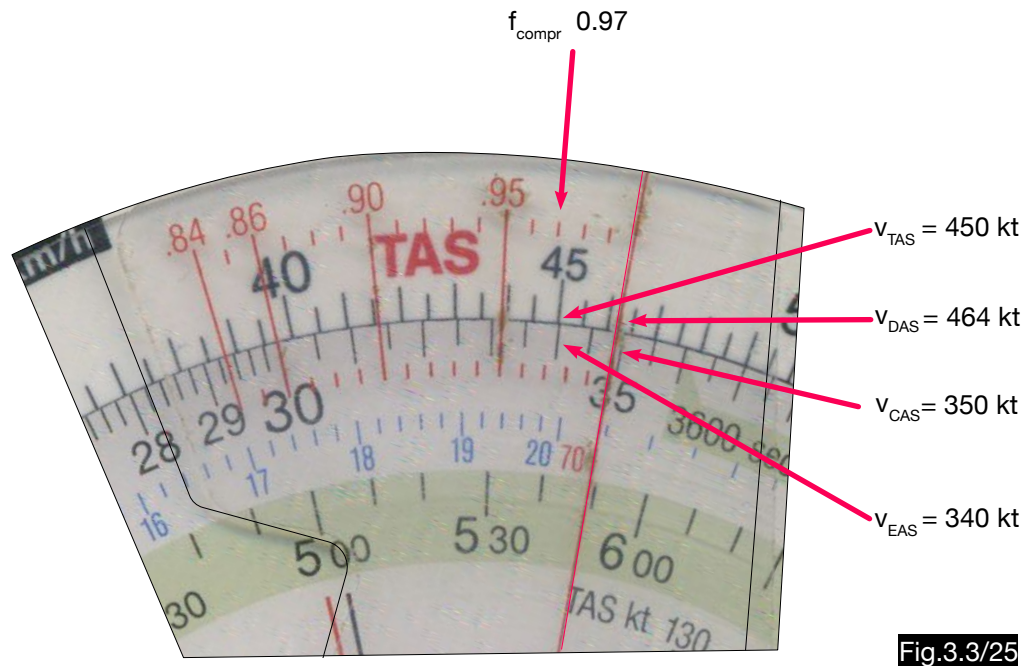


Fig.3.3/24



### 3.3.4.7. Compressibility correction with the computer ARC-2

The following example describes the handling of the computer ARC-2 for the calculation of the true airspeed considering the remaining error due to the compressibility of the air with the help of an approximate solution:

$V_{CAS}$	300 kt
pressure altitude	30,000 ft
C.O.A.T.	- 40°C

- 1 The value C.O.A.T = - 40°C is set in the AIRSPEED window opposite the pressure altitude 30,000 ft (see Fig.3.3/26).
- 2 An uncorrected  $v_{TAS} = 495$  kt is read from the outer scale opposite the 30 ( $v_{CAS} = 300$  kt). The figure 495 is divided by one hundred, three is subtracted from the result and the result is 1.95 ( $4.95 - 3 = 1.95$ ). This procedure is described next to the COMPRESSIBILITY CORRECTION window (TAS/100 - 3 divisions).
- 3 In the window COMPRESSIBILITY CORRECTION the arrow is moved from position 42 by the value 1.95 to position 43.95 (see Fig.3.3/27).
- 4 Opposite the 30 ( $v_{CAS} = 300$  kt) a  $v_{TAS} = 479$  kt can now be read (see Fig.3.3/27). The exact result is  $v_{TAS} = 468$ , so the approximate solution here has an error of 2.4 % ( $479 : 468 = 1.024$ ).



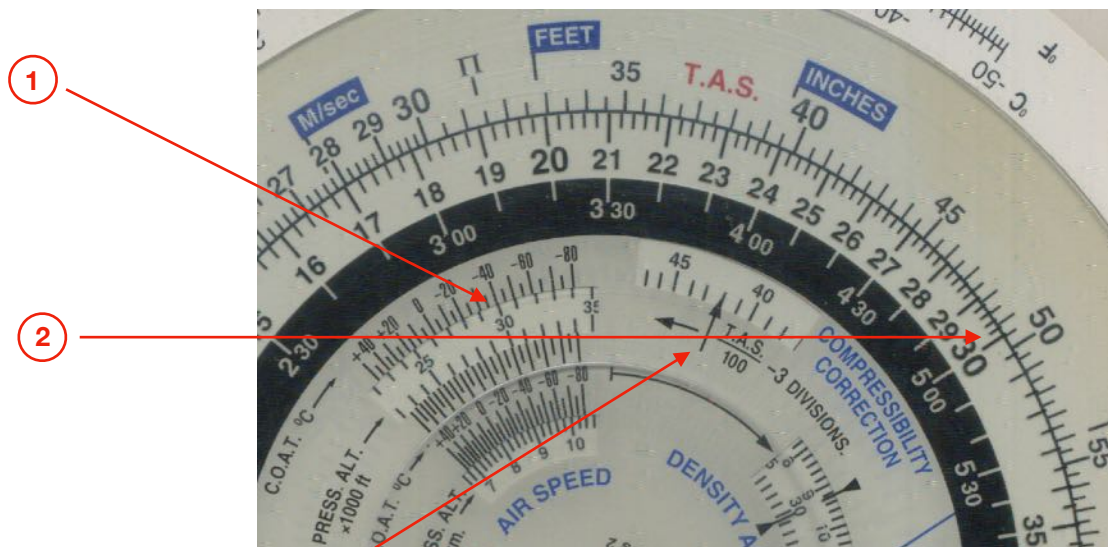


Fig.3.3/26

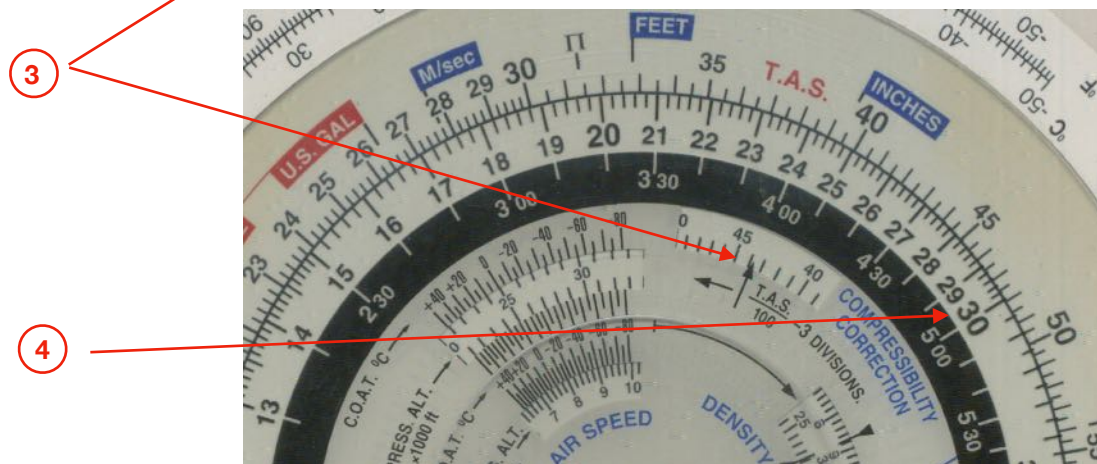


Fig.3.3/27

### 3.3.4.8. Compressibility correction with the computer B-26

For speeds between 500 and 600 kt and altitudes above 35,000 ft, the computer B-26 offers an approximate solution for the calculation of  $v_{TAS}$  under the condition of standard atmospheric temperatures, which corrects both the systematic error and the remaining error due to compressibility. The following example is used to check the accuracy:

pressure altitude	40,000 ft
$v_{CAS}$	550 kt

Using a calculator of the CR series (Huber patent) whose accuracy was proven at high Mach numbers under 3.3.2, the Mach number  $M = 1.67$  is determined for this example. Under ISA conditions, the sound speed is in 40,000 ft

$$a = \sqrt{\kappa R T} = \sqrt{1,4 \times 287 \times 216.5} = 294.9 \text{ m/s} = 573.3 \text{ kt}$$

For the  $v_{TAS}$  one gets:

$$v_{TAS} = M a = 1,67 \times 573.3 = 957.4 \text{ kt}$$

To solve this problem, the operating manual prescribes a computer setting according to Fig.3.3/28.

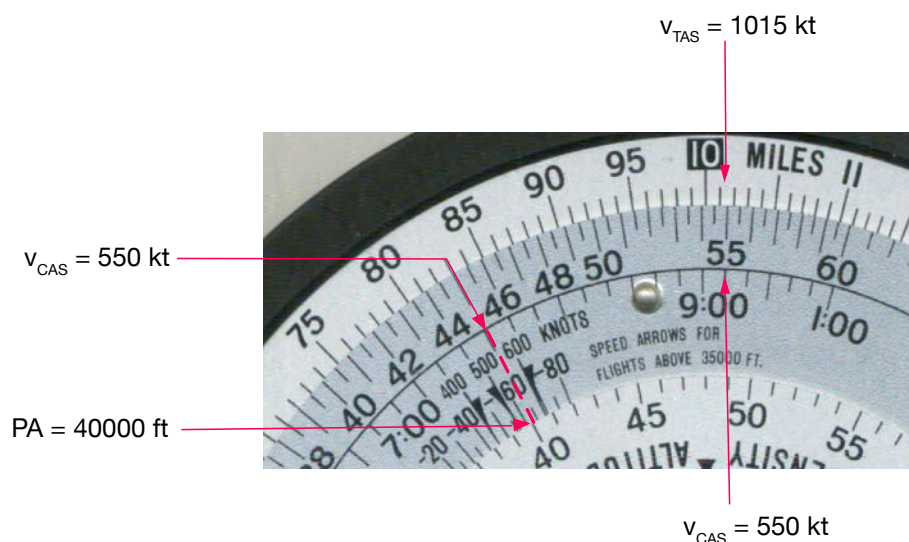


Fig.3.3/28

The result  $v_{TAS} = 1015 \text{ kt}$  deviates by 6% from the exact value ( $1015 : 957 = 1.06$ ).



### 3.3.4.9. Determination of true airspeed according to Huber

Fig.3.3/29 and 30 schematically show the structure of a Huber computer.

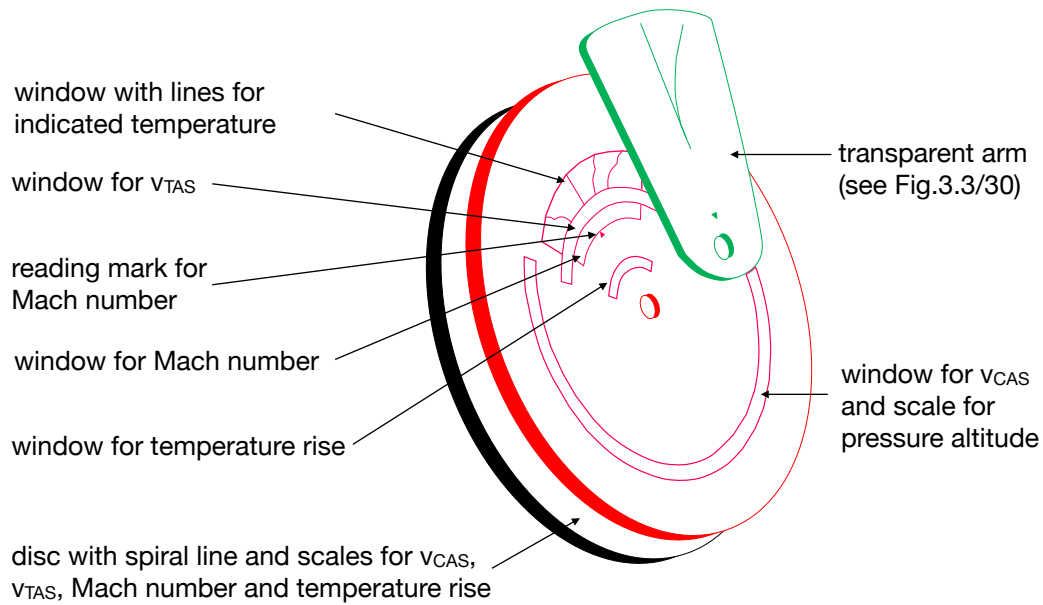


Fig.3.3/29

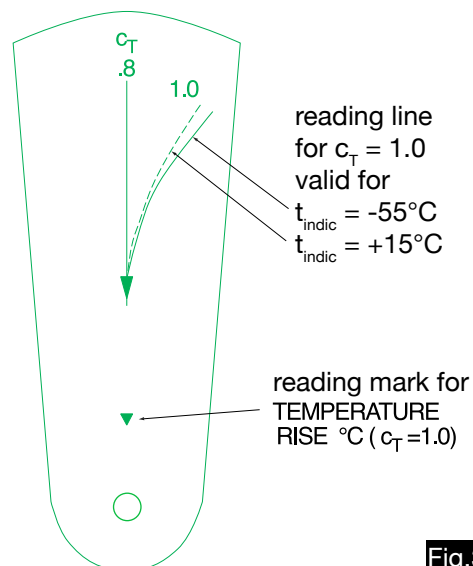


Fig.3.3/30

Chapter 3.3.2. describes how the Mach number is first calculated from dynamic pressure ( $v_{CAS}$ ) and pressure altitude using the Huber method. The true airspeed can be calculated from Mach number and sound speed (i.e. temperature). The Huber calculator uses the indicated temperature ( $t_{RAT}$ ) in the interest of simplest handling. The construction of the  $v_{TAS}$  scale is first carried out for arbitrarily selected values of the indicated temperature (here  $t_{RAT} = 0^\circ\text{C}$ ) and the temperature recovery coefficient (here  $c_T = 0.8$ ). With this factor, the temperature rise  $\Delta t$  is reduced by 20% so that the following formula applies for the correct temperature  $T_{SAT}$  at a displayed temperature of  $0^\circ$  (273 K):

$$T_{SAT} = 273 - 0.8 \Delta t$$

The  $v_{TAS}$  can then be determined using this equation:

$$v_{TAS/0} = a \quad M = M \sqrt{\kappa R T_{SAT}} = M \sqrt{\kappa R (273 - 0.8 \Delta t)}$$

The calculation is somewhat problematic, since the temperature rise itself is a function of the  $v_{TAS}$ . Therefore, for each Mach number one must first introduce an estimated  $\Delta t$  value into the calculation and with the received speed determine an improved  $\Delta t$  value, with which the calculation is repeated (iteration). For the Mach numbers 0.65, 0.7 and 0.85 this correlation is demonstrated (see Table.3.3/3 and Fig.3.3/31).

M	temperature rise $\Delta t$ derived by iteration	$v_{TAS/0} = M \sqrt{\kappa R (273 - 0.8 \Delta t)}$
0.65	21.5	208 m/s = 404 kt
0.7	25.0	223 m/s. = 433 kt
0.85	35.0	266 m/s = 518 kt

Tab.3.3/3

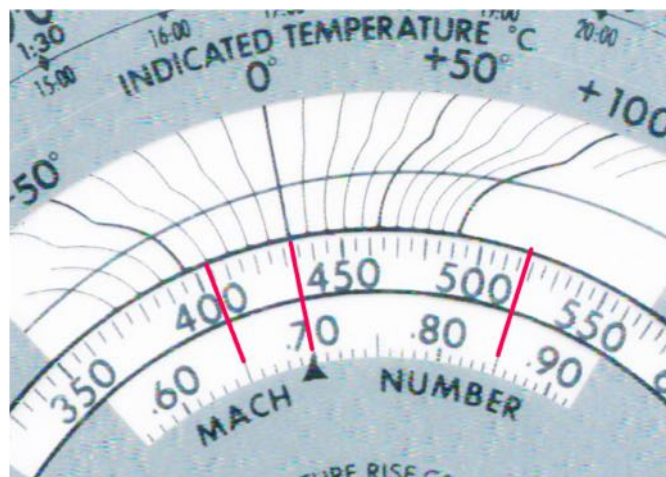


Fig.3.3/31

The  $v_{TAS}$  can be used to determine the line for the indicated temperature  $100^{\circ}\text{C}$  ( $373\text{ K}$ ) in the same way:

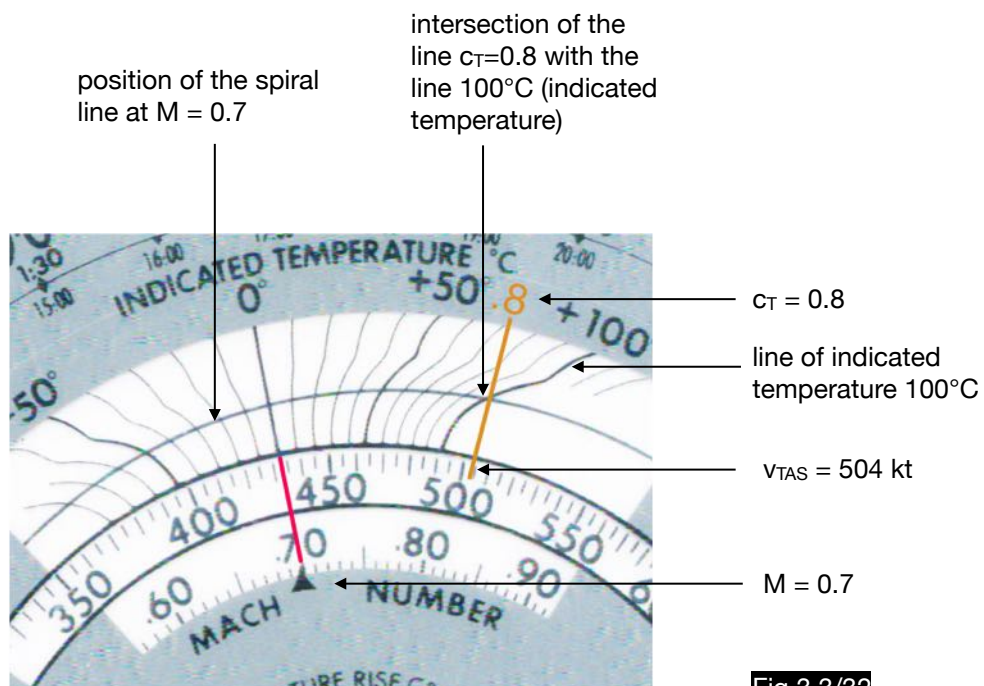
$$v_{TAS/100} = M \sqrt{\kappa R (373 - 0.8 \Delta t)}$$

Table 3.3/4 lists three true air speeds that allow the construction of the  $100^{\circ}$  line in the INDICATED TEMPERATURE field.

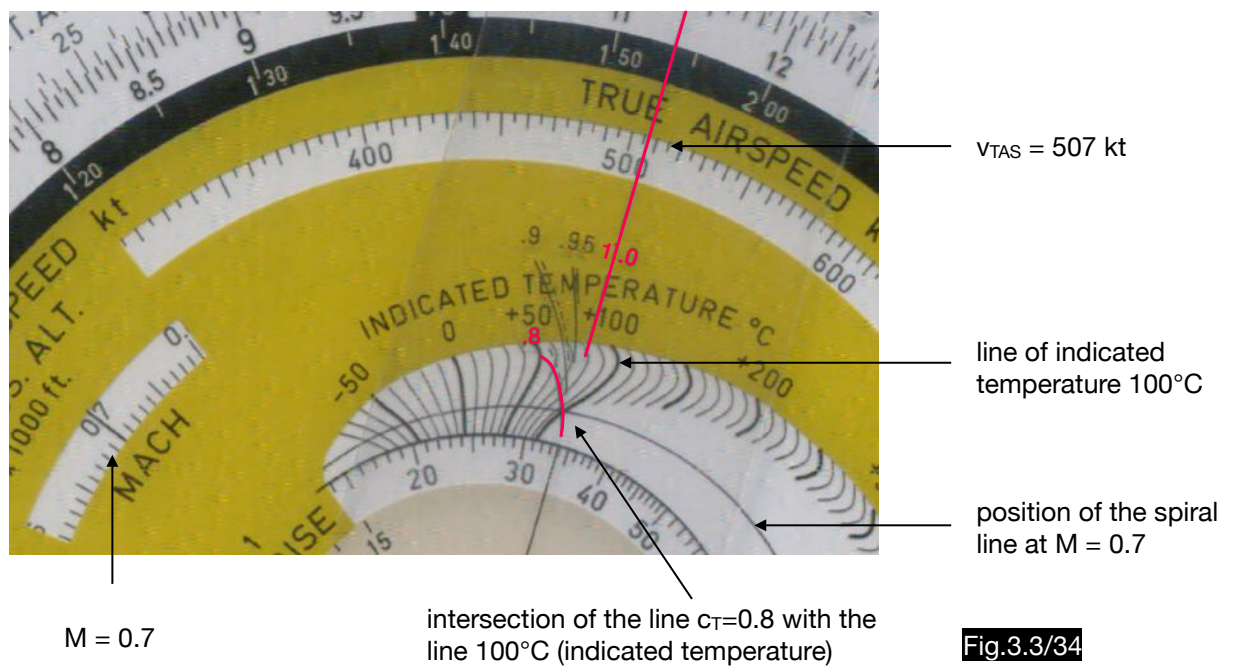
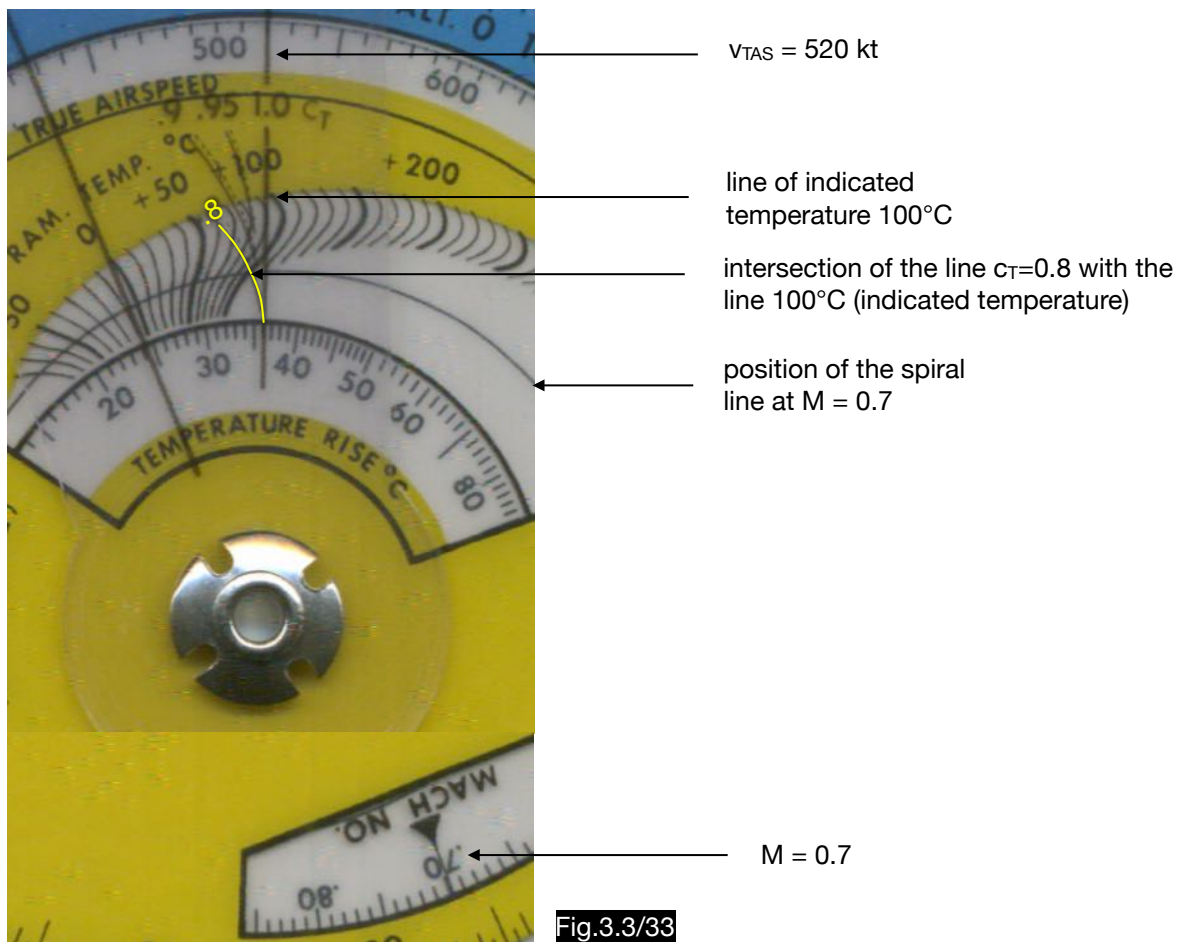
M	temperature rise $\Delta t$ derived by iteration	$v_{TAS/100} = M \sqrt{\kappa R (273 - 0.8 \Delta t)}$
0.65	29.0	244 m/s = 473 kt
0.7	33.5	261 m/s = 507 kt
0.85	48.0	311 m/s = 605 kt

**Tab.3.3/4**

Fig.3.3/32 shows the example for  $M = 0.7$  from Tab.3.3/4 in the computer CR-3, which comes to the result  $v_{TAS} = 504\text{ kt}$ . With the computer AL 6 (see Fig.3.3/33) a true airspeed of  $520\text{ kt}$  and with the computer AVIAT JET 647 (see Fig.3.3/34)  $507\text{ kt}$  is determined.



**Fig.3.3/32**



As can be seen from Fig.3.3/34, the correction field of this calculator is based on the indicated temperature 0°C and the factor  $c_T = 1$  for which applies:

$$T_{TAT} = T_{RAT} = T_{INDIC}$$

The following example illustrates how this simplifies the calculation of the temperature curve  
 $T_{INDIC} = 100^\circ\text{C}$ :

$M=0.64$	$t_{TAT\ 1} = 0^\circ\text{C} \ (273\ \text{K})$	$t_{TAT\ 2} = 100^\circ\text{C} \ (373\ \text{K})$	$c_T = 1.0$
----------	--	--	-------------

$$T_{TAT} = T_{SAT} (1 + 0.2 M^2) \quad (\text{see equ. 3.1/13})$$

$$v_{TAS} = a \quad M = M \sqrt{\frac{\kappa R T_{SAT}}{1}} = \sqrt{\frac{\kappa R T_{TAT}}{\frac{1}{M^2} + 0.2}}$$

$$\Delta v = v_{TAS2} - v_{TAS1} = \sqrt{\frac{\kappa R T_{TAT2}}{\frac{1}{M^2} + 0.2}} - \sqrt{\frac{\kappa R T_{TAT1}}{\frac{1}{M^2} + 0.2}} = \sqrt{\frac{\kappa R}{\frac{1}{M^2} + 0.2}} \left( \sqrt{T_{TAT2}} - \sqrt{T_{TAT1}} \right)$$

$$\Delta v = \sqrt{\frac{1.4 \ (287)}{\frac{1}{0.64^2} + 0.2}} \left( \sqrt{373} - \sqrt{273} \right) = 34.4\ \text{m/s} = 66.9\ \text{kt}$$

One point of the line  $t_{indic} = 100^\circ\text{C}$  is calculated with it. In Fig.3.3/35 the result of this example has to be confirmed:

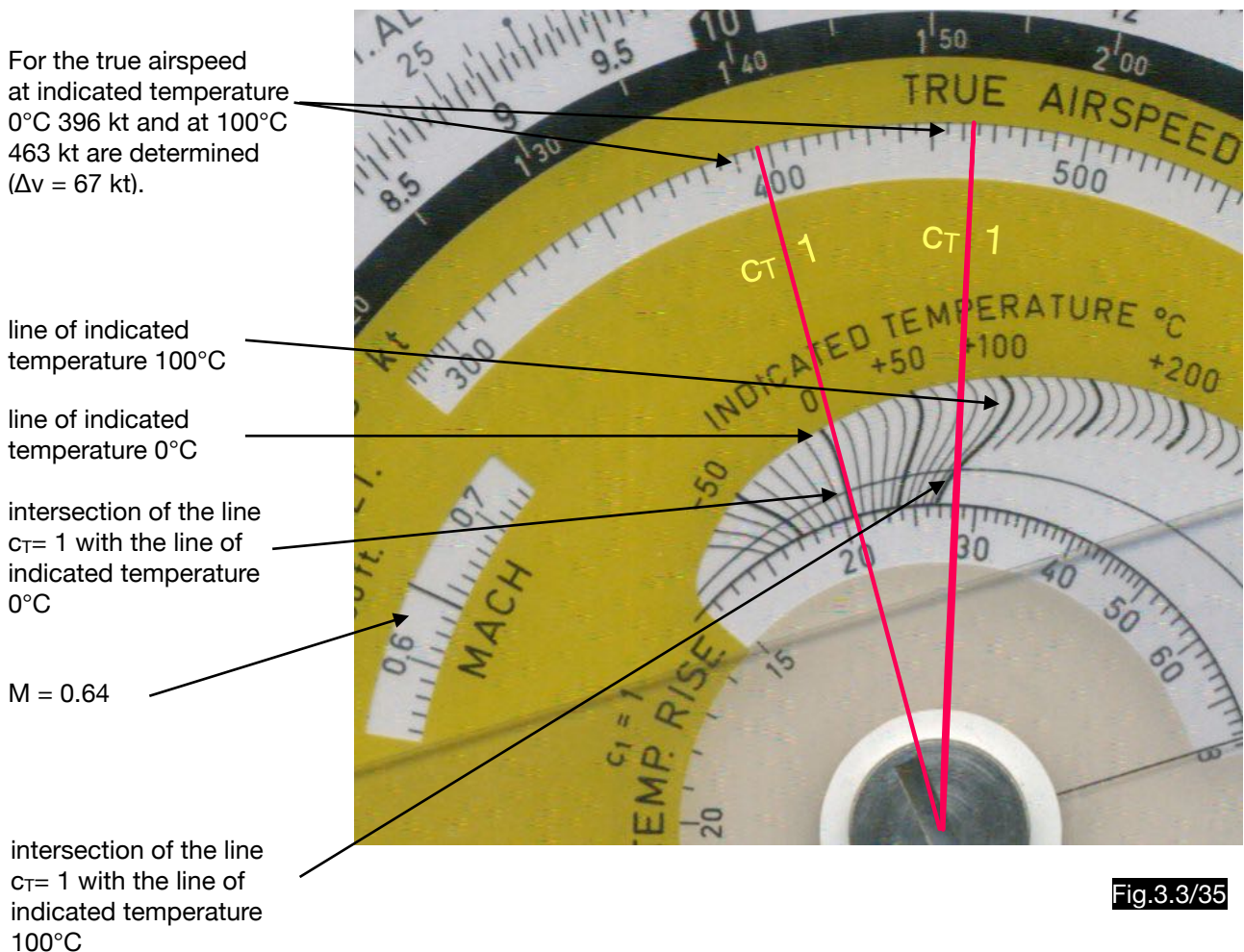


Fig.3.3/35



A diagram based on the indicated temperature  $0^{\circ}\text{C}$  and the factor  $c_T = 1$  shows Fig.3.3/36, which can be used to correct the true airspeeds determined for a certain Mach number when other temperatures are present. The diagram shown in a circular form in the computer moves over a spiral line which replaces the Mach number lines (abscissa axes). In Fig.3.3/37 it can be seen that the turning points of the temperature lines (which are a consequence of the changed flow conditions at  $M > 1$ ) do not lie on the concentric arc marked in red, but on the spiral line. In this way, the corrected speed can be found below the intersection of the spiral with the temperature line.

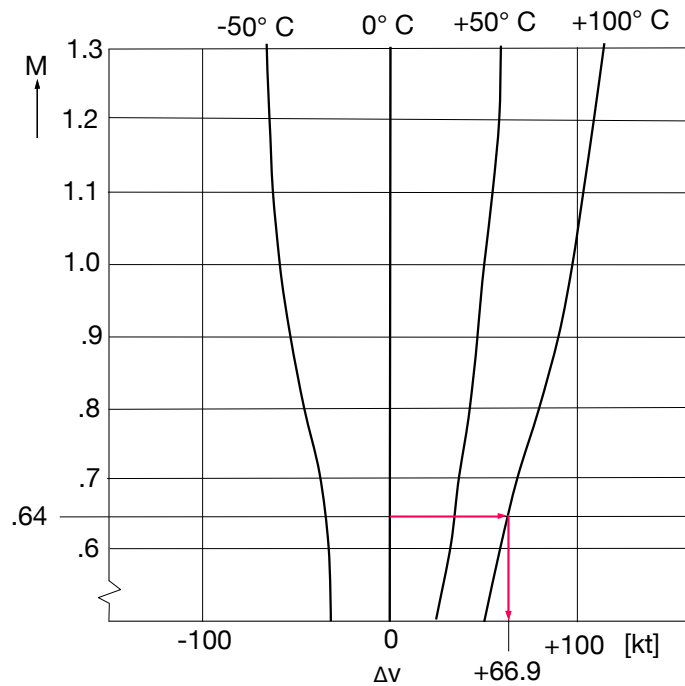


Fig.3.3/36

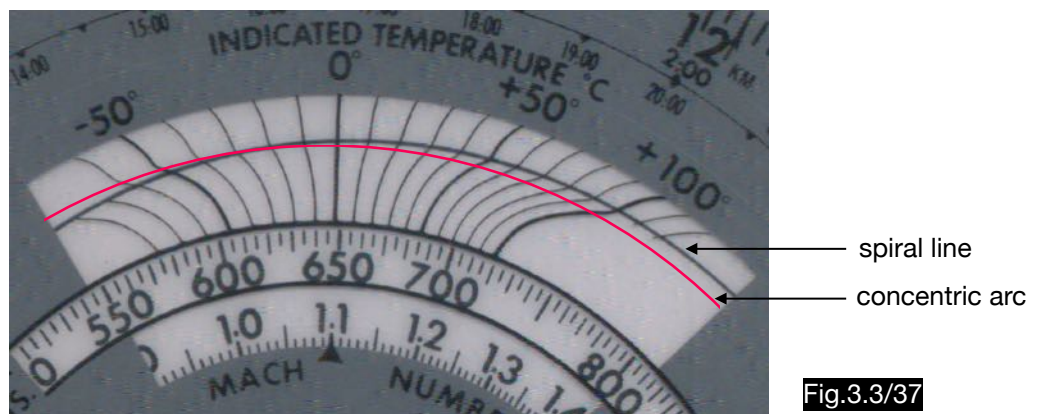


Fig.3.3/37

About the construction of the reading lines for the thermometer constant  $c_T$  on the transparent arm (see Fig.3.3/30) the following must be said. On the calculators there is this explanation for the double reading line at  $c_T = 1$ :

--- STD.SEA-LEVEL TEMP. (+ 15°C) ---  
 — STD.STRATOSPHERE TEMP. (- 55°C) —

Huber writes about this in his patent specification:

„These pairs of curves could be plotted as single curves computed for a temperature which is an average of the temperatures occurring most frequently during the flight schedule. This method, of course, would not be as minutely accurate as the double curve, but the computer setting would be simpler, and subject to less confusion.“

Although the double reading lines only become effective at Mach numbers above 1, most computer manufacturers have not made use of this simplification option. The problem is worthy of further investigation.

Imagine two airplanes of the same type flying side by side at the same speed, but with thermometers with different "TEMPERATURE RECOVERY COEFFICIENTS", namely  $c_T = 0.8$  and  $1.0$ . The temperature rise dependent on the true airspeed is  $\Delta t = 50^\circ\text{C}$ . In a first example the true temperature of the outside air is  $0^\circ\text{C}$ . In the aircraft with the error-free thermometer ( $c_T = 1.0$ ) an outside air temperature of  $+50^\circ\text{C}$  ( $0$  plus  $\Delta t$ ) is indicated and in the other aircraft  $+40^\circ\text{C}$  ( $0 + 50 \times 0.8$ ). In a second example the true temperature of the outside air is  $-50^\circ\text{C}$ . In the aircraft with the error-free thermometer ( $c_T = 1.0$ ) an outside air temperature of  $0^\circ\text{C}$  ( $-50^\circ\text{C}$  plus  $50^\circ\text{C} = 0^\circ$ ) is indicated and in the other aircraft  $-10^\circ\text{C}$  ( $-50 + 50 \times 0.8$ ). On the computer, which must supply the same  $v_{TAS}$  for both cases, this is as follows:

To determine the true airspeed, the line  $c_T = 0.8$  or  $c_T = 1$  must be brought over the intersection of the spiral line and the temperature line. In a warm atmosphere (as in the first example) the dashed line should be used. If the determination field for the speed scale was designed for  $c_T = 0.8$ , the  $1.0$  - line must be constructed so that the distance of the lines  $0.8$  and  $1.0$  along the spiral line corresponds to the distance of the temperature lines  $+40^\circ$  and  $+50^\circ$  (see Fig. 3.3/38).

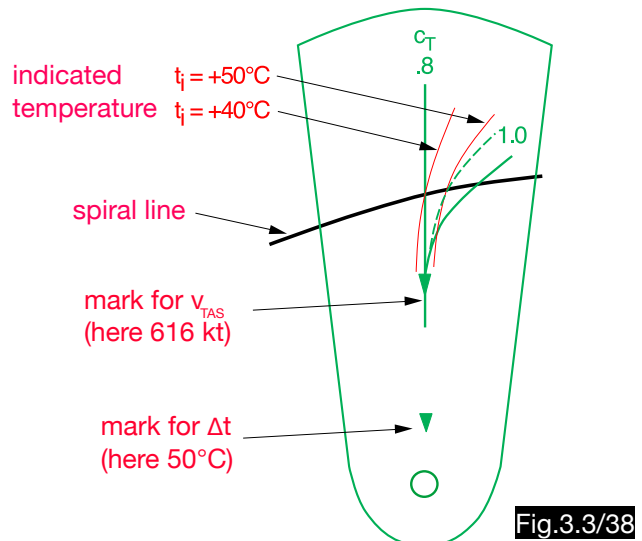


Fig.3.3/38

In the second example, a cold atmosphere with a C.O.A.T. =  $-50^\circ\text{C}$  is assumed under otherwise identical conditions. The temperature rise of  $50^\circ\text{C}$  results in a display of  $0^\circ\text{C}$  ( $50 - 50$ ) in the aircraft with the  $c_T = 1$  thermometer and  $-10^\circ\text{C}$  ( $-50 + 40$ ) in the other. At the computer this is shown as follows: Since this is a cold atmosphere, the non-dashed  $c_T = 1$  - line must be used, which lies further to the right of the  $0.8$  - line than the dashed one. This takes account of the fact that the distance between the temperature lines in the determination field increases from right to left i.e. from the warm to the cold side (see Fig. 3.3/39).

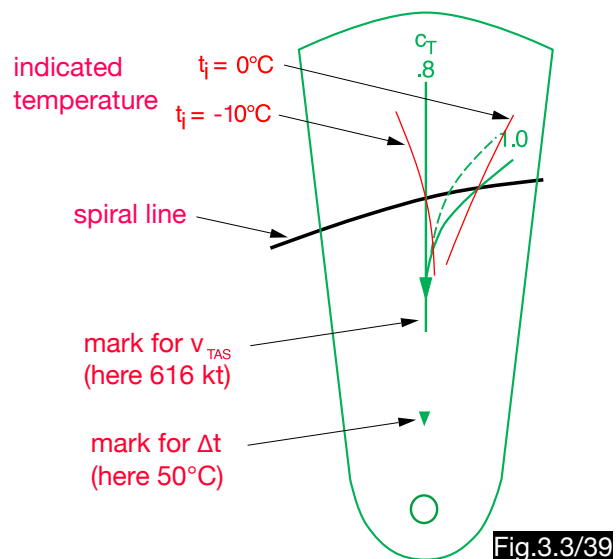


Fig.3.3/39

## 4. Annex

### 4.1. Formula symbols and abbreviations used

a	sound speed	$N_{\text{magn}}$	meridian of the magnetic north pole
$\alpha_{\text{DA}}$	drift angle	$N_{\text{true}}, N_{\text{T}}$	meridian of the geographical north pole
$\alpha_{\text{WCA}}$	wind correction angle	$\omega_z$	angular speed (vertical axis)
ASIR	airspeed indicator reading	OAT	outer air temperature
CAS	calibrated airspeed	p	static pressure
CH	compass heading	P.E.T.	point of equal times
C.O.A.T.	corrected outer air temperature	P.N.R.	point of no return
CRS	course	q	dynamic pressure
$C_T$	temperature recovery coefficient	QDM	magnetic bearing to
$\delta$	wind direction	QDR	magnetic bearing from
$\Delta t$	temperature rise	QFE	pressure at airfield elevation
$\Delta V_{\text{kompr}}$	compressibility error of the airspeed indicator	QNH	pressure adjusted to MSL
D	distance	QTE	true bearing from
DA	drift angle	QUJ	true bearing to
DEV	deviation	$\rho$	air density
DF	direction finder	R	radius, gas constant
DL	difference of latitude	RAS	rectified airspeed
DLo	difference of longitude	RAT	ram air temperature
$\varepsilon$	wind angle	RB	relative bearing
E	east	R.H.	right hand
EAS	equivalent airspeed	ROC	rate of climb
$\varphi$	average (mid) geographical latitude	RWY	runway
F	Fahrenheit	s, sec	second
$f_{\text{compr}}$	factor for TAS correction due to compressibility	S	south
GS	ground speed	SE	south east
h	hour	SAT	static air temperature
H	altitude, height	$\theta$	rhumb line (loxodrome) course
IAS	indicated airspeed	T, t	time
ILS	instrument landing system	TAS	true airspeed
I.C.A.N.	International Commission for Air Navigation	TAT	total air temperature
ICAO	International Civil Aviation Organization	$TB_{\text{from}}$	true bearing from
ISA	International Standard Atmosphere	$TB_{\text{to}}$	true bearing to
K	Kelvin	TC	true course
kt	knots, nautical mile per hour	TH	true heading
L.H.	left hand	TT	true track
LOP	line of position	u	wind speed
M	Mach number	v	airspeed
$MB_{\text{from}}$	magnetic bearing from	VAR	variation
$MB_{\text{to}}$	magnetic bearing to	$V_G$	ground speed
MC	magnetic course	$V_{GA}$	average ground speed
Min, min	minute	VOR	omnidirectional radio range
MH	magnetic heading	$V_{TAS}$	true airspeed
m.p.h.	stat. miles per hour	$V_{TAS_{\text{eff}}}$	effective true airspeed
ML	mid. latitude	$V_v$	vertical speed
MSL	mean sea level	$V_W$	wind speed
MT	magnetic track	$V_{WC}$	cross wind component
N	north	$V_{WL}$	longitudinal wind component
$N_{\text{comp}}$	north direction of the compass	w	vertical speed (ROC), wind speed, ground speed
NDB	non direction beacon	W	west, wind point
NM	nautical miles	WCA	wind correction angle
		WS	wind speed, ship speed

## 4.2. References to the illustrations

### Abbreviations used:

- D Pictures drawn by the author. Information in brackets indicates the origin (if nothing is noted these are drawings of items from the author's collection).  
S Scans of items from the author's collection.

Figure	Source	Figure	Source
1.1/1-4	D	1.28/1	D (Immler: Grundlagen der Flugzeugnavigation)
1.2/1-2	S	1.29/1	D
1.2/3-7	D	1.30/1-4	D,S
1.2/8	S	1.31/1-2	D,S (Clemens Richter: Ultraleicht Flieger)
1.2/9	D	1.32/1-4	D,S
1.2/10-12	D	1.33/1-2	D,S
1.2/13	D	1.34/1-2	D,S
1.2/14-17	D	1.34/3	D (US patent 3,276,682)
1.2/18-19	D (Weems: Air Navigation)	1.34/4-7	D,S
1.2/20-25	D (US patent 2,013,603)	1.35/1-4	D,S
1.3/1	D	1.35/5	(Bennewitz: Flugzeuginstrumente)
1.3/2	S	1.35/6	D (internet)
1.3/3-5	D	1.35/7-9	D
1.4/1	S	1.36/1	D
1.4/2-3	D	1.36/2-7	D (GB patent 581,906)
1.4/4-6	D	2.1/1-4	D
1.5/1	D	2.2/1-5	D
1.5/2	D (GB patent 150103)	2.3/1-2	D
1.5/3	D	2.3/3	S (US patent 2,823,857)
1.5/4-9	D (GB patent 427117)	2.3/4	D
1.6/1	S	2.3/5	S
1.6/2	D (US patent 2,334,135)	2.3/6-8	D (US patent 3,276,682)
1.7/1	S	2.3/9-10	D (US patent 4,555,852)
1.7/2-6	D (FR patent 621.521)	2.3/11-14	D
1.8/1-17	D (patent GB 430,449 and GB 395,806)	2.3/15	S
1.9/1-3	collection Jochen Brandt	2.3/16-17	D
1.9/4-10	D	2.3/18-19	D,S (Weems: Air Navigation)
1.10/1	S	2.3/20-23	D
1.10/2-3	D	2.4/1-10	D,S
1.11/1	<a href="http://www.knightson.co.uk">http://www.knightson.co.uk</a>	2.4/11-14	D
1.12/1-4	D,S	2.5/1-4	D
1.13/1	S	2.6/1-11	D,S
1.13/2-12	D	2.7/1-8	D
1.14/1-10	D (Deutsche Nationalbibliothek Signatur: 1936B2684)	2.8/1	D
1.15/1-4	D (collection A. Piel)	2.9/1-10	D
1.16/1-2	D	2.10/1-9	D (US patent 2,405,113)
1.17/1-5	D	2.11/1-6	D
1.18/1-4	D	3.1/1-19	D,S
1.19/1-4	D,S (Annalen der Hydrographie und maritimen Meteorologie Nov./Dez. 1918)	3.2/1-5	D
1.20/1-3	D ( <a href="https://arc.aiaa.org/doi/10.2514/6.2010-158">https://arc.aiaa.org/doi/10.2514/6.2010-158</a> )	3.3/1-39	D
1.21/1-2	D (FR patent 626.697)		
1.22/1-4	D		
1.23/1-3	D,S		
1.24/1-5	D (E.Jünger: Luftfahrt ist not!)		
1.25/1-9	D,S		
1.26/1-3	D (Weems: Air Navigation)		
1.27/1-3	S		